Neap:

Trial Examination 2014

VCE Specialist Mathematics Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	5	5	58
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 20 pages. Formula sheet of miscellaneous formulas.

Answer sheet for multiple-choice questions.

Instructions

Write **your name** and your **teacher's name** in the space provided above on this page and in the space provided on the answer sheet for multiple-choice questions.
All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The maximum value of y reached by the ellipse with equation $\frac{(x+3)^2}{5} + \frac{(y-3)^2}{6} = k,\ k > 0$ is

A. $3 + 6\sqrt{k}$

B. $-3 + \sqrt{6k}$

C. $\sqrt{6k}$

D. $-3 + \sqrt{5k}$

E. $3 + \sqrt{6k}$

Question 2

The graph of $f(x) = \frac{1}{x^2 + bx - c}$ where b and c are real constants, has two vertical asymptotes if

A. $b^2 < 4c$

B. $b^2 > 4c$

C. $b^2 = -4c$

D. $b^2 < -4c$

E. $b^2 > -4c$

Question 3

Given that n is an integer, the graph of $y = \cot(2x)$ has vertical asymptotes at

A. $x = \frac{n\pi}{4}$

B. $x = \frac{n\pi}{2}$

C. $x = \frac{(2n+1)\pi}{4}$

D. $x = \frac{(2n+1)\pi}{2}$

E. $x = n\pi$

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Question 4

If $\cos(x) = -\frac{1}{10}$, $\frac{\pi}{2} < x < \pi$, the exact value of $\operatorname{cosec}(x)$ is

A. $\frac{3\sqrt{11}}{10}$

B. $-\frac{10}{3\sqrt{11}}$

C. $\frac{10}{3\sqrt{11}}$

D. $-\frac{3\sqrt{11}}{10}$

E. -10

Question 5

If $h(x) = f(g(x))$, where f and g are twice differentiable functions, then $h''(x)$ is equal to

A. $f''(g(x))$

B. $f''(g(x))g''(x)$

C. $f''(g(x))(g'(x))^2$

D. $f''(g(x))g'(x) + f'(g(x))g''(x)$

E. $f''(g(x))(g'(x))^2 + f'(g(x))g''(x)$

Question 6

If $z = x + yi$, the values of x and y such that $(2 - i)z = i$ are

A. $x = -\frac{1}{5}$ and $y = \frac{2}{5}$

B. $x = \frac{1}{5}$ and $y = -\frac{2}{5}$

C. $x = -\frac{1}{5}$ and $y = -\frac{2}{5}$

D. $x = \frac{1}{5}$ and $y = \frac{2}{5}$

E. $x = -1$ and $y = 2$

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Question 7

The region of the complex plane defined by $(1 + i)z + (1 - i)\bar{z} = 6$ can be represented by the Cartesian equation

A. $x = 3$

B. $y = 3$

C. $y = x + 3$

D. $y = x - 3$

E. $y = 3 - x$

Question 8

If $z = \cos(\theta) + i\sin(\theta)$, then $z^n - \frac{1}{z^n}$ is equal to

A. $2\cos(n\theta)$

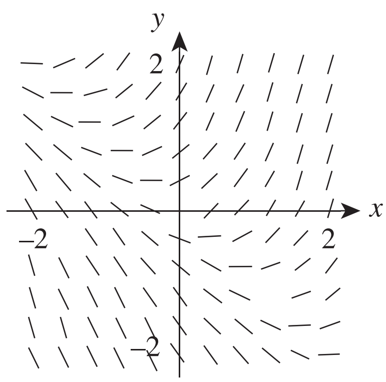
B. $-2\cos(n\theta)$

C. $2i\sin(n\theta)$

D. $-2i\sin(n\theta)$

E. 0

Question 9



The differential equation represented by the direction field above is

A. $\frac{dy}{dx} = x + 1$

B. $\frac{dy}{dx} = x + y$

C. $\frac{dy}{dx} = \frac{x}{y}$

D. $\frac{dy}{dx} = \log_e(y)$

E. $\frac{dy}{dx} = x^2$

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Question 10

$\frac{2x-1}{x^2+6x+9}$ expressed in partial fractions has the form

A. $\frac{A}{(x+3)} + \frac{B}{(x+3)^2}$

B. $\frac{A}{(x+3)} + \frac{B}{(x-3)}$

C. $\frac{A}{(x+3)} + \frac{B}{(x+3)}$

D. $\frac{A}{(x+3)} + \frac{Bx+C}{(x+3)^2}, B \neq 0$

E. $\frac{A}{(x+3)^2} + \frac{B}{(x+3)^2}$

Question 11

When added to a quantity of water, 8 grams of a chemical dissolves at a rate equal to 10% of the amount of undissolved chemical per minute.

If y grams is the amount of dissolved chemical at time t minutes, then y satisfies the differential equation

A. $\frac{dy}{dt} = 8 - y$

B. $\frac{dy}{dt} = y - 8$

C. $\frac{dy}{dt} = \frac{y-8}{10}$

D. $\frac{dy}{dt} = \frac{8-y}{10}$

E. $\frac{dy}{dt} = 8 - \frac{y}{10}$

Question 12

The region in the first quadrant bounded by the graph of $y = \sec(x)$, the line $x = \frac{\pi}{4}$ and both coordinate axes is rotated 360° about the x-axis.

The volume of this solid, in cubic units, is

A. $\frac{8\pi}{3}$

B. 2π

C. π

D. $\pi - 1$

E. $\frac{\pi^2}{4}$

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Question 13

Using a suitable substitution, $\int_{0}^{\frac{\pi}{3}} \frac{\cos(x)}{1 + \sin^2(x)}\, dx$ can be expressed in terms of u as

A. $\int_{0}^{\frac{\pi}{3}} \frac{1}{1 + u^2} du$

B. $\int_{0}^{\frac{\sqrt{3}}{2}} \frac{1}{1 + u^2} du$

C. $\int_{0}^{\frac{1}{2}} \frac{1}{1 + u^2} du$

D. $\int_{0}^{\frac{\sqrt{3}}{2}} \frac{1 - u^2}{1 + u^2} du$

E. $\int_{0}^{\frac{\sqrt{3}}{2}} \frac{\sqrt{1 - u^2}}{1 + u^2} du$

Question 14

The length of the projection of the line joining the two points $A(3, 17, -1)$ and $B(1, 6, 8)$ on the line parallel to the vector $\underset{\sim}{v} = \underset{\sim}{i} - 2\underset{\sim}{j} - 2\underset{\sim}{k}$ is equal to

A. $-\frac{2}{3}$

B. $\frac{2}{3}$

C. $\frac{2}{9}$

D. $-\frac{2}{9}$

E. $\frac{38}{3}$

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Question 15

Which one of the following sets of vectors are linearly dependent?

A. $\underset{\sim}{a} = \underset{\sim}{i} - 2\underset{\sim}{j}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} + \underset{\sim}{j}$

B. $\underset{\sim}{a} = \underset{\sim}{i}$, $\underset{\sim}{b} = \underset{\sim}{j}$ and $\underset{\sim}{c} = \underset{\sim}{k}$

C. $\underset{\sim}{a} = \underset{\sim}{i} - 3\underset{\sim}{j}$ and $\underset{\sim}{b} = -\underset{\sim}{i} + 2\underset{\sim}{j}$

D. $\underset{\sim}{a} = \underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$, $\underset{\sim}{b} = \underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{c} = \underset{\sim}{j}$

E. $\underset{\sim}{a} = -\frac{2}{3}\underset{\sim}{i} + \frac{1}{3}\underset{\sim}{j} + \frac{1}{3}\underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$

Question 16

From a fixed origin O, Natalie walks 300 metres north and then 200 metres in the direction N60°E to a point P.

If $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors in the east and north directions respectively, the position vector $\overrightarrow{OP}$ is equal to

A. $200\underset{\sim}{i} + 300\underset{\sim}{j}$

B. $100\sqrt{3}\underset{\sim}{i} + 200\underset{\sim}{j}$

C. $400\underset{\sim}{i} + 100\sqrt{3}\underset{\sim}{j}$

D. $100\sqrt{3}\underset{\sim}{i} + 400\underset{\sim}{j}$

E. $100\underset{\sim}{i} + (300 + 100\sqrt{3})\underset{\sim}{j}$

Question 17

The position of a particle at time t seconds is given by $\underset{\sim}{r}(t) = (t-1)\underset{\sim}{i} + 4(t-1)^2\underset{\sim}{j},\ t \geq 0$.

The Cartesian equation of the particle's path is

A. $y = \frac{\sqrt{x}}{2},\ x \geq -1$

B. $y = \frac{\sqrt{x}}{2},\ x \geq 1$

C. $y = 4x^2,\ x \geq -1$

D. $y = 4x^2,\ x \geq 1$

E. $y = 2x^2,\ x \geq -1$

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Question 18

A curve C is specified by the vector equation $\underset{\sim}{r}(t) = (t^3 - 2t^2 - 5)\underset{\sim}{i} + (t^4 + 2t^2 - 8t)\underset{\sim}{j}$.

The values of t for which C has a vertical tangent are

A. 0 only

B. 1 only

C. 0 and 1 only

D. 0 and $\frac{4}{3}$ only

E. 0, 1 and $\frac{4}{3}$

Question 19

A particle moves in a straight line such that at time t, its displacement from a fixed origin is x metres and its velocity is v m/s.

Given that $v = \sqrt{36x - 4x^2}$, the acceleration of the particle, in m/s^2, when $x = 9$ is

A. –9

B. –18

C. –36

D. 0

E. 18

Question 20

A man of mass 85 kg is standing in a lift which is moving with a downward acceleration of magnitude 3 m/s^2.

The magnitude of the force, in newtons, exerted by the floor of the lift on the man is

A. $85(g - 3)$

B. $85(g + 3)$

C. $85(3 - g)$

D. $85g$

E. 255

Question 21

A stone of mass 250 grams falls from rest under the action of gravity for a period of two seconds.

Neglecting the effect of air resistance, the stone's change in momentum, in kg m/s, over this two-second period is

A. 0

B. 2

C. 2.45

D. 4.9

E. 19.6

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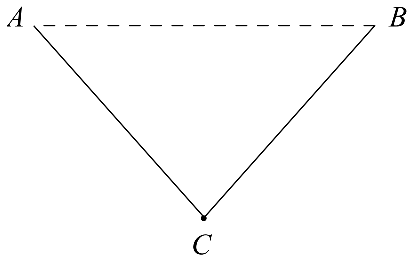
Question 22

A light inelastic string is attached to two points A and B which are in a horizontal line. A particle of mass m kg is attached to the string at C by means of a smooth ring and hangs in equilibrium. AC and BC each make an angle of α with the horizontal.



The tension in the string, measured in newtons, is

A. $\dfrac{mg}{2\sin(\alpha)}$

B. $\dfrac{mg}{2\cos(\alpha)}$

C. $\dfrac{m}{2\sin(\alpha)}$

D. $\dfrac{mg}{\sin(\alpha)}$

E. $\dfrac{mg}{\sin(\alpha)+\cos(\alpha)}$

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SECTION 2

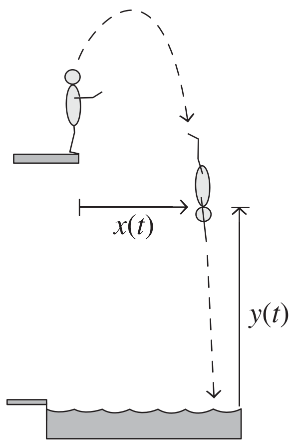
Instructions for Section 2

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (9 marks)

A diver performs a dive from the edge of a platform into a pool below.

At time t seconds after he dives, the diver's shoulders can be represented by the position vector $\underset{\sim}{r}(t) = x(t)\underset{\sim}{i} + y(t)\underset{\sim}{j},\ t \geq 0,$ where $x(t)$ metres is the horizontal distance from the front edge of the platform to the diver's shoulders and $y(t)$ metres is the vertical distance from the water's surface to his shoulders.



When the diver begins his dive, his shoulders are 11.6 metres above the water's surface.

The velocity, $\underset{\sim}{r}'(t)$, of the diver's shoulders during the dive is given by $\underset{\sim}{r}'(t) = 0.9\underset{\sim}{i} + (3.8 - 9.8t)\underset{\sim}{j}$, $0 \leq t \leq T$, where T is the time at which the diver's shoulders enters the water.

a. Find, correct to two decimal places, the maximum vertical distance from the water's surface to the diver's shoulders. 3 marks

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b. Find T, the time it takes for the diver's shoulders to enter the water. Give your answer correct to two decimal places. 2 marks

c. Use the definite integral $\int_0^T \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$ to find, correct to one decimal place, the total distance, d metres, travelled by the diver's shoulders from the time he dives from the platform until his shoulders enter the water. 2 marks

d. Find, correct to the nearest tenth of a degree, the angle α, $0° < \alpha < 90°$, between the path of the diver and the water's surface at the instant the diver's shoulders enter the water. 2 marks

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Question 2 (12 marks)

Consider $P(z) = z^4 + az^3 + bz + c$, where $z \in C$ and a, b and c are integers.

a. If $(z - ki)$, where $k \neq 0$ is a linear factor of $P(z)$, explain why $(z + ki)$ is also a linear factor of $P(z)$. 1 mark

Consider $P(z) = 0$.

b. Show that $b = ak^2$. 2 marks

c. Show that $b^2 + a^2c = 0$. 2 marks

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d. If $P(2) = 0$, show that b is an even number. 3 marks

Consider $W(z) = mz^3 + nz^2 + pz + q$, where $z \in C$ and m, n, p and $q \in R$.

e. If $W(u) = 0$, prove that $W(\bar{u}) = 0$. 4 marks

Question 3 (13 marks)

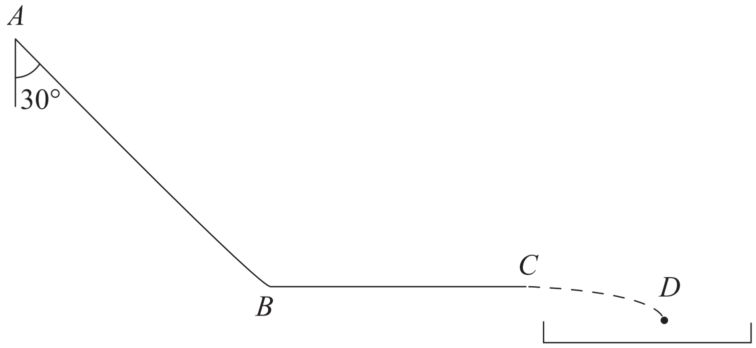
A waterslide consists of two sections.

The first section *AB* is a ramp of length 30 metres inclined at 30° to the vertical.

The second section *BC* is a horizontal chute of length 15 metres.

The end of this horizontal chute at point *C* is 1.5 metres above an entry pool.

A person of mass 60 kg, initially at rest at point *A*, slides on a mat from points *A* to *C* and then projects into the entry pool at point *D*.



The coefficient of friction between the mat and the surface of both sections of the waterslide is 0.25.

The effects of air resistance can be ignored throughout the person's ride.

a. Show that the person's acceleration, a m/s^2, at point *B* is given by $a = \frac{g(4\sqrt{3}-1)}{8}$. 3 marks

b. Show that the person's speed, v m/s, at point *B* is given by $v = \frac{\sqrt{30g(4\sqrt{3}-1)}}{2}$. 2 marks

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c. Show that the person's speed of projection, V m/s, from point C is given by $V = \sqrt{15g(2\sqrt{3} - 1)}$. 4 marks

d. Find, correct to one decimal place, the horizontal distance, x metres, travelled by the person in projecting from points C to D. 4 marks

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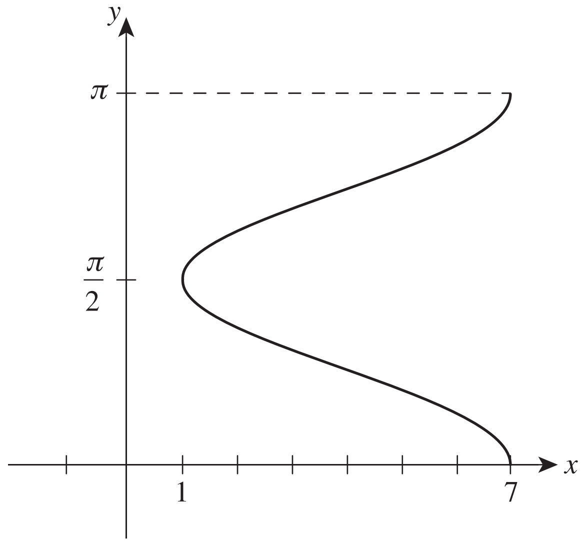
Question 4 (11 marks)

The graph of the relation $x = 3\cos(2y) + 4,\ 0 \le y \le \pi$, is shown below.



The curve is rotated 360° degrees about the y-axis to form a hollow liquid container.

All measurements are in metres.

a. Express the volume of this container, V m^3, as a definite integral and hence find the container's volume in exact form. 2 marks

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The container is filled with water at a rate of 2 m^3 per minute.

b. Find the exact rate at which the depth of water, h metres, is rising when the water level reaches $\frac{\pi}{4}$ metres. 5 marks

c. Find, correct to one decimal place, the time, in minutes, it takes for the depth of water to reach $\frac{\pi}{4}$ metres. 4 marks

Question 5 (13 marks)

Relative to a fixed origin O, the points P, Q and R have position vectors given respectively by:

$$\overrightarrow{OP} = 2\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}$$
$$\overrightarrow{OQ} = \underset{\sim}{i} + 3\underset{\sim}{j} + \underset{\sim}{k}$$
$$\overrightarrow{OR} = 3\underset{\sim}{i} - 6\underset{\sim}{j} + 6\underset{\sim}{k}$$

Let θ be the angle between $\overrightarrow{OQ}$ and $\overrightarrow{OR}$.

a. Show that $\theta = \cos^{-1}\left(-\frac{1}{\sqrt{11}}\right)$. 3 marks

b. Find the exact area of triangle OQR. 2 marks

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Relative to the fixed origin, O, a movable point, M, has a position vector given by
$\overrightarrow{OM} = (2 - 2\lambda)\underset{\sim}{i} + (1 + 4\lambda)\underset{\sim}{j} + (3 - 4\lambda)\underset{\sim}{k}$, where $\lambda \in R$.

c. Find the values of λ such that the length of the projection $\overrightarrow{RM}$ onto $\overrightarrow{RQ}$ is $\sqrt{110}$ units. 5 marks

d. Show that the points M, P and Q are collinear for $\lambda \in R$. 3 marks

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e. Hence show that triangle *OMR* has the same area as triangle *OQR*. 3 marks

END OF QUESTION AND ANSWER BOOKLET

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