SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 1

*(*TSSM's *2014 trial exam updated for the current study design)*

SOLUTIONS

Question 1

$$\frac{3-2x}{x^2-4x+3}=\frac{A}{x-3}+\frac{B}{x-1}$$
$$3-2x=A(x-1)+B(x-3)$$
$$x=1\Rightarrow B=-\frac{1}{2}\quad and\quad x=3\Rightarrow A=-\frac{3}{2}$$
$$\int_4^6\frac{3-2x}{x^2-4x+3}dx=-\frac{3}{2}\int_4^6\frac{1}{x-3}dx-\frac{1}{2}\int_4^6\frac{1}{x-1}dx$$
$$\int_4^6\frac{3-2x}{x^2-4x+3}dx=\left(-\frac{3}{2}\log_e|x-3|\right)_4^6-\left(\frac{1}{2}\log_e|x-1|\right)_4^6$$
$$\int_4^6\frac{3-2x}{x^2-4x+3}dx=-\frac{3}{2}(\log_e 3-\log_e 1)-\frac{1}{2}(\log_e 5-\log_e 3)$$
$$\int_4^6\frac{3-2x}{x^2-4x+3}dx=-\frac{3}{2}\log_e(3)-\frac{1}{2}\log_e\left(\frac{5}{3}\right)$$
$$\int_4^6\frac{3-2x}{x^2-4x+3}dx=-\frac{1}{2}\log_e(45)$$

M3+A1

Question 2

a. $\left|\vec{AB}\right| = \left|\vec{OB} - \vec{OA}\right| = \left|2\vec{i} - 2\vec{j} + \vec{k}\right| = \sqrt{4+4+1} = 3\,units$

M1+A1

b. $\vec{BA} \bullet \vec{CA} = \left(-2\vec{i} + 2\vec{j} - \vec{k}\right) \bullet \left(-4\vec{i} - 3\vec{j} + 2\vec{k}\right) = 8 - 6 - 2 = 0$

$\vec{BA} \perp \vec{CA}$

M2

c. $Area = \frac{1}{2} \times \sqrt{16+9+4} \times 3 = \frac{3}{2}\sqrt{29}\ sq\ units$

A1

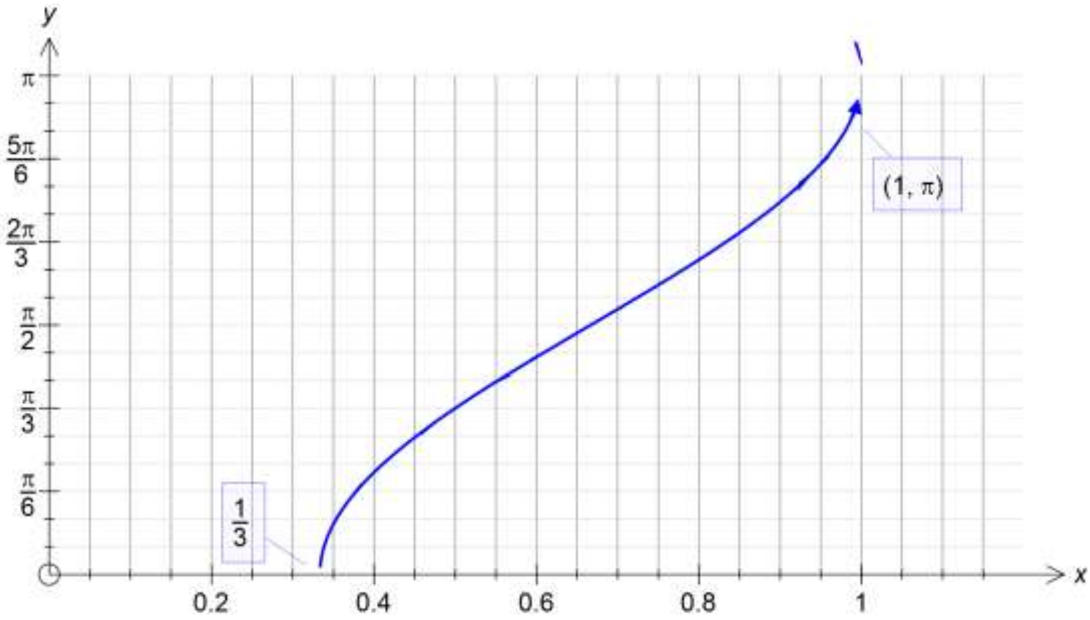
Question 3

a.

$$-1 \le 2 - 3x \le 1$$
$$-3 \le -3x \le -1$$
$$\frac{1}{3} \le x \le 1$$
$$Max\ \ domain\ \ is\ \ \left[\frac{1}{3}, 1\right]$$

M1+A1

b.



1 mark for shape and 1 mark for end-points

c.

$$y = \frac{\pi}{3} \Rightarrow x = \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(2-3x)^2}} \times -3$$

$$m_T = 2\sqrt{3}$$

$$m_N = \frac{-\sqrt{3}}{6}$$

M2+A1

Question 4

a. $(3i)^3 - 2(3i)^2 + 9(3i) - 18 = -27i + 18 + 27i - 18 = 0$

M1

b. $z = -3i$ is another solution for the equation.

$$(z - 3i)(z + 3i)(z - k) = z^3 - 2z^2 + 9z - 18$$
$$(z^2 + 9)(z - k) = z^3 - 2z^2 + 9z - 18$$
$$z^3 - kz^2 + 9z - 9k = z^3 - 2z^2 + 9z - 18$$
$$k = 2$$
$$Solutions\ are\ \ 3i\ ,\ -3i\ ,\ 2$$

M2+A1

Question 5

a.

$$2y - xy^2 + 5x = -6$$
$$2y' - (y^2 + 2xyy') + 5 = 0$$
$$y'(2 - 2xy) = y^2 - 5$$
$$\frac{dy}{dx} = \frac{y^2 - 5}{2(1 - xy)}$$

M2+A1

b.

$$y = 1 \Rightarrow x = -2$$
$$\frac{dy}{dx}(-2,-1) = \frac{1-5}{2(1+2)} = \frac{-2}{3}$$

M1+A1

Question 6

$$Volume = \pi \int_0^1 \left(x^2\right) dy$$

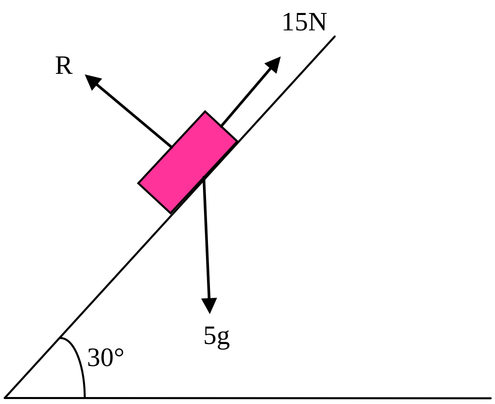
$$Volume = \pi \int_0^1 \left(\frac{4}{y+1} - 9 \right) dy$$

$$Volume = \pi \left(4\log_e(y+1) - 9y\right)_0^1 = \pi\left(4\log_e(2) - 9\right)$$

M3+A1

Question 7

a.



A2

b. $5g\sin(30°) - 15 = 5a$

$$5a = \frac{19}{2}$$

$$a = 1.9\ m/s^2$$

M1+A1

c. $R + 5g\cos(150°) = 0$

$$R = \frac{5\sqrt{3}}{2}g$$

M1+A1

Question 8

a. $\frac{dT}{dt} = -k(T-20)$

$$t = -\frac{1}{k}\log_e(T-20) + c$$

$$t = 0\,,\ T = 80 \Rightarrow c = \frac{1}{k}\log_e 60$$

$$t = -\frac{1}{k}\log_e(T-20) + \frac{1}{k}\log_e 60$$

$$t = 5\,,\ T = 70 \Rightarrow 5 = \frac{1}{k}\log_e\left(\frac{60}{50}\right)$$

$$k = \frac{1}{5}\log_e\left(\frac{6}{5}\right)$$

M2+A1

b. $10 = \frac{1}{k}\log_e\left(\frac{60}{T-20}\right)$

$$\frac{60}{T-20} = e^{10k}$$

$$T = 60e^{-10k} + 20$$

$$T = 60\times\left(\frac{5}{6}\right)^2 + 20 = 61\frac{2}{3}\,°C$$

M1+A1

