SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 2

(**TSSM's** ***2014 trial exam updated for the current study design)***

SOLUTIONS

SECTION 1: Multiple-choice questions (1 mark each)

Question 1

Answer: E

Explanation

$$y = \frac{-x}{2} + \frac{1}{2x}$$

Question 2

Answer: D

Explanation

$$\frac{d}{dx}\left(1 - 2\cos^{-1}\left(\frac{x}{2}\right)\right) = -2 \times \frac{-1}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \times \frac{1}{2} = \frac{2}{\sqrt{4-x^2}}$$

Question 3

Answer: C

Explanation

$$\frac{\bar{z}}{z} = \frac{3-2i}{3+2i} = \frac{(3-2i)^2}{9+4} = \frac{5-12i}{13}$$

Question 4

Answer: E

Explanation

$$Domain: -1 \le \frac{x-1}{2} \le 1 \Rightarrow -2 \le x-1 \le 2 \Rightarrow -1 \le x \le 3$$

$$Range: -\frac{\pi}{2} \le \sin^{-1}\left(\frac{x-1}{2}\right) \le \frac{\pi}{2}$$

Question 5

Answer: D

Explanation

$$\frac{1}{\sqrt{6}}\left(\vec{i} - 2\vec{j} - \vec{k}\right) \bullet \left(\vec{i} + 2\vec{j} - 3\vec{k}\right) = 1 - 4 + 3 = 0$$

Question 6

Answer: B

Explanation

$$Let \quad u = 1 - x^2$$

$$\frac{du}{dx} = -2x$$

$$-\frac{1}{2}\int_1^0 (1-u)u^{1/2}du = \frac{1}{2}\int_0^1 \left(u^{\frac{1}{2}} - u^{\frac{3}{2}}\right) du$$

Question 7

Answer: B

Explanation

$$x = -1 \Rightarrow y = 1 - \sqrt{3} \quad (in \ third \ quadrant)$$

$$2x + 2(y-1)y' = 0 \Rightarrow y' = -\frac{x}{y-1}$$

$$m = -\frac{-1}{-\sqrt{3}} = -\frac{1}{\sqrt{3}}$$

Question 8

Answer: A

Explanation

$1+\cot^2 t = \operatorname{cosec}^2 t$

$1+\left(\dfrac{y+1}{3}\right)^2 = \left(\dfrac{x-1}{2}\right)^2$

$\dfrac{(x-1)^2}{4} - \dfrac{(y+1)^2}{9} = 1$

Question 9

Answer: B

Explanation

$|z| > b$ is outside the circle.

Question 10

Answer: E

Explanation

$z = \left(4cis\left(\dfrac{4\pi}{3}\right)\right)^{\frac{1}{2}} \Rightarrow z = 2cis\left(\dfrac{2\pi}{3}\right),\ 2cis\left(\dfrac{-\pi}{3}\right)$

Question 11

Answer: C

Explanation

The angle is between 60° and 90°.

Question 12

Answer: C

Explanation

$$\underset{\sim}{b} \bullet \left(\underset{\sim}{a} - \underset{\sim}{b}\right) = \underset{\sim}{b} \bullet \underset{\sim}{a} - \underset{\sim}{b} \bullet \underset{\sim}{b} = \left|\underset{\sim}{b}\right| \left|\underset{\sim}{a}\right| \cos(\theta) - \left|\underset{\sim}{b}\right|^2 \neq \left|\underset{\sim}{b}\right| \left|\underset{\sim}{c}\right|$$

Question 13

Answer: B

Explanation

$$3 = 0 + \frac{1}{2} a \times 4 \Rightarrow a = \frac{3}{2}$$

$$v = 0 + 2 \times \frac{3}{2} = 3$$

$$p = mv = 12$$

Question 14

Answer: A

Explanation

$$\underset{\sim}{F} = 5\underset{\sim}{i} - \underset{\sim}{j}$$

$$2\underset{\sim}{a} = 5\underset{\sim}{i} - \underset{\sim}{j} \Rightarrow \underset{\sim}{a} = \frac{5}{2}\underset{\sim}{i} - \frac{1}{2}\underset{\sim}{j}$$

$$\left|\underset{\sim}{a}\right| = 2.5$$

Question 15

Answer: C

Explanation

Use Euler’s theorem.

Question 16

Answer: B

Explanation

$$\frac{dA}{dt} = 0 - \frac{A}{25}$$

Question 17

Answer: D

Explanation

The solution to the differential equation is cubic which is represented in the slope field.

Question 18

Answer: A

Explanation

$$g\sin(30°) - g\sin(15°)$$

Question 19

Answer: C

Explanation

$$V = \pi \int_0^3 y^{\frac{3}{2}} dy = \frac{18\pi\sqrt{3}}{5}$$

Question 20

Answer: A

Explanation

$$a = v\frac{dv}{dx} = \sqrt{9-x^2} \times \frac{1}{2\sqrt{9-x^2}} \times -2x = -x$$

$$v = \sqrt{5} \Rightarrow x = \pm 2$$

$$a = -2 \quad (as \quad x > 0)$$

Question 21

Answer: C

Explanation

$$2 = 20 + 4a \Rightarrow a = -\frac{9}{2}$$

$$F = 2 \times \frac{9}{2} = 9\,N$$

Question 22

Answer: E

Explanation

$$R - 60g = 60 \times \frac{g}{4}$$

$$R = 75g$$

SECTION 2

Question 1

a.

$$\left(\frac{x+3}{-2}\right)^2+\left(\frac{y-4}{3}\right)^2=1$$
$$\frac{(x+3)^2}{4}+\frac{(y-4)^2}{9}=1$$

M1+A1

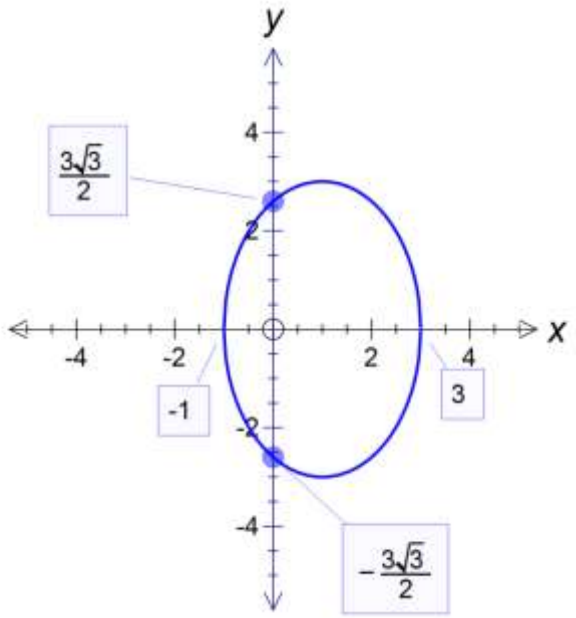
b.

$$y'=-\frac{9(x+3)}{4(y-4)}$$
$$solve\ \ \frac{9(x+3)}{4(y-4)}=\frac{3\sqrt{3}}{2}\quad and\quad \frac{(x+3)^2}{4}+\frac{(y-4)^2}{9}=1$$
$$\left(-3+\sqrt{3}\,,\frac{11}{2}\right)\quad and\quad \left(-3-\sqrt{3}\,,\frac{5}{2}\right)$$
$$-3\pm\sqrt{3}=3-2\cos\left(\frac{t}{2}\right)\quad for\quad 0\le t\le 4\pi$$
$$t=\frac{\pi}{3},\frac{11\pi}{3},\frac{5\pi}{3},\frac{7\pi}{3}$$

But we require gradient to be negative, so $t=\frac{5\pi}{3},\frac{11\pi}{3}$ only.

M2+A2

c.



2 marks for the intercepts

d.

$$V = \pi \int_{1}^{2.5} \left(9 - \frac{9}{4}(x-1)^2 \right) dx$$

A2

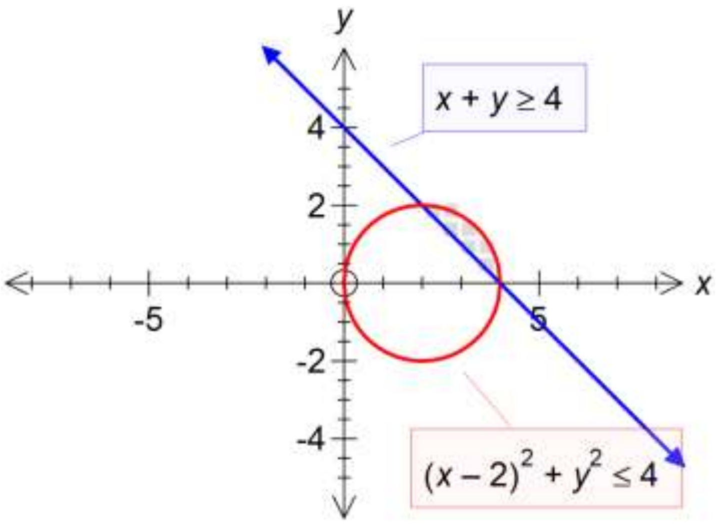
e.

$$V = \pi \int_{1}^{2.5} \left(9 - \frac{9}{4}(x-1)^2 \right) dx = \frac{351\pi}{32}$$

A1

Question 2

a.

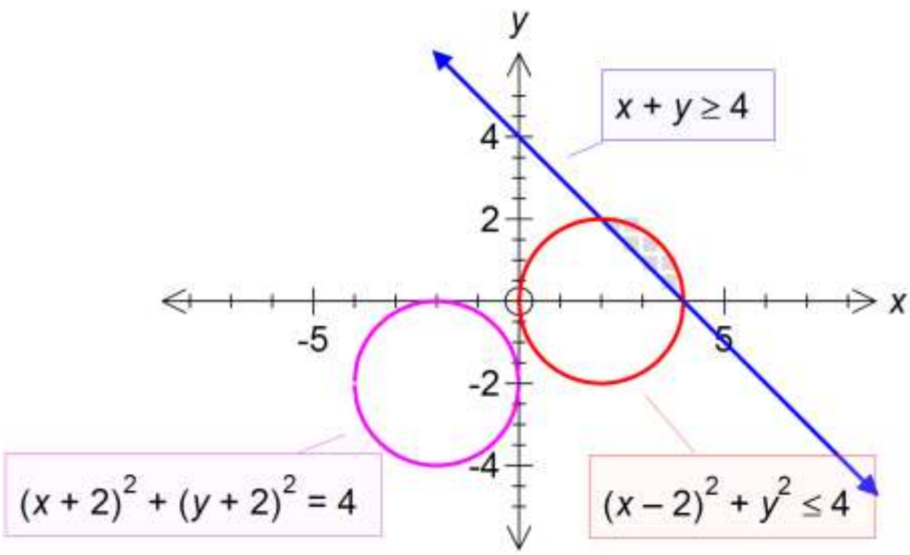


3 marks for line and circle, 1 mark for shading the intersection area

b. $Area = \int_2^4 \left(\sqrt{4-(x-2)^2} - (4-x)\right) dx = \pi - 2$

M2

c.



1 mark for centre and 1 mark for correct radius

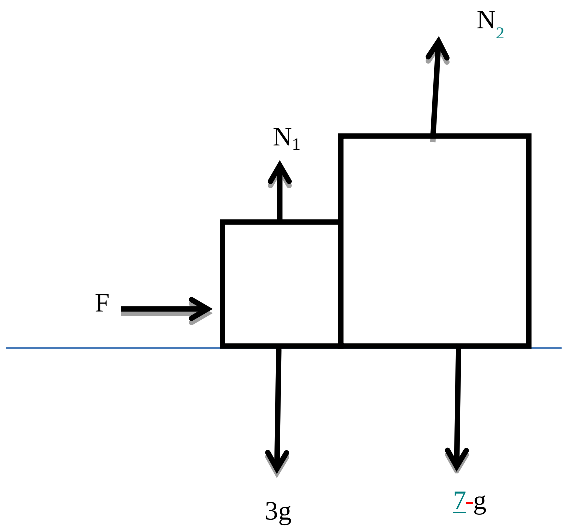
d.

$$C_1(-2\,,-2) \quad and \quad C_2(2\,,0)$$
$$C_1C_2 = \sqrt{(2+2)^2+(2-0)^2} = \sqrt{20} = 2\sqrt{5}$$
$$\text{Min} \quad \text{distance} = 2\sqrt{5}-2-2 = 2\sqrt{5}-4$$

M1+A1

Question 3

a.



4 marks

b.

$$F = 10a \Rightarrow a = \frac{F}{10}$$

M1 + A1

c.

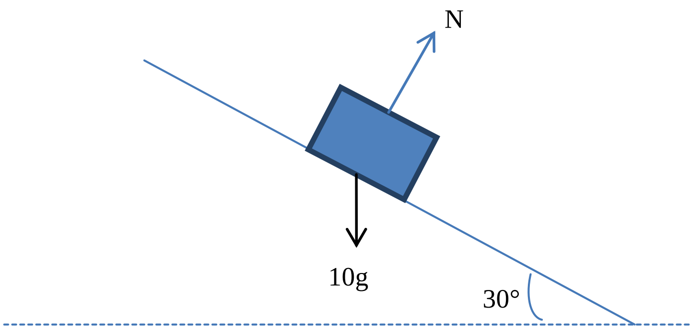
$$7\times\frac{F}{10} = 0.7F$$

M1+A1

d. $a = \frac{120}{10} = 12m/s^2$

A1

e.



A2

f. Acceleration perpendicular to the block is 0

Let acceleration parallel to the incline be a, then

$\text{mg}\sin(30°) = \text{ma}$

$\text{a} = \text{g}\sin(30°) = 4.9\ \text{m/s}^2$

Thus resultant acceleration $= 4.9\ \text{m/s}^2$

M2+A1

g.

$\text{N} = \text{mg}\cos(30°)$

$\text{N} = 10 \times 9.8 \times \frac{\sqrt{3}}{2} = 84.87N$

M1+A1

Question 4

a.

$$\left(t^3 - 9t + 8\right)\underset{\sim}{i} + t^2 \underset{\sim}{j} = (2 - t^2)\underset{\sim}{i} + (3t - 2)\underset{\sim}{j}$$

$$t^3 - 9t + 8 = 2 - t^2 \Rightarrow t^3 + t^2 - 9t + 6 = 0$$

$$and \quad t^2 = 3t - 2 \Rightarrow t^2 - 3t + 2 = 0$$

$$t = 2\,,1$$

$$At \quad t = 1, \quad t^3 + t^2 - 9t + 6 = -1 \neq 0$$

$$Thus \quad t = 2$$

The particles collide after 2 seconds.

M3+A1

b.

$$\dot{\underset{\sim}{r}}_A(t) = (3t^2 - 9)\underset{\sim}{i} + 2t\underset{\sim}{j}$$

$$\dot{\underset{\sim}{r}}_A(2) = 3\underset{\sim}{i} + 4\underset{\sim}{j}$$

$$Speed_A = \sqrt{9 + 16} = 5$$

$$\dot{\underset{\sim}{r}}_B(t) = -2t\underset{\sim}{i} + 3\underset{\sim}{j}$$

$$\dot{\underset{\sim}{r}}_B(2) = -4\underset{\sim}{i} + 3\underset{\sim}{j}$$

$$Speed_B = \sqrt{16 + 9} = 5$$

The particles collide when their speed is 5m/s.

M3+A1

c. $\dot{\underset{\sim}{r}}_A(t) \bullet \dot{\underset{\sim}{r}}_B(t) = -12 + 12 = 0$

A1

d. The particles are travelling at right angles at the time of collision.

A2

e.

$$\ddot{\underset{\sim}{r}}_B(t) = -2\underset{\sim}{i}$$

$$\ddot{\underset{\sim}{r}}_B(2) = -2\ m/s^2$$

M1+A1

Question 5

a.

$$\log_e N = 6 - 3e^{-0.4t}$$

$$\frac{1}{N}\frac{dN}{dt} = -3e^{-0.4t} \times -0.4 \Rightarrow \frac{1}{N}\frac{dN}{dt} = 1.2e^{-0.4t}$$

$$LHS = 1.2e^{-0.4t} + 0.4\left(6 - 3e^{-0.4t}\right) - 2.4 = 0$$

M2

b.

$$\log_e N = 6 - 3e^0$$

$$N = e^3$$

$$N = 20$$

A1

c.

$$\log_e N = 6 - 3e^{-0.4t}$$

$$As \quad t \to \infty, \quad \log_e N = 6 \Rightarrow N = e^6 \Rightarrow N = 403$$

M1+A1

d.

$$\log_e N = 6 - 3e^{-0.4t}$$

$$\frac{dN}{dt} = 1.2N\left(\frac{6 - \log_e N}{3}\right) = 0.4N(6 - \log_e N)$$

$$\frac{d^2N}{dt^2} = 0.4N \times \frac{-1}{N}\frac{dN}{dt} + (6 - \log_e N) \times 0.4\frac{dN}{dt}$$

$$\frac{d^2N}{dt^2} = -1.6N(6 - \log_e N) + 1.6N(6 - \log_e N)^2$$

M1+A1

e.

$$\frac{d^2N}{dt^2} = 0 \Rightarrow N = 148$$

$$(2.7\ ,148)$$

or (3,148) to the nearest integers.

M1+A1