VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

State Government Victoria

2014 Specialist Mathematics GA 3: Examination 2

GENERAL COMMENTS

The 2014 Specialist Mathematics examination 2 comprised 22 multiple-choice questions (worth 22 marks) and five extended-answer questions, worth a total of 58 marks. Most students made substantial attempts at all questions in Section 2.

This year there were three 'show that' questions (Questions 2bii., 4a. and 5aiii.), where students were required to establish a result given on the exam. In quite a few responses there was a lack of algebraic detail. Students need to provide a convincing sequence of steps in 'show that' questions to obtain full marks.

The requirement to show working in questions worth more than one mark was complied with well this year, particularly in Questions 1a., 3a., 4b. and 4d., where most students showed working leading to their results.

Exact answers were generally given, except when a numerical approximation was required.

The examination revealed areas of strength and weakness in student performance.

Areas of strength included:

- clarity in the presentation of solutions
- the use of CAS technology to differentiate functions (Question 1a.) and evaluate definite integrals (Questions 1dii. and 4c.)
- facility with complex number questions – most of Question 2 was done quite well
- the ability to set up a definite integral to represent a volume of revolution – Question 1di.

Areas of weakness included:

- poor presentation of some graphical features, such as circles drawn roughly (Question 2biii.), poor asymptotic behaviour of curves (Question 1c.) and graphs not drawn over the required domain (Question 1c.)
- not reading questions properly – a significant number of students gave an answer for $\overrightarrow{OP}$ instead of $\overrightarrow{AP}$ in Question 1c. In Question 3a., a large number of students failed to express $\underset{\sim}{a}$ in terms of the parallel and perpendicular resolutes, as required by the question
- omission of the integration constant when integrating – Question 4d.
- poor understanding of the term 'equation of motion' – Questions 5ai. and 5aii.

2014 Examination Report

SPECIFIC INFORMATION

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding errors resulting in a total less than 100 per cent.

Section 1

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	2	85	6	5	2	0	Asymptotes are $y = \pm\frac{2}{3}(x-3)$
2	2	3	19	5	71	1	Completed square form: $(x-3)^2 + 2(y+2)^2 = 1$
3	8	20	3	66	3	0	$f(x) = 1 - \frac{3(x-3)}{(x-3)(x+2)}$
4	2	1	90	3	4	0	
5	85	3	5	3	4	0	$z^2 = 8\,\text{cis}\left(\frac{3\pi}{2}\right)$
6	16	6	7	65	6	0	
7	7	4	9	4	75	0	The sum of the roots must be a real number.
8	4	10	69	5	11	1	
9	17	56	17	6	4	1	The circle had centre $(3,2)$ and radius 2. Only the line in option B intersected twice. Options A and C did not intersect. Options D and E were tangents.
10	64	18	9	3	4	1	
11	5	3	8	77	6	0	$y_1 = 2 + 0.1 \times \left(1^3 - 1 \times 2\right)$
12	8	6	64	15	6	1	
13	19	4	8	4	65	0	
14	7	9	72	4	8	0	The vertical line segments on the line $y = x$, so options A and C were possibilities. Negative gradients for $y = 0$ and $x > 0$ then gave option C.
15	5	69	12	8	5	0	
16	77	11	6	4	2	0	$-8 + mn + 3 = 0$ and $4^2 + m^2 + 3^2 = 10^2$ gives $m = 5\sqrt{3}$, $n = \frac{1}{\sqrt{3}}$
17	6	24	6	62	2	0	
18	8	18	9	19	45	1	Resolve forces horizontally: $2\sin(60^\circ) + 4\sin(60^\circ) - 5 \simeq 0.2$ Resolve forces vertically: $1 + 2\cos(60^\circ) - 4\cos(60^\circ) = 0$
19	5	18	49	23	4	1	$\lvert \underset{\sim}{F} \rvert = 5 \times \lvert \ddot{\underset{\sim}{r}}(t) \rvert = 5 \times \sqrt{36 + 28\sin^2(2t)}$ max $\lvert \underset{\sim}{F} \rvert = 5 \times \sqrt{36 + 28} = 40$

Question	% A	% B	% C	% D	% E	% No Answer	Comments
20	5	66	11	5	14	1	
21	52	14	11	17	5	1	$\frac{d\left(\frac{1}{2}v^2\right)}{dx} = -4x$, $\frac{1}{2}v^2 = -2x^2 + 50$, $x = 3$, speed $= 8$
22	7	6	25	12	49	1	

Section 2

Question 1a.

Marks	0	1	2	3	Average
%	3	3	7	87	**2.8**

$f'(x) = \frac{-9}{(x+2)^2(x-4)^2} \times (2x-2) = 0$, stationary point is $(1, -1)$

This question was answered very well. However, a significant number of students expressed $f(x)$ in partial fraction form before differentiating and this sometimes introduced errors. Other errors involved incorrect results for $f'(x)$. A small number of students isolated the quadratic denominator and used it to find the turning point.

Question 1b.

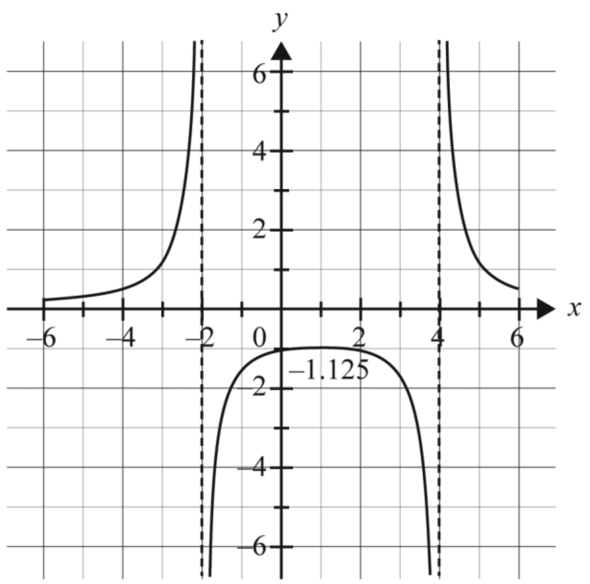
Marks	0	1	2	Average
%	3	18	79	**1.8**

$x = -2,\ x = 4,\ y = 0$

This question was answered well. The most frequent error was the omission of the asymptote $y = 0$.

Question 1c.

Marks	0	1	2	3	Average
%	2	7	30	61	**2.5**



Many students answered this question well; however, some gained only partial marks where graphs were not drawn over the correct domain, where the *y*-intercept was not found or where the curve swung away from an asymptote.

Question 1di.

Marks	0	1	2	Average
%	6	9	85	**1.8**

$$V=\int_0^3 \frac{81\pi}{(x+2)^2(x-4)^2}\,dx$$

This question was answered very well. Most errors involved the omission of π, or the failure to square $f(x)$ in the integrand.

Question 1dii.

Marks	0	1	Average
%	23	77	**0.8**

12.85

This question was answered well by students who managed to set up the integral for volume correctly.

Question 2ai.

Marks	0	1	2	Average
%	3	17	80	**1.8**

$$z_1 = 2\sqrt{3}\,\text{cis}\left(-\frac{\pi}{3}\right)$$

Overall, most students answered this question well, but incorrect arguments such as $\frac{\pi}{3}$ or $\frac{\pi}{6}$ were common. The argument $\frac{5\pi}{3}$ was also accepted.

Question 2aii.

Marks	0	1	Average
%	33	67	**0.7**

$$\frac{2\pi}{3}$$

This question was answered reasonably well, but many students did not express their answer as an angle in the interval $(-\pi, \pi]$. A number of students gave the entire expression for $(z_1)^4$ as an answer.

Question 2aiii.

Marks	0	1	2	Average
%	14	20	66	**1.6**

$-2\sqrt{3}$ and $\sqrt{3}+3i$

This question was answered reasonably well. Most students could find the conjugate root, but a number could not obtain $-2\sqrt{3}$, often omitting the negative sign. A number of students gave factors instead of the roots.

Question 2bi.

Marks	0	1	Average
%	15	85	**0.9**

4

The majority of students answered quite well. A number of students obtained incorrect answers involving i.

Question 2bii.

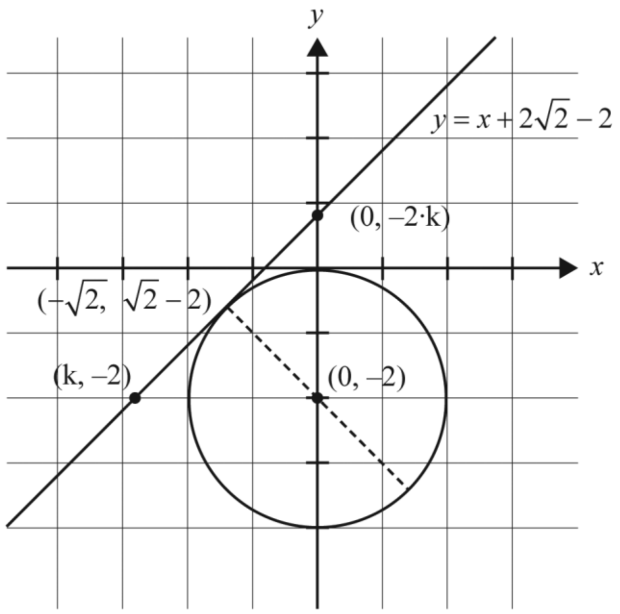
Marks	0	1	2	Average
%	20	24	57	**1.4**

$(x+i(y+2))(x-i(y+2))=4 \Rightarrow x^2+(y+2)^2=4$

This ‘show that’ question was only moderately well done. Most students could express the relation in terms of *x* and *y*, but a large number could not follow through with enough mathematical detail to show the given result. Some students substituted the incorrect forms $z=x+y$ and $\overline{z}=x-y$.

Question 2biii.

Marks	0	1	2	Average
%	9	10	81	**1.7**



Circle centre (0, –2), radius 2

Many students answered this question quite well, although some circles were drawn poorly. Some students drew circles with the incorrect centre, and others drew shapes other than circles.

Question 2c.

Marks	0	1	2	3	Average
%	73	14	3	9	**0.5**

$\frac{(-2-k)-(-2)}{0-k}=1$, using right-angled triangle trigonometry, $\frac{2}{(-2-k)-(-2)}=\cos(45°)$, $k=-2\sqrt{2}$

Most students struggled with this question. Students needed to equate the gradient of the tangent to 1 in order to proceed. Most students attempted to differentiate the equation to the circle implicitly, but ended up with too many variables. Other more successful attempts involved finding the equation of the tangent and equating expressions for the *y*-intercept, and setting up to solve the line and circle equations simultaneously, then equating the discriminant to zero.

Question 3a.

Marks	0	1	2	3	4	5	Average
%	14	4	8	10	23	41	**3.5**

$\underset{\sim}{a}.\hat{\underset{\sim}{b}}\hat{\underset{\sim}{b}} = \frac{1}{9}\left(2\underset{\sim}{i} - 2\underset{\sim}{j} - \underset{\sim}{k}\right)$ is the parallel resolute, $\underset{\sim}{a} - \underset{\sim}{a}.\hat{\underset{\sim}{b}}\hat{\underset{\sim}{b}} = \frac{25}{9}\underset{\sim}{i} + \frac{20}{9}\underset{\sim}{j} + \frac{10}{9}\underset{\sim}{k}$ is the perpendicular resolute,

$\underset{\sim}{a} = \frac{1}{9}\left(2\underset{\sim}{i} - 2\underset{\sim}{j} - \underset{\sim}{k}\right) + \frac{25}{9}\underset{\sim}{i} + \frac{20}{9}\underset{\sim}{j} + \frac{10}{9}\underset{\sim}{k}$

Many students made arithmetic errors finding the resolutes, and a significant number omitted the final line, where $\underset{\sim}{a}$ was to be expressed as the sum of the two vector resolutes. A significant number of students unsuccessfully attempted to find the resolutes from first principles, instead of applying the standard formulas.

Question 3bi.

Marks	0	1	2	Average
%	16	9	75	**1.6**

$\overrightarrow{AP} = \alpha\left(\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{a}\right)$

The majority of students answered this question well. However, the main error was to use $\alpha\left(\underset{\sim}{c} + \frac{1}{2}\underset{\sim}{a}\right)$, brought about by confusion of the direction of $\underset{\sim}{a}$. A small number of students did not realise that they needed to work with $\frac{1}{2}\underset{\sim}{a}$.

Question 3bii.

Marks	0	1	Average
%	45	55	**0.6**

$\overrightarrow{AP} = -\underset{\sim}{a} + \beta\left(\underset{\sim}{a} + \underset{\sim}{c}\right)$

A significant number of students misread this question and gave a correct answer for $\overrightarrow{OP}$ instead of $\overrightarrow{AP}$.

Question 3biii.

Marks	0	1	2	Average
%	50	14	37	**0.9**

$\alpha\left(\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{a}\right) = -\underset{\sim}{a} + \beta\left(\underset{\sim}{a} + \underset{\sim}{c}\right)$, $\beta - 1 = -\frac{1}{2}\alpha$ and $\alpha = \beta$, $\alpha = \frac{2}{3}$ and $\beta = \frac{2}{3}$

Few students could correctly equate coefficients to find the values of α and β .

Question 4a.

Marks	0	1	Average
%	21	79	**0.8**

Similar triangles, or other, gives $r = \frac{1}{2}h$, $V = \frac{1}{3} \times \pi \times \left(\frac{1}{2}h\right)^2 \times h$, which gives the result.

A number of students substituted $r = \frac{1}{2}$ to try to show the result.

Question 4b.

Marks	0	1	2	3	4	Average
%	13	5	17	7	58	**2.9**

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$, $\frac{dh}{dt} = \frac{4}{\pi h^2} \times \left(0.02\pi - 0.01\pi\sqrt{h}\right)$, where $h = 0.25$, $\frac{dh}{dt} = 0.96$

Most students attempted this question by using the correct form of the chain rule, but many only used the 'rate in' or the 'rate out', instead of the difference between the rates.

Question 4c.

Marks	0	1	2	Average
%	58	10	32	**0.8**

$t=\int_0^1 \frac{\pi h^2}{4\left(0.02\pi-0.01\pi\sqrt{h}\right)}dh$, $t=7.4$

Many students did not attempt this question. A number attempted to find t in terms of h, instead of using a definite integral. Some attempted to integrate $\frac{dh}{dt}$ with respect to h.

Question 4d.

Marks	0	1	2	3	4	5	Average
%	21	7	9	10	40	12	**2.8**

$\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}$, $\frac{dx}{dt}=\frac{16}{\pi\left(x+2\right)^2}\times 0.05\pi$, $\frac{dt}{dx}=\frac{\left(x+2\right)^2}{0.8}$, $x=\sqrt[3]{2.4t+8}-2$

Many students managed the early steps of working, but only a minority could find x correctly in terms of t. Students who did not express $\frac{dh}{dV}$ as a complete square had more difficulty in ultimately expressing x in terms of t. A number of equivalent technology-derived results for $x\left(t\right)$ were also accepted.

Question 5ai.

Marks	0	1	Average
%	31	69	**0.7**

$T_1-2g=2a$

Many students answered this question well, but some took the direction of acceleration to be opposite to that required by the question. A significant number of students only resolved forces instead of writing an equation of motion.

Question 5aii.

Marks	0	1	2	Average
%	33	7	60	**1.3**

$3g\sin\theta-T_2=3a$, $5g\sin\theta+T_2-T_1=5a$

This question was answered well. Some students made similar errors to those made in Question 5ai.

Question 5aiii.

Marks	0	1	Average
%	49	51	**0.5**

Adding the equations $T_1-2g=2a$, $3g\sin\theta-T_2=3a$ and $5g\sin\theta+T_2-T_1=5a$ gives

$3g\sin\theta+5g\sin\theta-2g=10a \Rightarrow a=\frac{g}{10}\left(8\sin\theta-2\right)$, $a=\frac{g}{5}\left(4\sin\theta-1\right)$

Many students did not show adequate working in this 'show that' question. A number of students made a the subject in their equations of motion, thereby introducing complicated fractions.

Question 5aiv.

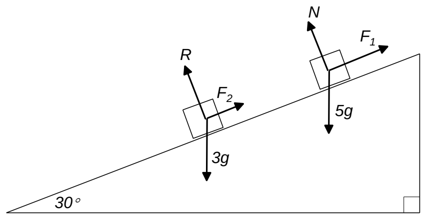
Marks	0	1	Average
%	37	64	**0.7**

$0 = \frac{g}{5}\left(4\sin\theta - 1\right),\ \theta = 14.5^\circ$

A number of students did not attempt this question. A small number of students gave an answer in radians.

Question 5bi.

Marks	0	1	2	Average
%	34	20	46	**1.1**



A large number of students did not distinguish between the different normal reactions and different friction forces acting on each block. Some students used the coefficient of friction to denote a friction force. A minority of students had friction forces acting down the plane.

Question 5bii.

Marks	0	1	2	Average
%	32	10	58	**1.3**

$5g\sin 30^\circ - 5g\cos 30^\circ \times 0.01 = 5a,\ a = 4.82$

$3g\sin 30^\circ - 3g\cos 30^\circ \times 0.11 = 3a,\ a = 3.97$

Many students answered this question well. The major error was inaccurate arithmetic.

Question 5biii.

Marks	0	1	2	3	Average
%	43	8	12	36	**1.4**

$\frac{1}{2}\times 3.97\times t_1^2 + 3 = \frac{1}{2}\times 4.82\times t_1^2,\ t_1 = 2.66$

Many students did not attempt this question. A few students worked on the basis that when the blocks collide, their velocities (and not displacements) would be the same. Other students had the + 3 on the incorrect side of their equation, which equated displacement expressions for each block. A number of students gave $t_1 = 2.67$ because of poor rounding.