Engage Education Foundation

Units 3 and 4 Specialist Maths: Exam 1

Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 1 hour writing time

Structure of book:

Number of questions	Number of questions to be answered	Number of marks
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers and rulers.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- No calculator is allowed in this examination.

Materials supplied:
- This question and answer booklet of 11 pages.

Instructions:
- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

Instructions

Answer all questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Questions

Question 1

Let $A(1,3,-2)$, $B(2,-1,3)$ and $C(3,1,2)$ be three points in 3-dimensional space.

a. Determine if the points A, B and C are collinear.

2 marks

b. Find a point $D(x,y,z)$ whose position vector bisects the angle between the position vectors of points A and C (and is coplanar to points A and C).

2 marks

c. Find θ, the angle between the position vectors of points A and B.

2 marks

d. Find the vector resolute of the position vector of point C in the direction perpendicular to the position vector of point B.

4 marks

Total: 10 marks

Question 2

Find the equation of the normal to the graph of $3x^2y + 9y^2 - 3xy = 24x$ at the point (3, 2).

3 marks

Question 3

Let $z = \cos(\theta) + i\sin(\theta)$.

a. Find an expression for $|i\bar{z} - z|$.

2 marks

b. Find the largest value of α such that $\operatorname{Arg}(i\bar{z} - z)$ is defined for all $\theta \in (-\alpha, \alpha)$.

2 marks

c. Given that $\theta \in (-\alpha, \alpha)$, as found in part b, find $\operatorname{Arg}(i\bar{z} - z)$.

2 marks

Total: 6 marks

Question 4

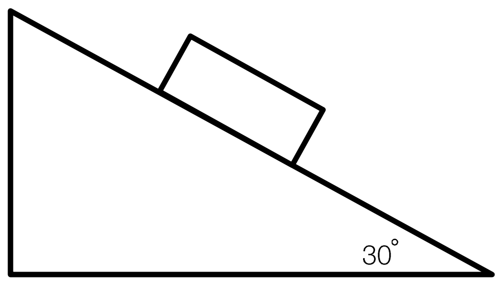
Evaluate $\int_{3}^{7} \frac{3x^2+2x^3-12x}{\sqrt{x-3}}\, dx$. You may leave your answer in form $\frac{a}{b} + \frac{c}{d} + e$.

4 marks

Question 5

A box with mass 3 kg is being pulled up a hill inclined 30^0 to the horizontal by a force parallel to the hill with a magnitude of M newtons. The coefficient of friction between the box and the hill is $\frac{1}{\sqrt{3}}$.

a. Label all forces acting on the box on the diagram below.



2 marks

b. Find the force required for the box to be on the point of moving up the hill.

3 marks

c. If the same force as in part b. was used on a different hill with no friction, what would be the acceleration of the box up the frictionless hill?

2 marks

Total: 7 marks

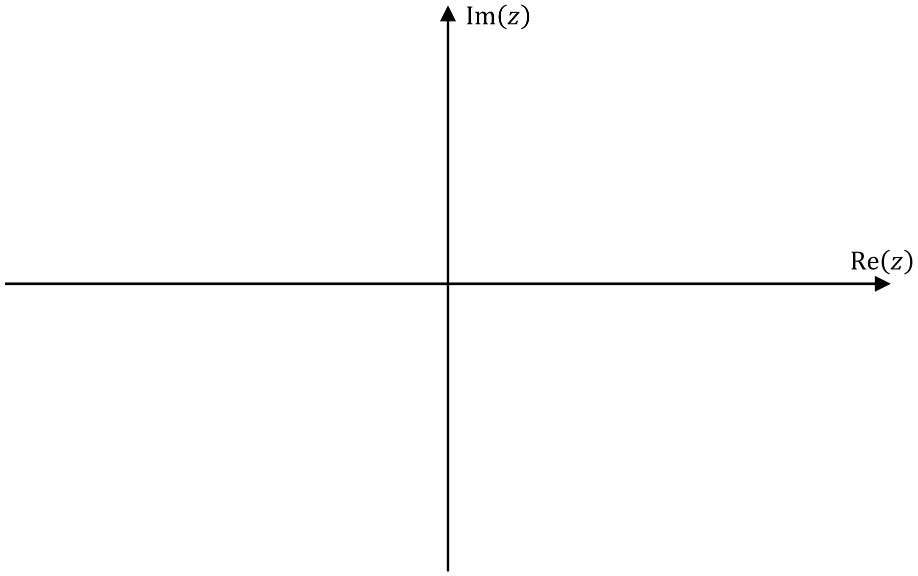
Question 6

The region bounded by the curve $y = \frac{x+3}{\sqrt{x^2+1}}$, the lines $x = 0, x = 2$ and the x-axis is rotated about the x-axis. Find the exact volume of this solid of revolution.

4 marks

Question 7

Sketch the region defined by $\boldsymbol{S} = \left\{\boldsymbol{z}: \mathbf{0} \leq \boldsymbol{Arg}(\boldsymbol{z}) < \frac{\mathbf{3\pi}}{\mathbf{4}}\right\} \cap \{\boldsymbol{z}: |\boldsymbol{z} + \mathbf{2} - \boldsymbol{i}| < \mathbf{2}\}$ on the axes below.



3 marks

Question 8

The position of particles A and B at any time $\boldsymbol{t}$ seconds, $\boldsymbol{t} \geq \mathbf{0}$ is given by $\boldsymbol{r_A}(\boldsymbol{t}) = (\mathbf{12} - \mathbf{2}\boldsymbol{t})\boldsymbol{i} + (\mathbf{6}\boldsymbol{t} - \mathbf{2})\boldsymbol{j}$ and $\boldsymbol{r_B}(\boldsymbol{t}) = (\mathbf{5}\boldsymbol{t} - \boldsymbol{t}^{\mathbf{2}})\boldsymbol{i} + (\boldsymbol{t}^{\mathbf{2}} + \mathbf{6})\boldsymbol{j}$ respectively.

Find the time at which the particles collide.

3 marks

End of Booklet

Looking for solutions? Visit www.engageeducation.org.au/practice-examswww.engageeducation.org.au/practice-exams

To enrol in one of our Specialist Mathematics lectures head to: www.engageeducation.org.au/lectures

Formula sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin A$
sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	$\mathbb{R}$
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$

$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)

$|z| = \sqrt{x^2 + y^2} = r$

$-\pi < \text{Arg}\, z \leq \pi$

$z_1 z_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$

$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\frac{d}{dx}(e^{ax}) = ae^{ax}$

$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\frac{d}{dx}(\log_e x) = \frac{1}{x}$

$\int \frac{1}{x} dx = \log_e|x| + c$

$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$

$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$

$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$

$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$

$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$

$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$

$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$

$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$

$\frac{d}{dx}(\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}}$

$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$

$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$

$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\left(v\frac{du}{dx} - u\frac{dv}{dx}\right)}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x), x_0 = a$ and $y_0 = a$, then $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
constant (uniform) acceleration	$v = u + at,\ s = ut + \frac{1}{2}at^2,\ v^2 = u^2 + 2as,\ s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$

$\dot{\boldsymbol{r}} = \frac{d\boldsymbol{r}}{dt} = \frac{dx}{dt}\boldsymbol{i} + \frac{dy}{dt}\boldsymbol{j} + \frac{dz}{dt}\boldsymbol{k}$

$|\boldsymbol{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\boldsymbol{r}_1.\boldsymbol{r}_2 = r_1 r_2 \cos\theta = x_1x_2 + y_1y_2 + z_1z_2$

Mechanics

momentum $\boldsymbol{p} = m\boldsymbol{v}$

equation of motion $\boldsymbol{R} = m\boldsymbol{a}$

friction $F \leq \mu N$