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## **SPECIALIST MATHEMATICS**

### **TRIAL EXAMINATION 2**

**2015**

Reading Time: 15 minutes

Writing time: 2 hours

#### **Instructions to students**

This exam consists of Section 1 and Section 2.  
Section 1 consists of 22 multiple-choice questions and should be answered on the detachable answer sheet on page 26 of this exam. This section of the paper is worth 22 marks.  
Section 2 consists of 5 extended-answer questions, all of which should be answered in the spaces provided. Section 2 begins on page 10 of this exam. This section of the paper is worth 58 marks.  
There is a total of 80 marks available.  
Where more than one mark is allocated to a question, appropriate working must be shown.  
An exact answer is required to a question unless otherwise stated.  
Unless otherwise stated, diagrams in this exam are not drawn to scale.  
The acceleration due to gravity should be taken to have magnitude  $g \text{ m/s}^2$  where  $g = 9.8$ .  
Students may bring one bound reference into the exam.  
Students may bring into the exam one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory does not need to be cleared.  
Formula sheets can be found on pages 23-25 of this exam.

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## SECTION 1

### Question 1

The axes intercepts of the ellipse with equation  $x^2 - 10x + 25y^2 - 100y + 100 = 0$  are located at

- A. (0,2) only
- B. (0,1) and (0,3) only
- C. (0,2) and (5,0) only
- D. (3,0) and (7,0) only
- E. (0,1),(0,3),(3,0) and (7,0) only

### Question 2

For the graph of the function with rule  $f(x) = x + \frac{2}{x^2}$ , it is **not** true to say that

- A. all of its asymptotes pass through the origin
- B.  $f(x) > x$  for  $x \in \mathbb{R} \setminus \{0\}$
- C. there is one straight and one curved asymptote
- D.  $f(x) > 0$  for  $x \in \mathbb{R}^+$
- E. there is one point of discontinuity

### Question 3

The curve defined by  $x = 3\sec(\theta) - 1$  and  $y = \frac{4}{\cot(\theta)}$  can be written in Cartesian form as

- A.  $\frac{(x+1)^2}{3} - \frac{y^2}{4} = 1$
- B.  $\frac{(x+1)^2}{9} + \frac{y^2}{16} = 1$
- C.  $\frac{y^2}{4} - \frac{(x+1)^2}{3} = 1$
- D.  $\frac{(x+1)^2}{3} + \frac{y^2}{4} = 1$
- E.  $\frac{(x+1)^2}{9} - \frac{y^2}{16} = 1$

### Question 4

The function  $f$  has the rule  $f(x) = a \sin^{-1}(bx+1)$ , the domain  $[-6,0]$  and the range  $[-\pi, \pi]$ . Therefore,

- A.  $a = \frac{1}{2}$  and  $b = 3$
- B.  $a = 2$  and  $b = -\frac{1}{3}$
- C.  $a = 2$  and  $b = \frac{1}{3}$
- D.  $a = 4$  and  $b = -3$
- E.  $a = 4$  and  $b = 3$

**Question 5**

If the complex number  $z$  has modulus  $\sqrt{3}$  and lies on the imaginary axis of an Argand diagram then  $z$  could be equal to

- A.  $-3i$
- B.  $-\sqrt{3}i$
- C.  $3i$
- D.  $\sqrt{3}+i$
- E.  $1+\sqrt{3}i$

**Question 6**

Let  $z = \text{cis}\left(\frac{-\pi}{3}\right)$ .

The imaginary part of  $z + \frac{i}{\sqrt{3}}$  is

- A.  $-\frac{\sqrt{3}}{2}i$
- B.  $-\frac{\sqrt{3}}{2}$
- C.  $\frac{-1}{2\sqrt{3}}$
- D.  $\frac{-i}{2\sqrt{3}}$
- E.  $-\sqrt{3}$

**Question 7**

Given the two complex numbers  $z_1 = a\text{cis}(\theta)$  and  $z_2 = b\text{cis}(2\theta)$ , the argument of  $\frac{z_1^3}{z_2}$  is

- A.  $-\frac{3}{2}$
- B.  $\frac{3}{2}$
- C.  $\frac{\theta^2}{2}$
- D.  $\theta$
- E.  $5\theta$

**Question 8**

The solutions to the equation  $z^2 = 2 + 2\sqrt{3}i$  have principal arguments of

- A.  $-\frac{2\pi}{3}$  and  $\frac{\pi}{3}$   
 B.  $-\frac{5\pi}{6}$  and  $\frac{\pi}{6}$   
 C.  $-\frac{\pi}{6}$  and  $\frac{\pi}{6}$   
 D.  $\frac{\pi}{3}$  and  $\frac{4\pi}{3}$   
 E.  $\frac{\pi}{6}$  and  $\frac{7\pi}{6}$

**Question 9**

The circle with equation  $|z + 3 - i| = 2$  intersects twice with the line that has the equation

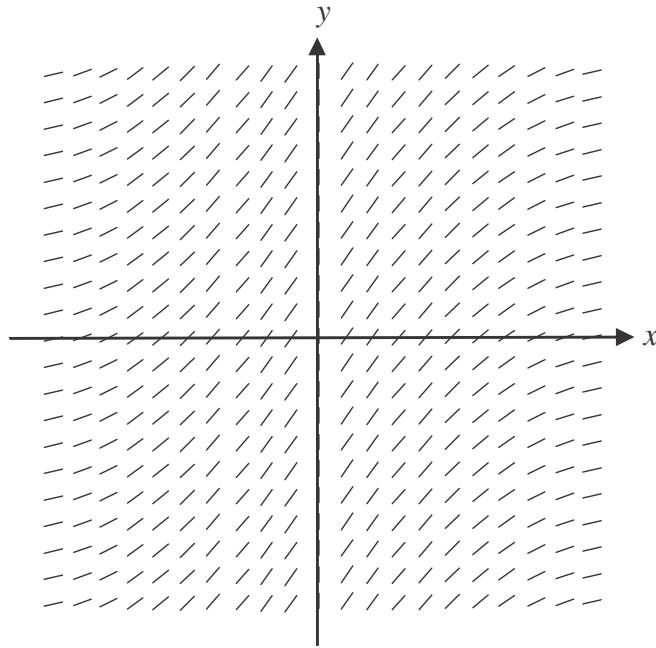
- A.  $\operatorname{Re}(z) = -1$   
 B.  $|z - 3| = |z - i|$   
 C.  $\operatorname{Im}(z) = -1$   
 D.  $|z + 3i| = |z + i|$   
 E.  $|z - 1| = |z - 3i|$

**Question 10**

A cylindrical tank is filled with liquid to a depth of  $h$  metres so that the volume of liquid  $V \text{ m}^3$  in the tank is given by  $V = 5\pi h$ . Liquid flows into the tank at the rate of  $0.1 \text{ m}^3$  per hour and leaks out at the rate of  $0.02\sqrt{h} \text{ m}^3$  per hour.

A differential equation that relates the variables  $h$  and  $t$  is

- A.  $\frac{dh}{dt} = \frac{1}{50\pi} - \frac{1}{\sqrt{h}}$   
 B.  $\frac{dh}{dt} = \frac{5 - \sqrt{h}}{250\pi}$   
 C.  $\frac{dh}{dt} = \frac{5 - \sqrt{h}}{250\pi h}$   
 D.  $\frac{dh}{dt} = 5\pi(0.1 - 0.02\sqrt{h})$   
 E.  $\frac{dh}{dt} = \frac{0.1 - 0.02\sqrt{h}}{5\pi h}$

**Question 11**

The differential equation that could represent the direction field shown above is

- A.  $\frac{dy}{dx} = \frac{1}{x}$   
 B.  $\frac{dy}{dx} = \frac{1}{x^2}$   
 C.  $\frac{dy}{dx} = \frac{1}{y^2}$   
 D.  $\frac{dy}{dx} = \frac{1}{x^2 y}$   
 E.  $\frac{dy}{dx} = \frac{1}{x - y}$

**Question 12**

For the differential equation  $\frac{dy}{dx} = x^2 y + 3x$ ,  $y = 1$  when  $x = 2$ . If Euler's method is used with a step size of 0.1, then the approximation for  $y$  when  $x = 2.2$  is

- A. 2  
 B. 3.312  
 C. 3.512  
 D. 3.4  
 E. 4

**Question 13**

Using a suitable substitution,  $\int_1^2 \frac{x-1}{\sqrt{1+2x}} dx$  can be expressed as

- A.  $\frac{1}{4} \int_3^5 \frac{u-3}{\sqrt{u}} du$
- B.  $\frac{1}{2} \int_1^2 \frac{u-1}{\sqrt{u}} du$
- C.  $\int_1^3 \sqrt{u}(u-3) du$
- D.  $\int_1^2 \frac{u-2}{4\sqrt{u}} du$
- E.  $\int_3^5 \frac{u-3}{\sqrt{u}} du$

**Question 14**

A particle is acted on by a force of 8 newtons acting in a direction due west and a force of 10 newtons acting in a direction S30°E.

The magnitude, in newtons, of the resultant force acting on the particle is

- A. 6
- B.  $\sqrt{180-80\sqrt{3}}$
- C.  $\sqrt{180-40\sqrt{3}}$
- D.  $2\sqrt{21}$
- E.  $2\sqrt{31}$

**Question 15**

The position vector of a particle is given by  $\underline{r}(t) = (2t^3 + 3)\underline{i} + t^2 \underline{j} - (1 - t^3)\underline{k}$ ,  $t \geq 0$ .

The magnitude of the acceleration of the particle in  $\text{m/s}^2$  at  $t=2$  is

- A.  $2\sqrt{10}$
- B.  $19\sqrt{2}$
- C.  $\sqrt{38}$
- D.  $2\sqrt{181}$
- E.  $\sqrt{598}$

**Question 16**

Given that the angle between the vectors  $\underline{a} = 2\underline{i} + \sqrt{2}\underline{j} - 2\underline{k}$  and  $\underline{b} = -2\underline{i} + 2\underline{j} + \sqrt{2}\underline{k}$  is  $\theta$ , then

$\sin(\theta)$  is

- A.  $-\frac{\sqrt{29}}{5}$   
 B.  $-\frac{\sqrt{21}}{5}$   
 C.  $\frac{\sqrt{21}}{5}$   
 D.  $\frac{2}{5}$   
 E.  $\frac{3}{5}$

**Question 17**

The velocity vector of a particle that starts its journey at the origin, is given by

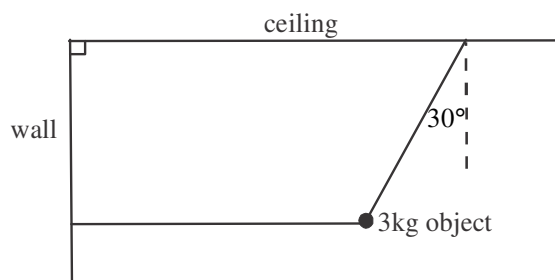
$$\underline{v}(t) = e^{3t}\underline{i} - 4\underline{j} + \sin(t)\underline{k}, \quad t \geq 0.$$

The position vector of the particle is given by

- A.  $\underline{r}(t) = \frac{e^{3t}}{3}\underline{i} - 4\underline{j} - \cos(t)\underline{k}$   
 B.  $\underline{r}(t) = \frac{e^{3t}}{3}\underline{i} - \cos(t)\underline{k}$   
 C.  $\underline{r}(t) = 3e^{3t}\underline{i} - \cos(t)\underline{k}$   
 D.  $\underline{r}(t) = \frac{e^{3t} - 1}{3}\underline{i} - 4t\underline{j} + (1 - \cos(t))\underline{k}$   
 E.  $\underline{r}(t) = \frac{e^{3t} + 1}{3}\underline{i} - 4\underline{j} - \cos(t)\underline{k}$

**Question 18**

An object of mass 3kg is connected to two light inextensible strings which hold it in equilibrium. One of the strings is horizontal and is attached to a wall. The other string is at an angle of  $30^\circ$  to the vertical and is attached to the ceiling.

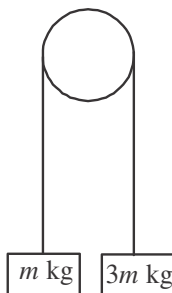


The tension in the horizontal string, in newtons, is

- A.  $\sqrt{3}$
- B. 3
- C.  $3\sqrt{3}$
- D.  $\sqrt{3}g$
- E.  $3g$

**Question 19**

A light inextensible string passes over a smooth pulley. Two particles of mass  $m$  kg and  $3m$  kg are attached to the ends of the string.



The system is released from rest.

The speed of the  $3m$  kg particle will be 24.5m/s after

- A. 1.25 seconds
- B. 2 seconds
- C. 2.5 seconds
- D. 3 seconds
- E. 5 seconds

**Question 20**

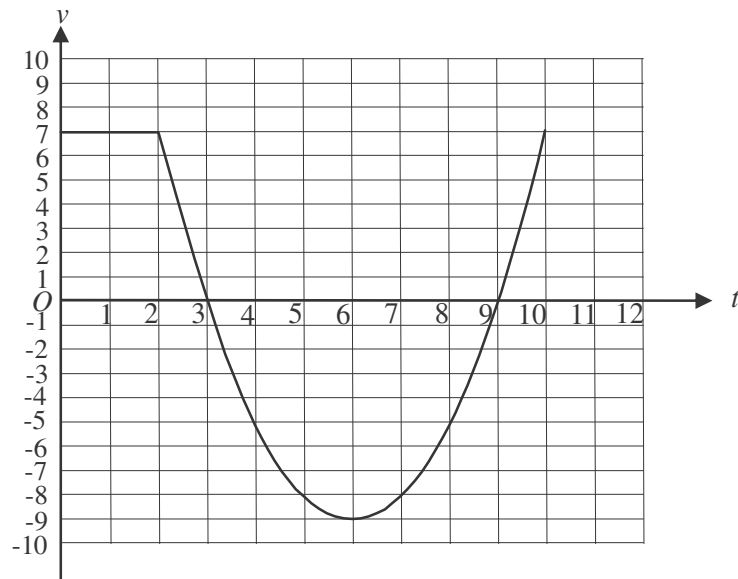
A particle of mass 4kg is moving in a straight line with a velocity of 6m/s when it is acted on by a force of 9 newtons acting in the opposite direction to the motion of the particle. The particle will come to rest after travelling a further distance of

- A. 8m
- B. 16m
- C. 20m
- D. 24.5m
- E. 40.5m



**Question 21**

The velocity-time graph is shown below for a particle travelling in a straight line with velocity  $v$  m/s at time  $t$  seconds.



For  $t \in [2, 10]$  the velocity of the particle is given by  $v(t) = (t-6)^2 - 9$ .

The distance travelled, in metres, by the particle for  $t \in [0, 10]$  is

- A.  $\frac{10}{3}$
- B.  $\frac{46}{3}$
- C.  $\frac{160}{3}$
- D.  $\frac{170}{3}$
- E.  $\frac{226}{3}$

**Question 22**

A particle moves in a straight line with an acceleration, in  $\text{m/s}^2$ , given by  $a = 3 - x^2$  where  $x$  is its displacement, in metres, from a fixed origin at  $O$ .

If the particle is at rest at  $O$ , then the particle will also be at rest where

- A.  $x = \sqrt{3}$  only
- B.  $x = -3$  only
- C.  $x = 3$  only
- D.  $x = \pm\sqrt{3}$  only
- E.  $x = \pm 3$  only

**SECTION 2****Question 1** (11 marks)

A curve is defined by the parametric equations  $x = t + \frac{2}{t} + 1$  and  $y = t - \frac{2}{t}$  where  $t \in \mathbb{R} \setminus \{0\}$ .

- a.** Show that the Cartesian equation of the curve is  $\frac{(x-1)^2}{8} - \frac{y^2}{8} = 1$ . 2 marks

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- b.** Find the values of  $t$  when the gradient of the curve is 2. 3 marks

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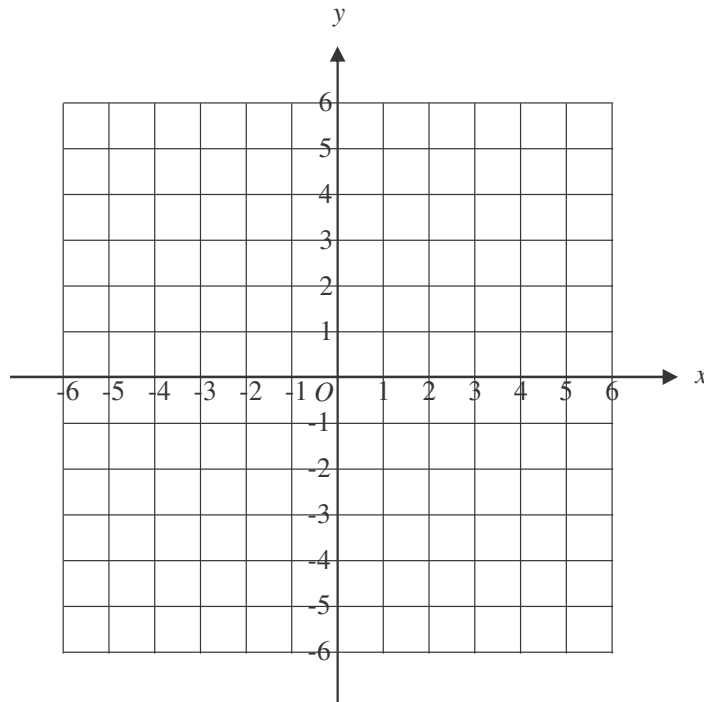


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- c. Sketch the graph of  $\frac{(x-1)^2}{8} - \frac{y^2}{8} = 1$ , showing clearly the asymptotes and any axes intercepts. 3 marks



The region enclosed by the graph of  $\frac{(x-1)^2}{8} - \frac{y^2}{8} = 1$  and the line  $x = 5$  is rotated about the  $x$ -axis to form a solid of revolution.

- d. Find the volume of this solid of revolution. 3 marks

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**Question 2** (13 marks)

Let  $z_1 = \sqrt{3} + i$ .

- a.** Express  $z_1$  in polar form. 2 marks

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- b.** Verify that  $z_1$  is one root of the equation  $z^4 = -8 + 8\sqrt{3}i$ . 1 mark

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- c.** If  $\text{Arg}(z_1^p) = \frac{\pi}{2}$ ,  $p \in R$ , find all the possible values of  $p$ . 2 marks

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Line  $L$  lies in the complex plane and has the equation  $|z - 2z_1| = |z|$ .

- d. i.** Show that the Cartesian equation of  $L$  is given by  $\sqrt{3}x + y = 4$ . 2 marks

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- ii.** Show that  $\bar{z}_1 + \frac{2}{\sqrt{3}}$  satisfies the relation  $|z - 2z_1| = |z|$ . 2 marks

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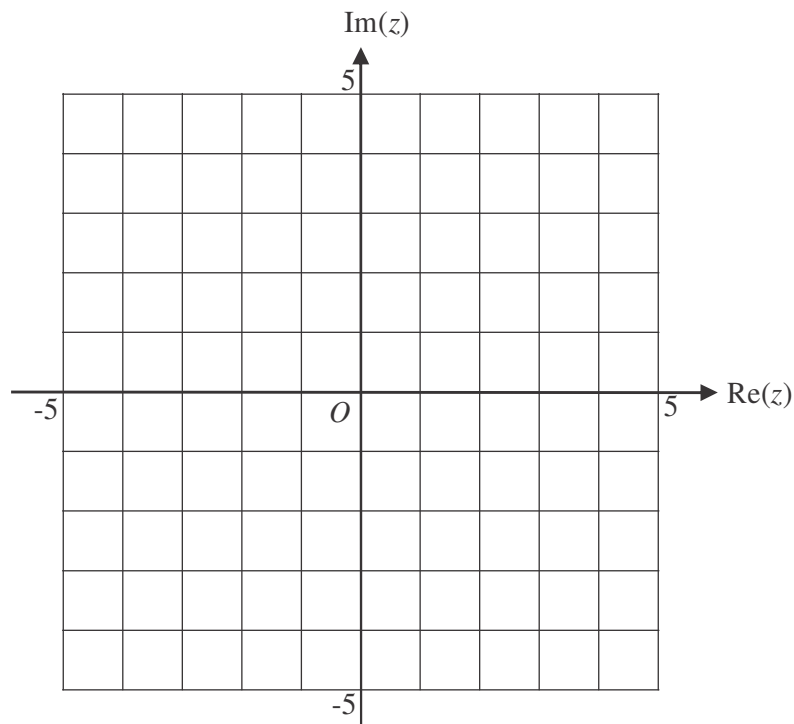
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- e. i. Sketch  $L$  and the graph of  $\text{Arg}(z) = \frac{\pi}{6}$  on the argand diagram below. 2 marks



- ii. The area in the first quadrant enclosed by the  $y$ -axis and the graphs of  $L$  and  $\text{Arg}(z) = \frac{\pi}{6}$ , is equal to the area enclosed by the graphs of  $L$ ,  $\text{Arg}(z) = \frac{\pi}{6}$  and  $\text{Re}(z) = k$  where  $k > 2$ . Find the value of  $k$ . 2 marks

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**Question 3** (12 marks)

The position vectors of two drones  $R$  and  $S$ , relative to a tower at  $O$ , are given respectively by

$$\underline{r}(t) = \sin(\pi t)\underline{i} + \cos(\pi t)\underline{j} \quad \text{and} \quad \underline{s}(t) = \cos(\pi t)\underline{i} + \sqrt{3}\sin(\pi t)\underline{j}, \quad t \geq 0.$$

The unit vectors  $\underline{i}$  and  $\underline{j}$  are directed to the east and north of  $O$  respectively.

The displacement components are measured in kilometres and time,  $t$ , is measured in hours.

- a.** Show that the Cartesian equation of the path of drone  $S$  is  $x^2 + \frac{y^2}{3} = 1$ . 2 marks

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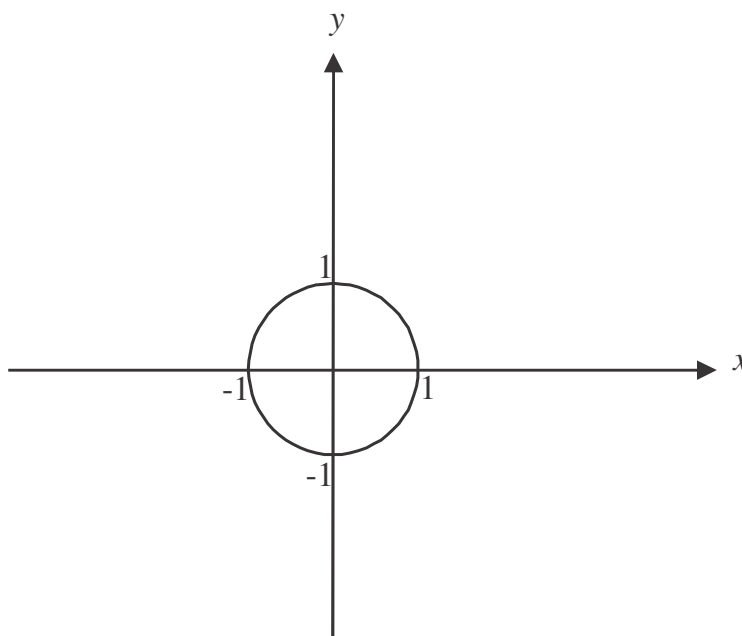


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The path followed by drone  $R$  is shown on the set of axes below.



- b.** Sketch the path followed by drone  $S$  on the set of axes above. 2 marks

- c.** Find the starting position of each of the drones, expressing them in Cartesian coordinates. 2 marks

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- d.** Show that the drones never meet. 2 marks

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- e.** Find the speed of drone  $R$ , in km/hr, at  $t = 3$ . 1 mark

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- f.** Find the magnitude of the acceleration of drone  $R$ , in  $\text{km/hr}^2$ , at  $t = 2$ . 1 mark

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- g.** Show that at  $t = \frac{1}{2}$ , the drones are travelling in directions that are perpendicular to each other.

2 marks

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**Question 4** (10 marks)

In a high rise development, the number of apartments,  $N$ , that are sold  $t$  days after a marketing campaign begins is modelled by the equation  $\tan\left(\frac{N-50\pi}{100}\right) = \frac{t}{30} - 4$ ,  $t \geq 0$ .

- a.** Find the number of apartments that are pre-sold, that is, sold before the marketing campaign begins. Express your answer to the nearest integer. 1 mark

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- b.** According to this model, what is the limiting number of apartments that can be sold. Express your answer to the nearest integer. 1 mark

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- c.** Using implicit differentiation to find  $\frac{dN}{dt}$ , verify that  $\tan\left(\frac{N-50\pi}{100}\right) = \frac{t}{30} - 4$  satisfies the differential equation  $\cos^2\left(\frac{N-50\pi}{100}\right) - 0.3\frac{dN}{dt} = 0$ . 2 marks

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The graph of the function  $N(t)$  has a point of inflection at the point  $(a,b)$ .  
Also, an alternative form of the differential equation given in part c., is

$$\frac{dN}{dt} = \frac{10}{3 \left( \tan^2 \left( \frac{N-50\pi}{100} \right) + 1 \right)}$$

- d. Using this alternative form of the differential equation or otherwise, find  $\frac{d^2N}{dt^2}$  and hence find the values of  $a$  and  $b$ .

4 marks

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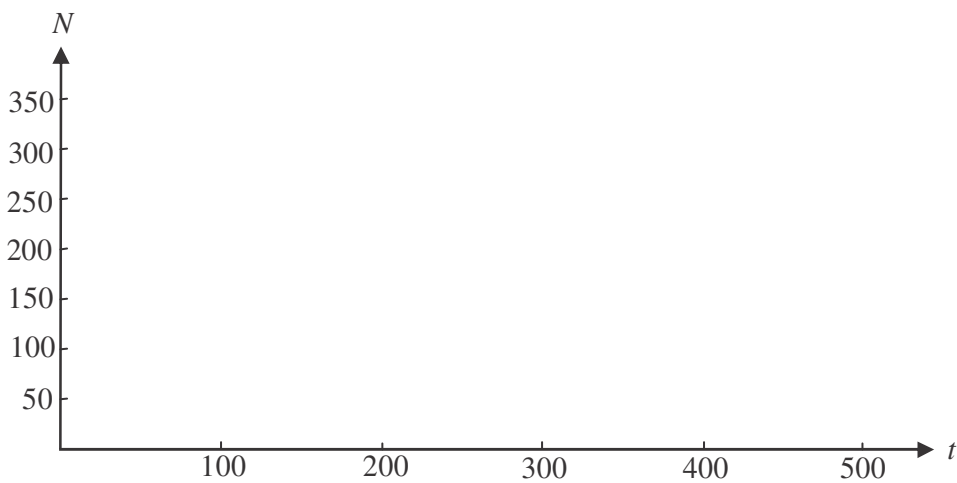
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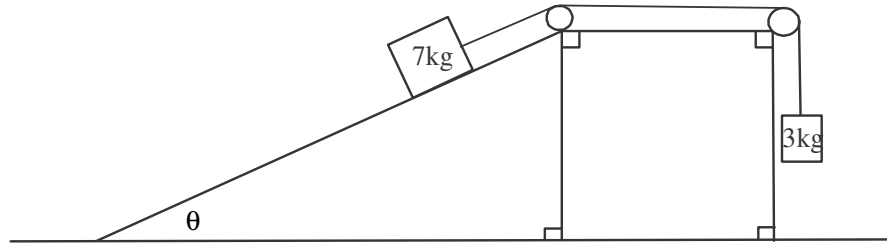
- e. Sketch the graph of the function  $N(t)$  on the set of axes below for  $t \in [0,500]$ .

2 marks



**Question 5** (12 marks)

A mass of 7kg is on a smooth plane inclined at  $\theta$  degrees to the horizontal where  $\theta < 90^\circ$ . It is connected by a light inextensible rope which passes over a smooth pulley, to a mass of 3kg which is hanging vertically, as shown in the diagram below.



- a. i.** Write down an equation of motion for the mass of 7kg when it is moving down the plane. 1 mark

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- ii.** Write down an equation of motion for the mass of 3kg when it is moving vertically upwards. 1 mark

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- iii.** Show that  $a = \frac{g(7 \sin(\theta) - 3)}{10}$ . 1 mark

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- iv. If the system is to be in equilibrium, find the value of  $\theta$  expressed in degrees correct to one decimal place. 1 mark

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- v. If the 3kg mass is to be moving vertically upwards, find the possible values of  $\theta$  expressed in degrees correct to one decimal place. 1 mark

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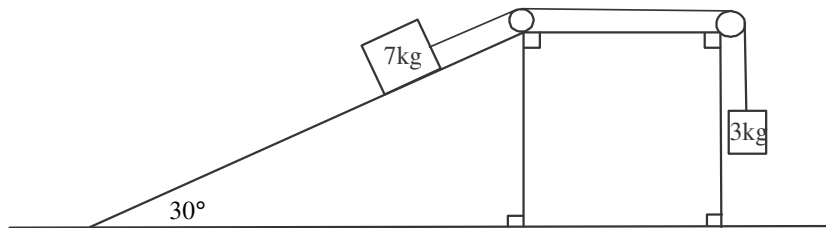


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The smooth inclined plane is replaced with a rough plane that is inclined at an angle of  $30^\circ$  to the horizontal as shown in the diagram below.



The coefficient of friction between the rough plane and the 7kg mass is 0.106. The 3kg mass and the 7kg mass are at rest.

- b. Show that the 7kg mass is not on the point of moving down the plane. 2 marks

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The rope breaks.

**c.** Find the acceleration of the

- i.** 7kg mass down the slope giving your answer in  $\text{m/s}^2$ , correct to the nearest  $0.01\text{m/s}^2$ .

2 marks

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- ii.** 3kg mass vertically downwards.

1 mark

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The distance that the 7kg mass has to travel down the inclined plane to reach the horizontal surface is three times the distance that the 3kg mass has to travel vertically to reach the horizontal surface.

- d.** Using your answers to part **c.**, find the ratio of the speed of the 3kg mass to the speed of the 7kg mass as they each reach the horizontal surface after the rope breaks.

2 marks

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## Specialist Mathematics Formulas

### Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc \sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab \cos C$

### Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
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### Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

function	sin <sup>-1</sup>	cos <sup>-1</sup>	tan <sup>-1</sup>
domain	[-1, 1]	[-1, 1]	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	[0, π]	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

### Algebra (Complex numbers)

$$z = x + yi = r(\cos\theta + i\sin\theta) = r\operatorname{cis}\theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$-\pi < \operatorname{Arg} z \leq \pi$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \quad (\text{de Moivre's theorem})$$

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**Calculus**

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:  $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

quotient rule:  $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

Euler's method: If  $\frac{dy}{dx} = f(x)$ ,  $x_0 = a$  and  $y_0 = b$ ,

$$\text{then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + hf(x_n)$$

acceleration:  $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration:  $v = u + at$      $s = ut + \frac{1}{2}at^2$      $v^2 = u^2 + 2as$      $s = \frac{1}{2}(u+v)t$



### Vectors in two and three dimensions

$$\underline{r} = x \underline{i} + y \underline{j} + z \underline{k}$$

$$|\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underline{r}} = \frac{d \underline{r}}{dt} = \frac{dx}{dt} \underline{i} + \frac{dy}{dt} \underline{j} + \frac{dz}{dt} \underline{k}$$

### Mechanics

momentum:  $\underline{p} = m \underline{v}$

equation of motion:  $\underline{R} = m \underline{a}$

friction:  $F \leq \mu N$

# SPECIALIST MATHEMATICS

## TRIAL EXAMINATION 2

### MULTIPLE- CHOICE ANSWER SHEET

STUDENT NAME:.....

### INSTRUCTIONS

Fill in the letter that corresponds to your choice. Example:  A  B  C  D  E

The answer selected is B. Only one answer should be selected.

- |   |   |
|---|---|
| 1. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E  | 12. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E |
| 2. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E  | 13. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E |
| 3. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E  | 14. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E |
| 4. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E  | 15. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E |
| 5. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E  | 16. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E |
| 6. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E  | 17. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E |
| 7. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E  | 18. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E |
| 8. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E  | 19. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E |
| 9. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E  | 20. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E |
| 10. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E | 21. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E |
| 11. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E | 22. <input type="radio"/> A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> E |