itute.com
Online & home tutors Registered business name: itute ABN: 96 297 924 083

Specialist Mathematics

2015

Trial Examination 1
(1 hour)

Instructions

Answer **all** questions. Do **not** use calculators.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working or explanation **must** be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1

Consider the set of complex numbers $S = \left\{ z : \frac{1}{\overline{z}} - \frac{1}{z} = i \right\}$.

a. Find the Cartesian equation representing S in the x-y plane. 2 marks

b. For $z \in S$, find the possible values of $\operatorname{Re}(z)$. 1 mark

c. For $z \in S$, find the possible values of $\operatorname{Im}(z)$. 1 mark

d. Sketch the graph of S on the Argand plane. 2 marks

Question 2

Consider polynomial $P(z)=z^3-2iz^2+2z-2i$. Let α, β and γ be the roots of $P(z)=0$.

a. Show that $\alpha+\beta+\gamma=2i$. 1 mark

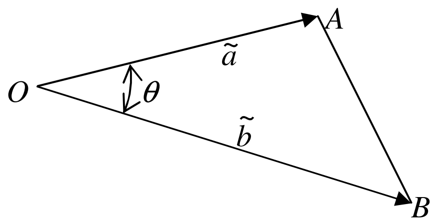
b. Show that $(i-\alpha)(i-\beta)(i-\gamma)=i$. 1 mark

c. Use part a and part b to find the value of $(\alpha+\beta-\gamma)(\beta+\gamma-\alpha)(\gamma+\alpha-\beta)$. 2 marks

Question 3

Triangle OAB is defined by two non-parallel and non-zero vectors $\overrightarrow{OA}=\tilde{a}$ and $\overrightarrow{OB}=\tilde{b}$.

The area of the triangle is given by $|\tilde{a}||\tilde{b}|\sin\theta$ where θ is the angle between vectors $\tilde{a}$ and $\tilde{b}$.



Express the area of ΔOAB in terms of dot products of $\tilde{a}$ and $\tilde{b}$. 3 marks

Question 4

Let $\underset{\sim}{p} = \frac{2}{3}\underset{\sim}{i} + \frac{1}{3}\underset{\sim}{j} + \frac{2}{3}\underset{\sim}{k}$, $\underset{\sim}{q} = -\frac{1}{3}\underset{\sim}{i} - \frac{2}{3}\underset{\sim}{j} + \frac{2}{3}\underset{\sim}{k}$ and $\underset{\sim}{r} = \frac{2}{3}\underset{\sim}{i} - \frac{2}{3}\underset{\sim}{j} - \frac{1}{3}\underset{\sim}{k}$ be vectors in 3-dimensional space defined by perpendicular unit vectors $\underset{\sim}{i}$, $\underset{\sim}{j}$ and $\underset{\sim}{k}$.

a. Show that $\underset{\sim}{p}$, $\underset{\sim}{q}$ and $\underset{\sim}{r}$ are unit vectors. 1 mark

b. Show that $\underset{\sim}{p}$, $\underset{\sim}{q}$ and $\underset{\sim}{r}$ are perpendicular to each other. 2 marks

c. Show that $\underset{\sim}{s} = \underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ can be expressed as $\underset{\sim}{s} = \underset{\sim}{p} + \underset{\sim}{q} + \underset{\sim}{r}$. 1 mark

d. Given $\underset{\sim}{t} = \frac{2}{\sqrt{3}}\underset{\sim}{p} - \sqrt{3}\,\underset{\sim}{q} + \frac{1}{\sqrt{3}}\underset{\sim}{r}$, find $\underset{\sim}{s} \,.\, \underset{\sim}{t}$. 2 marks

Question 5

Consider $f:\left[\pi, \frac{3\pi}{2}\right] \to R,\ f(x) = \frac{1}{2}\cos(2x)$.

a. Sketch the graph of f^{-1}. Show and label endpoint(s) and intercept(s). 2 marks

b. Find the rule of f^{-1} in terms of the arccos (i.e. $\cos^{-1}$) function. 2 marks

c. Find the equation of the normal to the graph of f^{-1} at $x = 0$. 2 marks

Question 6

The solution curve to the differential equation $\frac{dy}{dx}-\frac{x^2}{2y}=0$ passes through $(2, 1)$.

a. Use Euler's method (first order approximation) to estimate the value of y at $x=3$.
Choose 0.5 as the step size. 2 marks

b. Show that $y=\sqrt{\frac{x^3-5}{3}}$ is the equation of the solution curve. 1 mark

Question 7

A solid of revolution is formed by revolving $y=f(x)$ about the x-axis, where $f(x)\geq 0$ for $0\leq x\leq 2$.

The volume of the solid is given by $V=\int_0^2 \pi\left(2x-x^2\right)dx$.

a. Find $f(x)$. 1 mark

b. Find the area enclosed by $y=f(x)$ and $y=-f(x)$. 2 marks

Question 8

A 0.4 kg particle moves with acceleration $\tilde{a} = \frac{g}{4}\left(\sqrt{3}\,\tilde{i} - \tilde{j}\right)$ in a vertical x-y plane, where $g = 9.8$ m s^{-2}.

$\tilde{i}$ is a horizontal unit vector in the positive x-direction, and $\tilde{j}$ is a unit vector pointing vertically upwards in the positive y-direction.

The particle starts from rest at position $\tilde{r} = 10\tilde{j}$.

a. Find the exact time t in seconds when the particle is at $\tilde{r} = 10\sqrt{3}\,\tilde{i}$. Express your answer in terms of g.

3 marks

b. Find the exact speed in m s^{-1} of the particle at $\tilde{r} = 10\sqrt{3}\,\tilde{i}$. Express your answer in terms of g. 2 marks

c. Find the magnitude of the resultant force (net force) in newtons at $t = 1$ second. 1 mark

Question 9

A particle is in equilibrium under the action of three forces $\tilde{F}_1$, $\tilde{F}_2$ and $\tilde{F}_3$.

$\tilde{F}_1 = \sqrt{7}$ newtons, $|\tilde{F}_3| = 3|\tilde{F}_2|$, and the angle between $\tilde{F}_2$ and $\tilde{F}_3$ is $120°$.

a. Find the exact magnitude of $\tilde{F}_3$ in newtons. 2 marks

b. $\tilde{F}_2$ and $\tilde{F}_1$ make an acute angle θ where $\sin\theta = \alpha\sqrt{\beta}$. Find the exact value of α and β. 2 marks

End of Exam 1