Year 2015

VCE

Specialist Mathematics

Trial Examination 2

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Victorian Certificate of Education 2015

STUDENT NUMBER

Figures										Letter:
Words										

SPECIALIST MATHEMATICS

Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 33 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple-choice questions and sign your name in the space provided.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The line $y = -2x + 4$ is an asymptote to the hyperbola $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$, then it is possible that

A. $a = 1$, $b = 2$, $h = -1$ and $k = 2$

B. $a = 1$, $b = 2$, $h = 2$ and $k = 4$

C. $a = 2$, $b = 4$, $h = -2$ and $k = 6$

D. $a = 2$, $b = 4$, $h = 1$ and $k = 2$

E. $a = 3$, $b = 6$, $h = 2$ and $k = 1$

Question 2

The ellipse $x^2 + 2px + 4y^2 - 4qy = r^2$ has a centre and semi-minor vertical axes respectively of

A. $\left(-p, \frac{q}{2}\right)$, $\frac{\sqrt{r^2 + p^2 + q^2}}{2}$

B. $(-p, q)$, $\frac{\sqrt{r^2 + p^2 + q^2}}{2}$

C. $\left(p, -\frac{q}{2}\right)$, $\frac{\sqrt{r^2 + p^2 + q^2}}{2}$

D. $\left(p, -\frac{q}{2}\right)$, $\sqrt{r^2 + p^2 + 4q^2}$

E. $\left(-p, \frac{q}{2}\right)$, $\sqrt{r^2 + p^2 + 4q^2}$

Question 3

The graph of $y=\dfrac{A}{3+bx-x^2}$ has one vertical asymptote at $x=-3$ and crosses the *y*-axis at $y=2$. Which of the following is **false?**

A. The graph has a minimum stationary point at $\left(-1,\dfrac{3}{2}\right)$

B. The *x*-axis is a horizontal asymptote.

C. The graph has another vertical asymptotes at $x=1$

D. $A=6$

E. $b=2$

Question 4

If a is a positive real constant, then the ellipse $\dfrac{(x+a)^2}{4a^2}+\dfrac{y^2}{a^2}=1$ and the hyperbola $\dfrac{x^2}{a^2}-\dfrac{(y-a)^2}{4a^2}=1$

A. do not intersect.

B. intersect at only one point.

C. intersect at exactly two points.

D. intersect at exactly three points.

E. intersect at exactly four points.

Question 5

Which of the following functions has a range of $[-a,a]$?

A. $f(x)=\dfrac{2a}{\pi}\cos^{-1}\left(\dfrac{x}{a}\right)$

B. $f(x)=\dfrac{2a}{\pi}\sin^{-1}\left(\dfrac{x}{a}\right)$

C. $f(x)=\dfrac{2a}{\pi}\tan^{-1}\left(\dfrac{x}{a}\right)$

D. $f(x)=\dfrac{a}{\pi}\cos^{-1}\left(\dfrac{x}{a}\right)$

E. $f(x)=\dfrac{a}{\pi}\sin^{-1}\left(\dfrac{x}{a}\right)$

Question 6

A particle moves so that its position vector is given by $\underset{\sim}{r}(t) = 4\sin(t)\underset{\sim}{i} + \cos(2t)\underset{\sim}{j}$ where the position in measured in metres and $t \geq 0$ is the time in seconds. The particle moves along part of

A. a straight line.

B. a parabola.

C. a circle.

D. an ellipse.

E. a hyperbola.

Question 7

A fourth degree polynomial $P(z)$ has roots $z = \pm\sqrt{a}\,i$ and $z = \pm\sqrt{b}$ where a and b are positive real constants. Then

A. $P(z) = z^4 + (a-b)z^2 - ab$

B. $P(z) = z^4 + (b-a)z^2 - ab$

C. $P(z) = z^4 + (a-b)z^2 + ab$

D. $P(z) = z^4 + (b-a)z^2 + ab$

E. $P(z) = z^4 - (a+b)z^2 + ab$

Question 8

Given the complex number $z = \sqrt{a} + \sqrt{2}\,i$ where a is a real positive number, if $|z|^3 = 27$, then a is equal to

A. 1

B. 5

C. 7

D. 25

E. $\left(3 - \sqrt{2}\right)^2$

Question 9

Given the complex number $z = a\,\text{cis}\left(\frac{\pi}{b}\right)$ where a and b are non-zero real numbers.

Then $\frac{1}{\overline{z}}$ is equal to

A. $z = a\,\text{cis}\left(\frac{\pi}{b}\right)$

B. $z = a\,\text{cis}\left(-\frac{\pi}{b}\right)$

C. $z = \frac{1}{a}\text{cis}\left(\frac{\pi}{b}\right)$

D. $z = \frac{1}{a}\text{cis}\left(-\frac{\pi}{b}\right)$

E. $z = \frac{1}{a}\text{cis}\left(\frac{b}{\pi}\right)$

Question 10

If $\frac{dy}{dx} = \cos\left(x^2\right)$ and $y = 1$ when $x = 1$, then the value of y when $x = 2$ can be found by evaluating

A. $\int_1^2 \left(\cos\left(u^2\right) - 1\right) du$

B. $\int_1^2 \left(\cos\left(u^2\right) + 1\right) du$

C. $\int_1^2 \cos\left(u^2\right) du - 1$

D. $\int_1^2 \cos\left(u^2\right) du + 1$

E. $\int_1^2 \sin\left(u^2\right) du$

Question 11

A cricket ball is smashed by a batsman off his toes. Its position vector is given by $\underset{\sim}{r}(t) = 15t\sqrt{2}\,\underset{\sim}{i} + \left(15t\sqrt{2} - 4.9t^2\right)\underset{\sim}{k}$ for $t \ge 0$, where $\underset{\sim}{i}$ is a unit vector in metres horizontally forward and $\underset{\sim}{k}$ is a unit vector in metres vertically upwards.
Students when analysing the motion of the cricket ball, stated some propositions

Alex stated that the cricket ball is hit with an initial velocity of $15\,\text{ms}^{-1}$ at an angle of 45^0.
Betty stated that the cricket ball hits the ground again after a time of 2.16 seconds.

Colin stated that the cricket ball reaches a maximum height of 22.96 metres.

David stated that the cricket ball first hits the ground at a distance of 91.84 metres from where it was hit.

Then

A. Only Alex and Betty are correct.

B. Only Alex, Betty and Colin are correct.

C. Only Betty and Colin are correct.

D. Only Colin and David are correct.

E. All of Alex, Betty, Colin and David are correct.

Question 12

The gradient of the normal to a curve at any point $P(x, y)$ is twice the square root of the gradient joining *P* and point $(-1,1)$. The coordinates of the points, *P* satisfy the differential equation

A. $\dfrac{dy}{dx} - 2\left(\dfrac{y-1}{x+1}\right)^2 = 0$

B. $\dfrac{dy}{dx} - 2\sqrt{\dfrac{y+1}{x-1}} = 0$

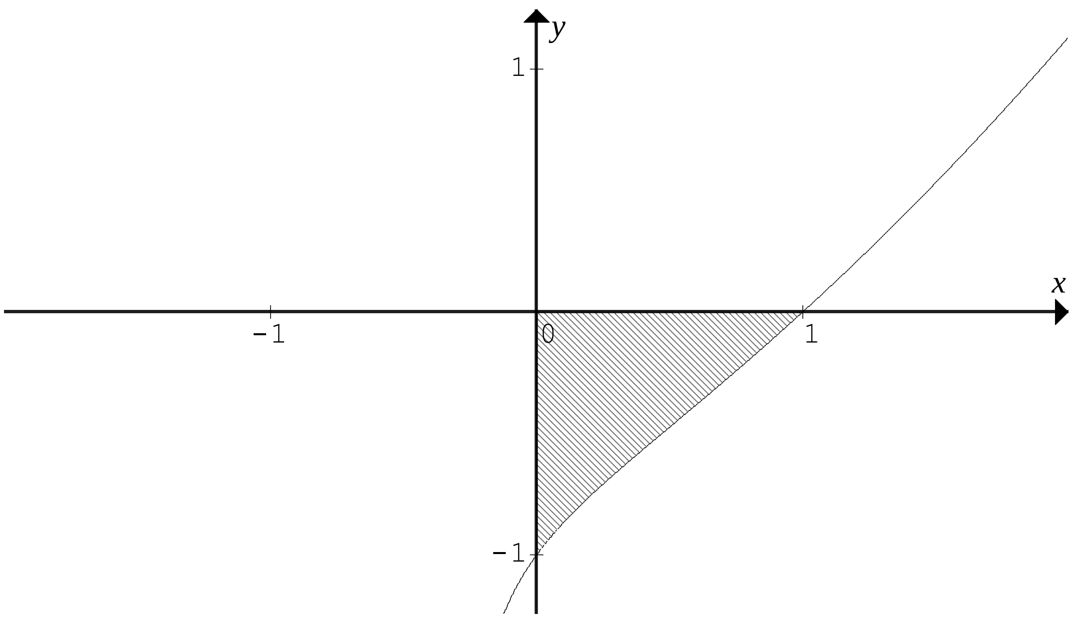
C. $\dfrac{dy}{dx} - 2\sqrt{\dfrac{y-1}{x+1}} = 0$

D. $2\dfrac{dy}{dx} - \sqrt{\dfrac{x+1}{y-1}} = 0$

E. $2\dfrac{dy}{dx} + \sqrt{\dfrac{x+1}{y-1}} = 0$

Question 13

Part of the graph with the equation $y = \dfrac{x^2 - 1}{\sqrt{3x+1}}$ is shown below.



The shaded area that is the area bounded by the curve and the coordinates axis, can be expressed as

A. $\dfrac{1}{3}\displaystyle\int_1^2 \dfrac{(4-u)(u+2)}{\sqrt{u}}\,du$

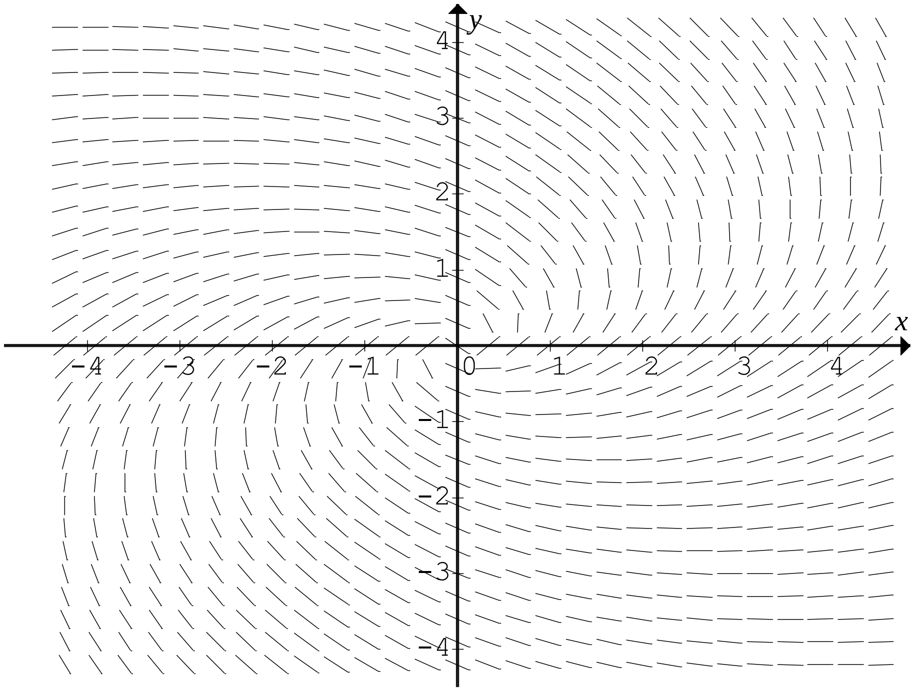
B. $\dfrac{1}{27}\displaystyle\int_1^2 \dfrac{(u^2-4)(u^2+2)}{u}\,du$

C. $\dfrac{1}{27}\displaystyle\int_1^4 \dfrac{(4-u^2)(u^2+2)}{u}\,du$

D. $\dfrac{1}{27}\displaystyle\int_1^4 \dfrac{(4-u)(u+2)}{\sqrt{u}}\,du$

E. $\dfrac{1}{27}\displaystyle\int_1^4 \dfrac{(u-4)(u+2)}{\sqrt{u}}\,du$

Question 14



The differential equation which best represents the above direction field is

A. $\dfrac{dy}{dx} = \dfrac{x-y}{2y-x}$

B. $\dfrac{dy}{dx} = \dfrac{x-y}{x-2y}$

C. $\dfrac{dy}{dx} = \dfrac{x+y}{2y-x}$

D. $\dfrac{dy}{dx} = \dfrac{x+y}{x+2y}$

E. $\dfrac{dy}{dx} = \dfrac{x+y}{x-2y}$

Question 15

The position vector of a 2 kg moving particle is given by $\underset{\sim}{r}(t) = 4\sin(t)\underset{\sim}{i} + \cos(2t)\underset{\sim}{j}$ where the position in measured in metres and $t \geq 0$ is the time in seconds. The maximum momentum in kg-m/s of the particle is

A. 8

B. 4

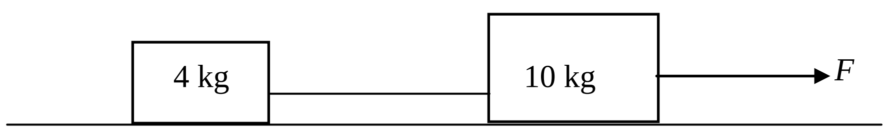
C. 2

D. 1

E. $2\sqrt{5}$

Question 16

Two boxes of masses 10 kg and 4 kg are connected by a light horizontal string and are on a horizontal table, as shown in the diagram below. The coefficient of friction between the 10 kg box and the table is 0.5.The contact between the 4 kg block and table is smooth. The 10 kg box is pulled by a force of F, parallel to the table. Which of the following is **false?**



A. If $F = 50$ newtons, the boxes move with a constant acceleration equal to $\frac{1}{14}$ m/s^2

B. If $F = 49$ newtons, the boxes are on the point of moving.

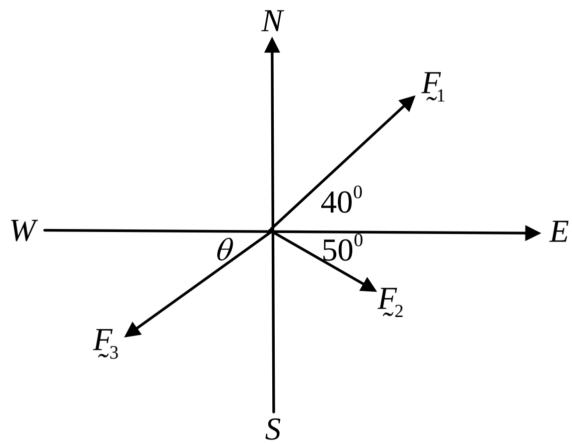
C. If $F = 48$ newtons, the boxes move with constant velocity.

D. If $F = 47$ newtons the boxes remain at rest.

E. If $F = 46$ newtons the boxes are not on the point of moving.

Question 17

A body is on a horizontal smooth plane and acted upon by three forces, $\underset{\sim}{F}_1, \underset{\sim}{F}_2$ and $\underset{\sim}{F}_3$.
A north-south west-east framework is shown.



Given that $\left|\underset{\sim}{F}_1\right| = 10$, $\left|\underset{\sim}{F}_2\right| = 5$ and let $F_3 = \left|\underset{\sim}{F}_3\right|$ and that the body moves in the north direction then

A. $F_3\cos(\theta) = 10.874$ and $F_3\sin(\theta) > 2.598$

B. $F_3\cos(\theta) = 10.874$ and $F_3\sin(\theta) < 2.598$

C. $F_3\sin(\theta) = 2.598$ and $F_3\sin(\theta) < 10.874$

D. $F_3\sin(\theta) = 10.874$ and $F_3\cos(\theta) > 2.598$

E. $F_3\sin(\theta) = 10.874$ and $F_3\cos(\theta) < 2.598$

Question 18

A tennis ball is thrown vertically upwards with a speed of 4 m/s from a point 1.6 metres above the ground. Then which of the following is most correct?

A. The ball hits the ground after 1.11 seconds.

B. The ball hits the ground after 0.29 seconds.

C. The ball reaches a maximum height of 2.4 metres above the ground.

D. The ball reaches its maximum height after 0.4 seconds.

E. The ball hits the ground with a speed of 3.92 m/s.

Question 19

A particle moves in such a way that its velocity vector at a time $t \geq 0$ is given by $4e^{\frac{t}{2}}\underset{\sim}{i} - 2\sin(2t)\underset{\sim}{j}$. Initially the position vector of the particle is $3\underset{\sim}{i}$. The position vector of the particle at a time t is given by

A. $8e^{\frac{t}{2}}\underset{\sim}{i} + \cos(2t)\underset{\sim}{j}$

B. $2e^{\frac{t}{2}}\underset{\sim}{i} - 4\cos(2t)\underset{\sim}{j}$

C. $\left(2e^{\frac{t}{2}} + 1\right)\underset{\sim}{i} + \left(\cos(2t) - 1\right)\underset{\sim}{j}$

D. $\left(8e^{\frac{t}{2}} - 5\right)\underset{\sim}{i} + \left(\cos(2t) - 1\right)\underset{\sim}{j}$

E. $\left(8e^{\frac{t}{2}} - 5\right)\underset{\sim}{i} + \left(1 - \cos(2t)\right)\underset{\sim}{j}$

Question 20

The acceleration in m/s^2 of a particle moving in a straight line is given by $x\cos(x)$, where x metres is its displacement from the origin O. Initially the body is at rest at the origin, then after one second, its speed in m/s, is closest to

A. 57.3

B. 1.66

C. 1.4

D. 1.0

E. 0.87

Question 21

The area between the graphs of $y=\frac{\pi x}{8}$ and $y=\sin^{-1}\left(\frac{x}{4}\right)$ where $x \ge 0$ is rotated about the *y*-axis, to form a volume of revolution. The volume obtained can be expressed as

A. $\pi\int_0^{\frac{\pi}{2}}\left(\frac{64x^2}{\pi^2}-16\sin^2(x)\right)dx$

B. $\pi\int_0^{\frac{\pi}{2}}\left(16\sin^2(x)-\frac{64x^2}{\pi^2}\right)dx$

C. $\pi\int_0^{\frac{\pi}{2}}\left(\frac{8x}{\pi}-4\sin(x)\right)^2 dx$

D. $\pi\int_0^{4}\left(\frac{\pi x}{8}-\sin^{-1}\left(\frac{x}{4}\right)\right)^2 dx$

E. $\pi\int_0^{4}\left(\frac{\pi^2 x^2}{64}-\left(\sin^{-1}\left(\frac{x}{4}\right)\right)^2\right)dx$

Question 22

A particle moves so that at a time *t* seconds, its velocity *v* m/s is given by

$v(t)=\frac{16}{\pi}\tan^{-1}\left(\frac{6-t}{9}\right)$. Over the first 10 seconds, the distance travelled by the particle in metres is closest to

A. 5

B. 13

C. 14

D. 295

E. 798

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (12 marks)

A curve is defined by the parametric equations

$x = 2(t - \sin(t))$

$y = 2(1 - \cos(t))$ for $t \in [0, 2\pi]$

a.i. Show that the gradient of the curve is given by $\cot\left(\frac{t}{2}\right)$.

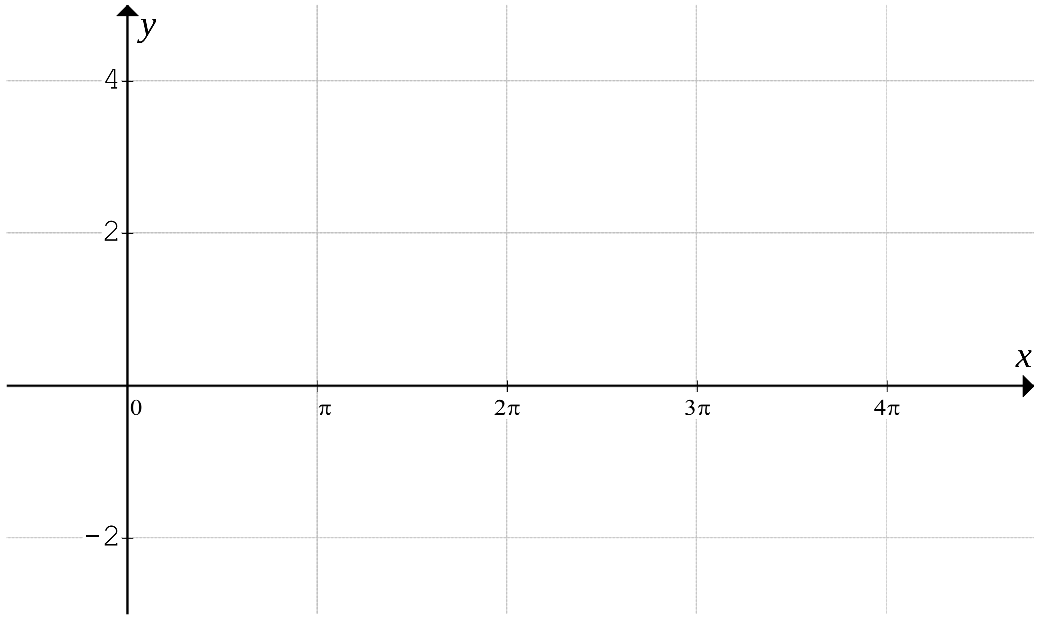
2 marks

ii. Find the coordinates on the curve where the gradient is parallel to the x-axis.

2 marks

b. Sketch the curve on the axis below, clearly indicating any turning points and the endpoints.

2 marks



c. A particle moves along the vector equation $\underset{\sim}{r}(t) = 2(t - \sin(t))\underset{\sim}{i} + 2(1 - \cos(t))\underset{\sim}{j}$ for $t \in [0, 2\pi]$. The speed of the particle at a time t can expressed as $u\sin(kt)$, find the values of u and k.

3 marks

d. The total area A of the region bounded by the curve and the x-axis can be expressed as $A = c\int_0^{2\pi} \sin^n(kt)\,dt$. Find the values of c, n and k.

2 marks

e. When the area A is rotated about the x-axis, it forms a solid of revolution. The volume V of this solid of revolution can be expressed as $V = p\int_0^{2\pi} \sin^m(kt)\,dt$. Write down the values of p and m.

1 mark

Question 2 (12 marks)

a. Given that $\sin\left(\frac{2\pi}{5}\right)=\frac{1}{4}\sqrt{2\left(5+\sqrt{5}\right)}$, show that $\cos\left(\frac{2\pi}{5}\right)=\frac{1}{4}\left(\sqrt{5}-1\right)$.

2 marks

b. Let $u=\frac{1}{2}\left(\sqrt{5}-1\right)+\frac{1}{2}\sqrt{2\left(5+\sqrt{5}\right)}\,i$

i. Express u in polar form.

1 mark

ii. Find $\text{Arg}\left(u^3\right)$.

1 mark

c. For what values of n is $\left(\sqrt{5}-1+\left(\sqrt{2\left(5+\sqrt{5}\right)}\right)i\right)^n$ a real number?

2 marks

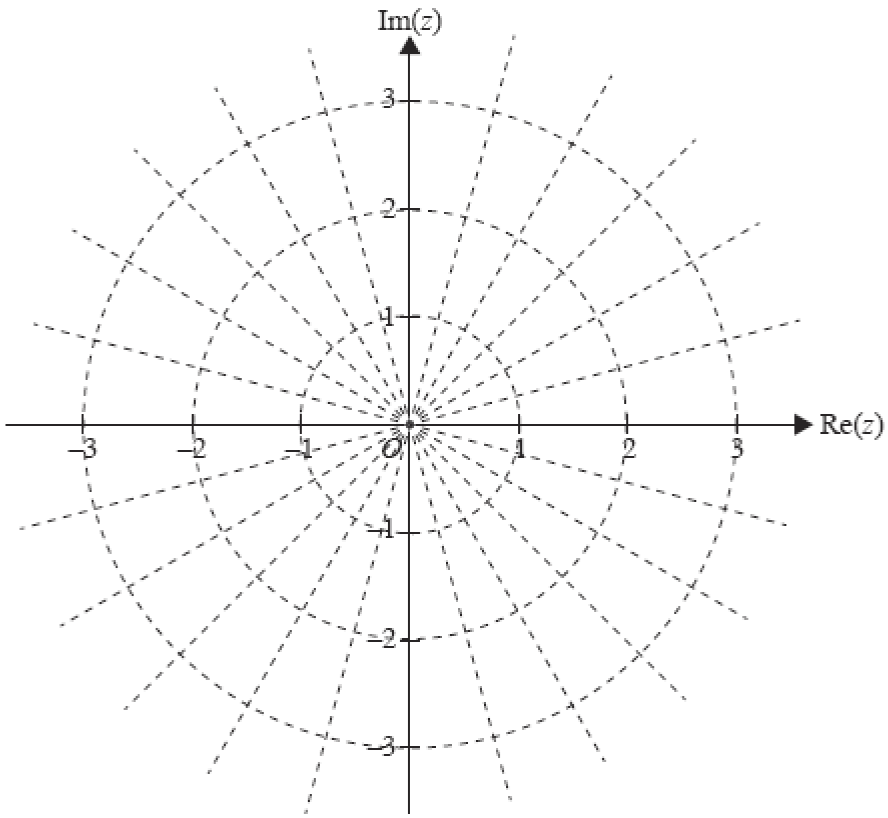
d. Solve the equation $z^5 - 32 = 0$, $z \in C$, stating all the roots in both rectangular and polar form.

3 marks

e. Let $S = \{z : |z| \le 2\}$, $R = \{z : 2\text{Re}(z) + (\sqrt{5}+1) \le 0\}$ and $T = \{z : \text{Arg}(z) \ge \frac{4\pi}{5}\}$

On the argand diagram below, sketch all the roots of $z^5 - 32 = 0$ and shade and describe the region corresponding to $\{z : z \in S \cap R \cap T\}$ where $z \in C$.

2 marks

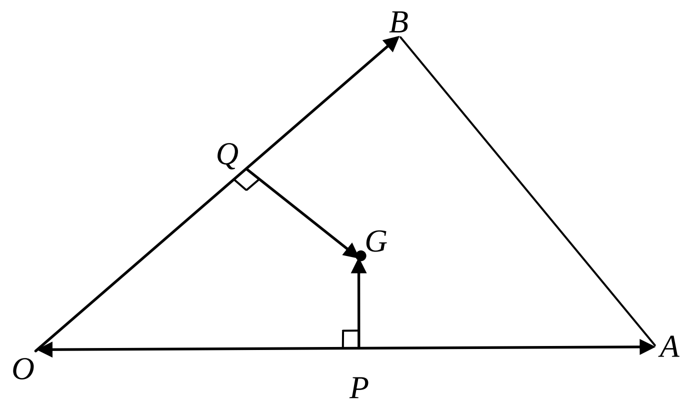


f. Find the area of the region $S \cap R \cap T$.

1 mark

Question 3 (10 marks)

a. In the triangle *OAB*, *P* and *Q* are the midpoints of *OA* and *OB* respectively and *G* is a point inside the triangle. The vectors $\overrightarrow{PG}$ and $\overrightarrow{QG}$ are perpendicular to the sides *OA* and *OB* respectively. Let $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$ and $\overrightarrow{OG} = \underset{\sim}{g}$.



i. Express $\overrightarrow{PG}$ and $\overrightarrow{QG}$ in terms of $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{g}$ and hence show that $\underset{\sim}{g}.\underset{\sim}{a} = \frac{1}{2}|\underset{\sim}{a}|^2$ and $\underset{\sim}{g}.\underset{\sim}{b} = \frac{1}{2}|\underset{\sim}{b}|^2$.

3 marks

ii. Let R be the midpoint of AB, show that RG is perpendicular to AB.

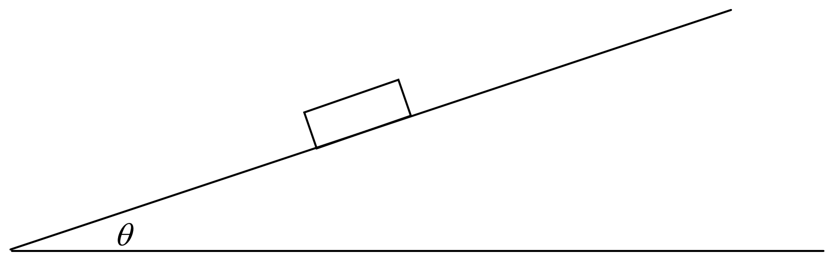
4 marks

b. Given the vectors $\underset{\sim}{u} = 3\underset{\sim}{i} - 2\underset{\sim}{j} - 4\underset{\sim}{k}$, $\underset{\sim}{v} = -2\underset{\sim}{i} + \underset{\sim}{j} + t\underset{\sim}{k}$, and $\underset{\sim}{w} = 9\underset{\sim}{i} - 7\underset{\sim}{j} - 8\underset{\sim}{k}$.
Find the value of the scalar t if the vectors $\underset{\sim}{u}$, $\underset{\sim}{v}$ and $\underset{\sim}{w}$ form a linearly dependant set of vectors.

3 marks

Question 4 (10 marks)

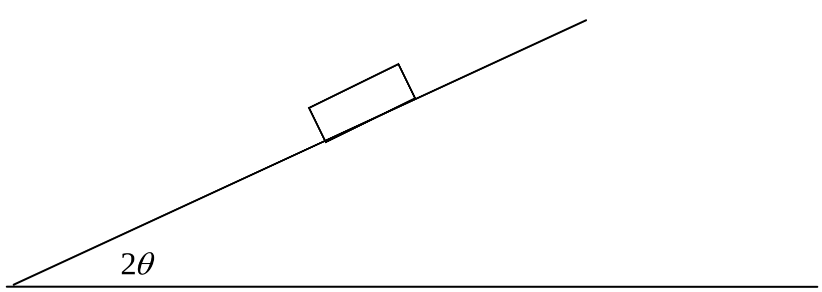
A block of mass m kg rests on a rough plane inclined at an angle of θ to the horizontal, where $0 < \theta < 30^0$. The coefficient of friction between the block and the plane is μ.



i. Show that $\mu = \tan(\theta)$. 2 marks

The plane is now raised and is inclined at an angle of 2θ to the horizontal.

ii. The block is now on the point of moving up the plane, when a horizontal force of P newtons is applied to the block. On the diagram below mark in labelling and describing all the forces acting on the block.



1 mark

iii. By resolving the forces, show that $P = mg\tan(3\theta)$.

6 marks

iv. If $P = 2mg$ find the value of θ in degrees and minutes.

1 mark

Question 5 (14 marks)

a. While serving in tennis, a tennis ball is thrown vertically upwards with a speed of 4 m/s from a point 1.6 metres above the ground. While travelling upwards the tennis ball is subjected to a resistance force of $0.00294v^2$ newtons, where v m/s is its speed at a time t seconds after it has been released. The mass of the tennis ball is 58.8 gm.

i. Show that the equation of motion of the tennis ball as it rises is given by

$$\ddot{x} = \frac{-\left(196 + v^2\right)}{20}$$

1 mark

ii. Using calculus, show that $v = 14\tan\left(\tan^{-1}\left(\frac{2}{7}\right) - \frac{7t}{10}\right)$

4 marks

iii. Hence find the time in seconds, correct to three decimal places when the tennis ball reaches its maximum height.

1 mark

iv. Write down a definite integral which gives the maximum height in metres, above ground level reached by the tennis ball.

2 marks

v. Determine correct to three decimal places, the maximum height in metres, above ground level that the tennis ball reaches.

1 mark

b. The position vector of the tennis ball after it has been hit by the server is given by $\underset{\sim}{r}(t)=36t\underset{\sim}{i}+4\sin(\pi t)\underset{\sim}{j}+h\cos(\pi t)\underset{\sim}{k}$ for $0\le t\le T$ where t is the time in seconds after the ball has been hit. The origin is the point from where the server stands when beginning to serve the ball. Dimensions are given in metres where $\underset{\sim}{i}$ is a unit vector of one metre horizontally forward, $\underset{\sim}{j}$ is a unit vector of one metre to the left and $\underset{\sim}{k}$ a unit vector of one metre vertically upwards.

i. Find the time T when the tennis ball hits the ground.

1 mark

ii. Find the position vector of the tennis ball when it strikes the ground.

1 mark

iii. Find the speed to the nearest km/hr at which the tennis ball was initially hit.

2 marks

iv. The net in tennis has a height of 1.07 metres and is 11.89 metres horizontally forward from the server. The ball just touches the top of the net as it passes over. Find correct to three decimal places the value of h, the initial height from which the tennis ball was served.

1 mark

END OF EXAMINATION

EXTRA WORKING SPACE

END OF QUESTION AND ANSWER BOOKLET

SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of triangle:	$\frac{1}{2}bc\sin(A)$
sine rule:	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule:	$c^2=a^2+b^2-2ab\cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$ $\cot^2(x)+1=\operatorname{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$ $\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$ $\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1,1]$	$[-1,1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$z = x + yi = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad -\pi < \text{Arg}(z) \le \pi$

$z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2)$ $\qquad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\qquad \underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \dfrac{d\underset{\sim}{r}}{dt} = \dfrac{dx}{dt}\underset{\sim}{i} + \dfrac{dy}{dt}\underset{\sim}{j} + \dfrac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

sliding friction: $F \le \mu N$

constant (uniform) acceleration:

$v = u + at$ $\qquad s = ut + \frac{1}{2}at^2$ $\qquad v^2 = u^2 + 2as$ $\qquad s = \frac{1}{2}(u+v)t$

acceleration: $\qquad a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\tfrac{1}{2}v^2\right)$

Calculus

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$

$$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$$

$$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$$

$$\frac{d}{dx}\left(\log_e\left(x\right)\right)=\frac{1}{x}$$

$$\int \frac{1}{x}dx=\log_e\left(x\right)+c,\ \text{ for }\ x>0$$

$$\frac{d}{dx}\left(\sin\left(ax\right)\right)=a\cos\left(ax\right)$$

$$\int \sin\left(ax\right)dx=-\frac{1}{a}\cos\left(ax\right)+c$$

$$\frac{d}{dx}\left(\cos\left(ax\right)\right)=-a\sin\left(ax\right)$$

$$\int \cos\left(ax\right)dx=\frac{1}{a}\sin\left(ax\right)+c$$

$$\frac{d}{dx}\left(\tan\left(ax\right)\right)=a\sec^2\left(ax\right)$$

$$\int \sec^2\left(ax\right)dx=\frac{1}{a}\tan\left(ax\right)+c$$

$$\frac{d}{dx}\left(\sin^{-1}\left(x\right)\right)=\frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$$

$$\frac{d}{dx}\left(\cos^{-1}\left(x\right)\right)=\frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$$

$$\frac{d}{dx}\left(\tan^{-1}\left(x\right)\right)=\frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$$

product rule: $\frac{d}{dx}\left(uv\right)=u\frac{dv}{dx}+v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

Euler's method If $\frac{dy}{dx}=f(x),\ x_0=a$ and $y_0=b$, then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf\left(x_n\right)$

END OF FORMULA SHEET

ANSWER SHEET

STUDENT NUMBER

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Words									

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