Neap:

Trial Examination 2015

# VCE Specialist Mathematics Units 3&4

## Written Examination 1

## Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

**Structure of Booklet**

| *Number of questions* | *Number of questions to be answered* | *Number of marks* |
|---|---|---|
| 9 | 9 | 40 |

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are not permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

**Materials supplied**

Question and answer booklet of 9 pages and a sheet of miscellaneous formulas.

Working space is provided throughout the booklet.

**Instructions**

Write your **name** and your **teacher's name** in the space provided above on this page.

All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2015 VCE Specialist Mathematics Units 3&4 Written Examination 1.

**Instructions**

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g$ m/s$^2$, where $g = 9.8$.

**Question 1 (4 marks)**

Find the equation of the tangent to the curve $y^2 = (2x + 3)^4$ at the point $\left(-\frac{1}{2}, 4\right)$. Give your answer in the form $y = mx + c$.

**Question 2 (3 marks)**

Solve the equation $\cos^2(x) = \sin^2(x) - 15\sin(x) + 8, 0 < x < 2\pi$.

**Question 3 (5 marks)**

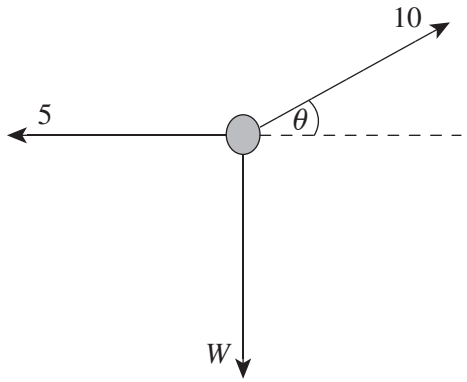
Consider $z = x + yi, x, y \in R, z \in C$.

**a.** Show that $\bar{z}^2 = x^2 - y^2 - 2xyi$. 2 marks

**b.** If $z^2 = \bar{z}^2$, show that $z$ must be either real or purely imaginary. 3 marks

**Question 4 (3 marks)**

The diagram below shows a particle of weight $W$ newtons that is held in equilibrium by two forces. The force of magnitude 5 newtons acts in a horizontal direction and the force of magnitude 10 newtons acts at an angle $\theta$ above the horizontal. All three forces act in the same vertical plane.




Find the mass of the particle.

**Question 5 (5 marks)**

A particle moves in a straight line so that its displacement, $x$, from a fixed origin, $O$, is related to its velocity, $v$, by the equation $v = 3\sqrt{4 - x^2}$.

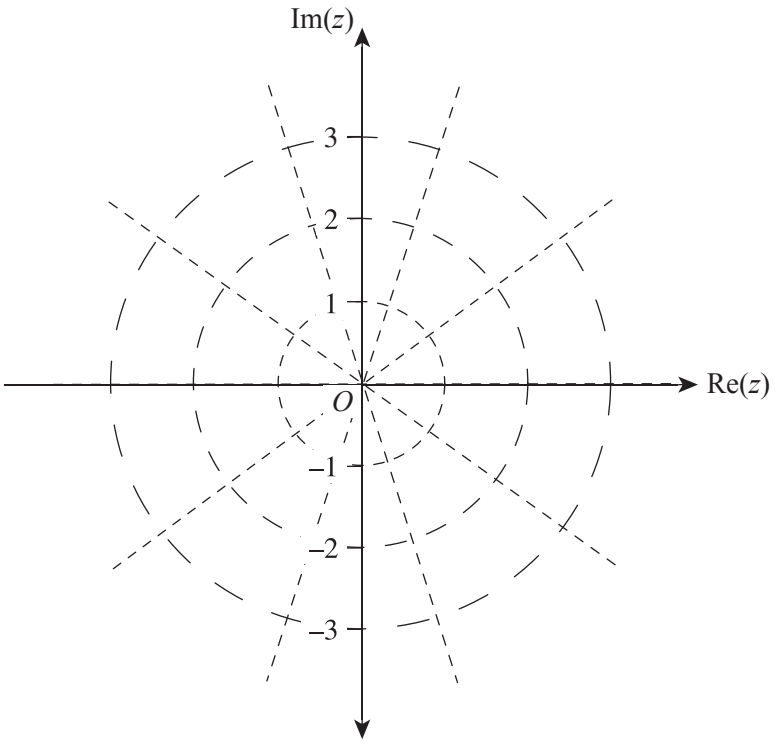
**a.** Show that the acceleration, $\ddot{x}$, of the particle is given by the differential equation $\ddot{x} = -9x$. 3 marks

**b.** Hence verify that the particle's displacement at time $t$ is given by $x = 2\sin(3t)$. 2 marks

**Question 6 (3 marks)**

Let $z = \cos(\theta) + i\sin(\theta), z \in C$.

**a.** On the Argand diagram below, plot the roots of the equation $z^5 = 1$. 1 mark




When plotted on an Argand diagram, the roots of the equation $z^5 = 1$ form a regular pentagon.

**b.** Find the area of the regular pentagon, giving your answer in the form $k\sin(\alpha)$. 2 marks

**Question 7 (6 marks)**

Find $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan^3(x)dx$. Give your answer in the form $a - b\log_e(c)$, where $a$, $b$ and $c$ are rational numbers.

**Question 8 (5 marks)**

The position vector of a particle at time $t$, $t \geq 0$ is defined by $\underset{\sim}{r}(t) = a\cos(2t)\underset{\sim}{i} + a\sin(2t)\underset{\sim}{j} + bt\underset{\sim}{k}$, $a, b > 0$.

**a.** Find the particle's velocity. 1 mark

**b.** Show that the particle moves in such a way that it always makes a fixed angle, $\theta$, with the $\underset{\sim}{k}$ direction. 4 marks

**Question 9 (6 marks)**

Consider the function $f$ defined by $f(x) = \dfrac{6}{x^2 + 2x + 2}$.

**a.** Show that the graph of $f$ has no vertical asymptotes. 1 mark

The graph of $f$ has an axis of symmetry at $x = k$.

**b.** Find the area enclosed by the graph of $f$, the $x$-axis and the lines $x = \dfrac{\sqrt{3} - 3}{3}$ and $x = k$. 5 marks

**END OF QUESTION AND ANSWER BOOKLET**