2015 VCE Specialist Mathematics 2 examination report

General comments

The 2015 Specialist Mathematics examination 2 comprised 22 multiple-choice questions (worth a total of 22 marks) and five extended-answer questions (worth a total of 58 marks). Most students made substantial attempts at the questions in Section 2.

This year there were five questions (Questions 3ci., 3d., 4c., 5di. and 5dii.) where students needed to establish a given result. It needs to be re-emphasised that in this sort of question all steps that lead to the given result must be clearly and logically set out. Students needed to provide a convincing and clear sequence of steps to obtain full credit.

The requirement to show working in questions worth more than one mark was complied with quite well this year, although a number of students did not show working for Questions 2aiv. and 2bi.

Exact answers were generally given where appropriate, although a few students moved from a correct exact answer to an incorrect decimal approximation (Questions 1c. and 3cii.).

The examination revealed areas of strength and weakness in student performance.

Areas of strength included:

- the use of CAS technology to solve equations that give both real and complex solutions, and evaluate definite integrals
- facility with complex number questions – most of Question 2 was attempted well
- the use of pencil to sketch graphs – few graphs were completed in pen, and most were relatively neat, but not always correct.

Areas of weakness included:

- poor presentation of graphs and graphs drawn over incorrect domains and ranges (Question 1d.), and circles drawn roughly (Question 2aii.)
- inaccurate transfer of graphs from a CAS screen to a scaled set of axes, preserving shape and endpoints (Question 1d.)
- failure to check whether all aspects of a question had been answered
- graphs drawn correctly but not labelled (Question 1d.)
- omission of the constant of integration or lack of detail showing its evaluation, particularly in a 'show that' question (Question 5dii.)
- dropping of $\underset{\sim}{i}$, $\underset{\sim}{j}$, $\underset{\sim}{k}$ in a vector question (Question 4c.), whereby an expression ends up as a mixture of vector and scalar quantities.

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Section 1

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	1	3	6	5	**84**	0	$\frac{x-2}{3}=\cos(t)$, $\frac{y-3}{2}=\sin(t)$, then eliminate t.
2	**70**	3	19	5	3	0	Creating a graph using CAS technology was the simplest way to answer this question.
3	15	5	10	**50**	19	0	
4	7	20	11	**43**	18	1	Options D and E had the correct asymptotes, but (5, 5) satisfied only option D.
5	2	**81**	7	7	3	0	z first simplifies to $z=\sqrt{2}\text{cis}\left(\frac{\pi}{12}\right)$
6	11	8	21	16	**43**	1	A sketch of each shows that option E is the line $x = 1$.
7	**63**	16	8	8	4	1	$z^{63}=\left(2\sqrt{3}\right)^{63}\text{cis}(21\pi)=\left(2\sqrt{3}\right)^{63}\text{cis}(\pi)$
8	9	11	**57**	6	17	1	
9	6	18	**47**	12	16	1	Option C simplifies to $r_2<1$
10	3	**56**	27	6	7	0	$u=3x+1$ results in option B.
11	2	5	18	**66**	8	0	
12	6	**76**	6	4	8	0	
13	7	15	26	**47**	6	1	The solution doesn't cross any gradient segments.
14	3	4	8	9	**76**	0	$\frac{d^2y}{dx^2}=-x\sin(x)+2\cos(x)$, therefore option E was correct.
15	**49**	4	10	7	29	1	Require $\underset{\sim}{F}.\hat{\underset{\sim}{w}}\ \hat{\underset{\sim}{w}}$
16	**23**	6	7	53	11	0	Being in equilibrium, the three vector forces defined add to $\underset{\sim}{0}$.
17	10	22	**48**	10	9	1	Application of the scalar product to $\overrightarrow{BA}$ and $\overrightarrow{BC}$ gives the required result.
18	4	**75**	8	10	2	0	Both $\underset{\sim}{i}$ and $\underset{\sim}{j}$ components must equate.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
19	8	20	65	4	2	0	$T-g=1\times 4.9$, $mg-T=m\times 4.9$, which solve to give m = 3 (option C).
20	4	60	11	12	13	1	$-11=5+a\times 16,\ a=-1$, $5\times 16-0.5\times 1\times 16^2=-48$, dist = 48
21	3	22	10	13	52	0	Horizontally: $F\cos(\theta)-\mu N=Ma$ Vertically: $N=Mg-F\sin(\theta)$, when combined give option E.
22	5	23	18	42	11	2	$\frac{dt}{dv}=-\frac{1}{\left(9.8+0.1v^2\right)}$, $t=-\frac{10}{\sqrt{98}}\tan^{-1}\left(\frac{v}{\sqrt{98}}\right)+c$, t = 0, v = 0 $c=\frac{10}{\sqrt{98}}\times\frac{\pi}{3}$ the required time, option D.

Section 2

Question 1a.

Marks	0	1	Average
%	25	75	0.8

$-\frac{\sin x\cos x}{y}$ or $-\frac{\sin 2x}{2y}$

This question was answered reasonably well. The main error was some answers were given only in terms of x. Some students moved from a correct answer involving x and y to an incorrect answer involving only x. Omission of the negative sign occurred occasionally.

Question 1bi.

Marks	0	1	Average
%	9	91	0.9

$\sqrt{2}$, 1

This question was well answered. Some students wrote 2 instead of $\sqrt{2}$ and others included $\pm$ alternatives in their answers.

Question 1bii.

Marks	0	1	Average
%	15	85	0.9

0, 0

This question was answered quite well. A few students had answers other than zero, while some students did not make it clear that both answers were zero.

Question 1c.

Marks	0	1	2	3	Average
%	5	9	29	57	2.4

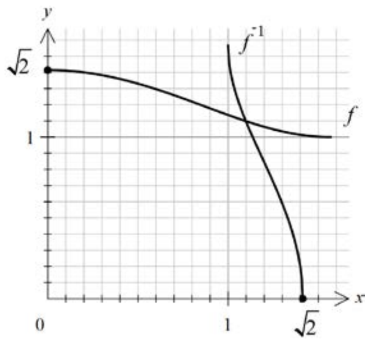
$f^{-1}(x) = \arcsin\sqrt{2-x^2}$, Domain $\left[1, \sqrt{2}\right]$, Range $\left[0, \frac{\pi}{2}\right]$

This question was answered fairly well. The main errors were the incorrect domain and/or range, or the omission of one or both. A small number of students gave the inverse relation by including $\pm$ in front of the square root.

Most students knew to interchange x and y as a first step.

Question 1d.

Marks	0	1	2	Average
%	35	29	36	1



This question was answered moderately well. Many students did not accurately transfer the graphs from a CAS screen to the axes provided. Of those who managed to draw the graphs correctly, a significant number did not label them. Incorrect location of endpoints and incorrect concavity were common.

Question 1e.

Marks	0	1	Average
%	34	66	0.7

1.099

A significant number of students did not give their answer correct to three decimal places.

Question 1fi.

Marks	0	1	2	Average
%	10	10	79	1.7

$$\int_0^1 \pi\left(2-\sin^2 x\right)dx$$

This question was answered reasonably well. Common errors included not squaring $f(x)$, incorrect terminals and the occasional omission of π and/or dx.

Question 1fii.

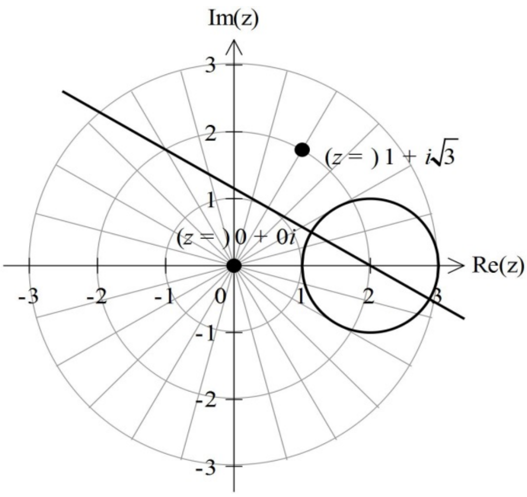
Marks	0	1	Average
%	29	71	0.7

5.4

This question was answered very well by students who set up the integral correctly. A number of students who set up the integral correctly did not include the π in their calculations and obtained 1.7.

Question 2ai.

Marks	0	1	2	Average
%	7	24	69	1.6



This question was answered quite well, but many students could not accurately position $1+i\sqrt{3}$, not realising it lay on the circle of radius 2. A number of students did not fully label both points.

Question 2aii.

Marks	0	1	2	Average
%	15	26	59	1.5

Refer to the diagram in Question 2ai.

Most students graphed the circle correctly, although some circles were poorly drawn.

A common error was to draw a straight line with a positive gradient. A number of students terminated their line at (2, 0). Few students seemed to realise that the required line was the perpendicular bisector of the line interval joining (0, 0) and $\left(1, \sqrt{3}\right)$.

Question 2aiii.

Marks	0	1	Average
%	38	62	0.6

$y=-\frac{1}{\sqrt{3}}x+\frac{2}{\sqrt{3}}$

This question was generally well answered. The most common error was the gradient given as positive.

Question 2aiv.

Marks	0	1	2	3	Average
%	31	14	12	44	1.7

$2+\frac{\sqrt{3}}{2}-\frac{1}{2}i,\ 2-\frac{\sqrt{3}}{2}+\frac{1}{2}i$

This question was reasonably well answered. Common errors were answers given in the wrong form and sign errors. Most students attempted to solve the equations of the line and circle simultaneously.

Question 2bi.

Marks	0	1	2	3	Average
%	44	25	10	21	1.1

$2\operatorname{cis}(\alpha),\ 2\operatorname{cis}(-\alpha)$

Most students attempted to apply the quadratic formula or complete the square, but few managed to find the values of z in polar form. Dealing with the discriminant proved to be a problem for many. A number of students left answers in cartesian form, and some erroneously converted correct cartesian form answers to $2\sqrt{2}\operatorname{cis}(\alpha)$ and $2\sqrt{2}\operatorname{cis}(-\alpha)$.

Question 2bii.

Marks	0	1	Average
%	77	23	0.3

$\frac{5\pi}{12}$

Many students did not attempt this question. Common errors were unsimplified expressions involving z, $-\frac{5\pi}{12}$ and $\frac{5\pi}{6}$.

Question 3a.

Marks	0	1	2	Average
%	20	16	64	1.5

$\frac{dy}{dx}=\frac{1}{2}\left(\frac{\sin t}{\cos^2 t}+\sin t\right)\times\frac{1}{\cos t}=\frac{\tan t}{2}\left(\sec^2 t+1\right)$ Equivalent forms of this answer were also allowed.

This question was reasonably well answered. Many students attempted a chain rule relation, but a number of these had x instead of t in what otherwise would have been a correct answer. Some students gave $\frac{dy}{dt}$ as their answer, while others first eliminated t to get y in terms of x, found $\frac{dy}{dx}$, and then expressed their answer in terms of t. The latter method was lengthy and more prone to errors. Some students did unnecessary further working out, attempted to simplify a correct answer and changed it to an incorrect answer.

Question 3b.

Marks	0	1	Average
%	36	64	0.7

$$\frac{7\sqrt{3}}{18}$$

This question was managed quite well by students who answered Question 3a. correctly.

Question 3ci.

Marks	0	1	Average
%	48	52	0.5

$$x=\sin t,\ y=\frac{1}{2}\sin t \times \tan t,\quad y=\frac{1}{2}\sin t \times \frac{\sin t}{\cos t},\quad y=\frac{1}{2}\sin t \times \frac{\sin t}{\sqrt{1-\sin^2 t}},\quad y=\frac{1}{2}x \times \frac{x}{\sqrt{1-x^2}}$$

Many responses to this question lacked the necessary steps. Some students assumed what was to be shown and then proceeded to show something else.

Question 3cii.

Marks	0	1	Average
%	45	55	0.6

$$\left[-\frac{\sqrt{3}}{2},\frac{\sqrt{3}}{2}\right]$$

This question was answered reasonably well. Common errors included $\left(-\frac{\sqrt{3}}{2},\frac{\sqrt{3}}{2}\right)$, $(-1, 1)$ and $\left(-\frac{\pi}{3},\frac{\pi}{3}\right)$.

Question 3d.

Marks	0	1	2	Average
%	24	15	61	1.4

Right-hand side

$$= \frac{2x^2}{\sqrt{1-x^2}} + x \times \frac{1}{2}\left(1-x^2\right)^{-\frac{1}{2}} \times -2x + \left(1-x^2\right)^{\frac{1}{2}} = \frac{x^2}{\sqrt{1-x^2}} + \frac{\sqrt{1-x^2}}{1} = \frac{x^2+1-x^2}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$$

Students were asked to simplify the right-hand side of the equation in this question. However, some students attempted to work on both sides of the given equation, usually integrating both sides. The second-last step of putting the two square root terms over a common denominator was occasionally omitted.

Question 3e.

Marks	0	1	2	3	Average
%	50	16	14	20	1.1

$$A = 4 \times \int_0^{\frac{\sqrt{3}}{2}} \frac{x^2}{2\sqrt{1-x^2}}\,dx = \left[\arcsin(x) - x\sqrt{1-x^2}\right]_0^{\frac{\sqrt{3}}{2}}, \quad A = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$$

Only a small number of students seemed to understand this 'hence' question. An antiderivative with terminals needed to be written down. The most common answer was an integral for the area, followed by its evaluation using CAS technology.

Question 4ai.

Marks	0	1	Average
%	13	87	0.9

150

Most students answered this question well. A number of students attempted to solve $\underset{\sim}{r}(t) = 60$, failing to realise that only the $\underset{\sim}{k}$ component was 60.

Question 4aii.

Marks	0	1	2	Average
%	52	15	33	0.8

47°

A significant number of students seemed not to know what 'angle of elevation' meant. A number found the complementary angle – the angle with the vertical. Others found angles made with the $\underset{\sim}{i}$ or $\underset{\sim}{j}$ directions. A small number of students tried to find the angle of elevation using the velocity vector. A common error was to assume that the helicopter, when at an altitude of 60 m, was directly above its initial location.

Question 4b.

Marks	0	1	Average
%	57	43	0.5

60

Most students didn't realise that the period of the horizontal motion was required. Many students gave lengthy answers to this question, which was not necessary.

Question 4c.

Marks	0	1	2	3	Average
%	19	9	21	52	2.1

$\dot{\underset{\sim}{r}}(t) = -\frac{5\pi}{6}\sin\left(\frac{\pi t}{30}\right)\underset{\sim}{i} + \frac{5\pi}{6}\cos\left(\frac{\pi t}{30}\right)\underset{\sim}{j} + \frac{2}{5}\underset{\sim}{k}$, $\ddot{\underset{\sim}{r}}(t) = -\frac{\pi^2}{36}\cos\left(\frac{\pi t}{30}\right)\underset{\sim}{i} - \frac{\pi^2}{36}\sin\left(\frac{\pi t}{30}\right)\underset{\sim}{j}$

$$\ddot{\underset{\sim}{r}}(t).\dot{\underset{\sim}{r}}(t) = \frac{5\pi^3}{216}\sin\left(\frac{\pi t}{30}\right)\cos\left(\frac{\pi t}{30}\right) - \frac{5\pi^3}{216}\cos\left(\frac{\pi t}{30}\right)\sin\left(\frac{\pi t}{30}\right) = 0$$

A significant number of students gave answers with x as the variable instead of t. Some students differentiated using CAS technology in degree mode, and a significant number omitted the $\underset{\sim}{k}$ component from the velocity. Often, $\underset{\sim}{i}$, $\underset{\sim}{j}$ or $\underset{\sim}{k}$ were just dropped in the midst of working. Some students simply asserted that $\ddot{\underset{\sim}{r}}(t).\dot{\underset{\sim}{r}}(t) = 0$, without setting out the scalar product to show it.

Question 4d.

Marks	0	1	2	Average
%	26	35	39	1.2

2.65

Most students knew that they needed to find $\left|\dot{\underset{\sim}{r}}(t)\right|$. Leaving out the $\underset{\sim}{k}$ component was a common error. Some students could not simplify $\left(-\frac{5\pi}{6}\sin\left(\frac{\pi t}{30}\right)\right)^2 + \left(\frac{5\pi}{6}\cos\left(\frac{\pi t}{30}\right)\right)^2$ using the Pythagorean identity.

Question 4e.

Marks	0	1	2	3	Average
%	26	16	6	53	1.9

20.6

This question was moderately well answered. Frequent errors occurred in finding $\underset{\sim}{r}(45)$, and many students attempted to find the distance using $\left|\underset{\sim}{r}(45)\right| - \left|\underset{\sim}{r}_{\text{tree}}\right|$ instead of $\left|\underset{\sim}{r}(45) - \underset{\sim}{r}_{\text{tree}}\right|$. Occasionally an exact value of $5\sqrt{17}$ was given instead of an answer correct to one decimal place.

Question 5a.

Marks	0	1	Average
%	22	78	0.8

$F = 250g\sin 10^\circ$, $F = 425$

This question was answered quite well. Most errors centred on incorrect trigonometry when resolving the weight force and calculators being in radian mode.

Question 5b.

Marks	0	1	2	Average
%	19	23	58	1.4

$250g\sin 10^\circ - 200 = 250a$, $a = 0.902$

This question was reasonably well answered. The most common error was to use the rounded value of 425 instead of $250g\sin 10^\circ$, when three decimal place accuracy was required.

Question 5c.

Marks	0	1	2	Average
%	15	11	75	1.6

$v^2 = 2 \times 0.902 \times 30$, $v = 7.36$

This question was generally answered well. A number of students found the time to descend 30 m first, then used the time to find the speed.

Question 5di.

Marks	0	1	2	Average
%	42	29	30	0.9

$v\dfrac{dv}{dx} = 1.4(7-v)$, $\dfrac{dx}{dv} = \dfrac{v}{1.4(7-v)}$, $1.4\dfrac{dx}{dv} = -1 + \dfrac{7}{7-v}$ division method

$$\begin{array}{r}
-1 \\
-v+7\overline{)\,v\quad} \\
\underline{v-7} \\
7
\end{array}$$

Many students managed to get the correct form for acceleration and inverted, but omitted the step to establish the final form, which could have been division or another valid method.

Question 5dii.

Marks	0	1	Average
%	48	52	0.5

$1.4x = \int -1 + \dfrac{7}{7-v}\,dv$, $1.4x = -v - 7\log_e(7-v) + c$, $0 = 0 - 7\log_e(7) + c$, $c = 7\log_e(7)$

$1.4x = -v - 7\log_e(7-v) + 7\log_e(7)$

In this ‘show that’ question, many students did not include the constant of integration or show its evaluation.

Question 5diii.

Marks	0	1	Average
%	47	53	0.6

2.7

A number of students did not attempt this question. A few students incorrectly substituted $v=-5$.

Question 5div.

Marks	0	1	2	3	Average
%	84	3	1	12	0.4

$\int_0^5 \frac{1}{1.4(7-v)}\,dv = 0.9$

Only a small number of students attempted this question. Some attempted to find the time using direct integration instead of a definite integral.