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Students Name:………………………………………

SPECIALIST MATHEMATICS UNITS 3 & 4

TRIAL EXAMINATION 1

2016

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 10 questions.
All questions should be answered in the spaces provided.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may **not** bring any notes or calculators into the exam.
Where more than one mark is allocated to a question, appropriate working must be shown.
An exact answer is required to a question unless otherwise specified.
Unless otherwise indicated, diagrams in this exam are not drawn to scale.
The acceleration due to gravity should be taken to have magnitude g m/s^2 where $g = 9.8$
Formula sheets can be found on pages 10-12 of this exam.

Question 1 (4 marks)

Let $z_1 = 2i$.

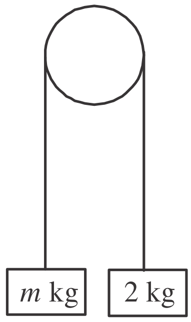
a. Simplify $\dfrac{\bar{z}_1}{1+i}$. 1 mark

b. Show that z_1 is a solution to the equation $z^3 - 3z^2 + 4z - 12 = 0$. 1 mark

c. Find all the solutions to this equation. 2 marks

Question 2 (4 marks)

A light inextensible string passes over a smooth pulley with particles of mass *m* kg and 2 kg attached to the ends of the string as shown.



When released, the acceleration of the 2 kg particle downwards is 1 ms^{-2}.

a. Find the value of *m*, in terms of *g*. 2 marks

The system is reset and the *m* kg particle is replaced by a 3 kg particle. The acceleration of the 2 kg particle is now *a* ms^{-2} upwards.

b. Find the value of *a*, in terms of *g*. 2 marks

Question 3 (3 marks)

Given that $f(x) = \arcsin(3x)$, find $f''\left(\frac{1}{6}\right)$.

Question 4 (3 marks)

Consider the relation $xy^2 + y\log_e(x) - 2y - 3 = 0, \quad x > 0$. Evaluate $\frac{dy}{dx}$ at the point (1, 3).

Question 5 (6 marks)

A company operates a telephone helpline for its clients. The waiting time, in minutes, for clients using the helpline is normally distributed with mean μ.
A random sample is taken of 100 of the company's clients who use the helpline and an approximate 95% confidence interval for μ is $15.02 < \mu < 16.98$.

a. Find the mean $\bar{x}$ and the standard deviation s for this sample. 2 marks

The company upgrades its telephone helpline by outsourcing it to a call centre business.
The waiting times, in minutes, for callers to this call centre are normally distributed with an advertised mean waiting time of 5.2 minutes and a standard deviation of 1.5 minutes.
After the upgrade, the company is suspicious that the waiting times on average are longer than those suggested by the advertising. It randomly samples 25 of its clients and finds that the mean waiting time is 5.8 minutes. Assume that the population standard deviation remains at 1.5 minutes.

b. **i.** Write down appropriate null and alternative hypotheses to test whether the mean waiting time is longer than that advertised by the call centre business. 1 mark

ii. Find the p value for this test. 2 marks

iii. Hence explain whether or not the null hypothesis should be rejected or not rejected at the 5% level of significance. 1 mark

Question 6 (3 marks)

Evaluate $\int_0^{\frac{\pi}{4}} \cos^2(x)\sin(2x)dx$.

Question 7 (3 marks)

The region in the first quadrant enclosed by the graph with equation $y = \frac{3}{\sqrt{x^2+4}}$, the coordinate axes and the line with equation $x = 2$, is rotated about the x–axis to form a solid of revolution.

a. Write down a definite integral that gives the volume of this solid of revolution. 1 mark

b. Find the volume of the solid of revolution. 2 marks

Question 8 (5 marks)

Consider the three vectors $\underset{\sim}{a} = 2\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$, $\underset{\sim}{b} = \underset{\sim}{i} + m\underset{\sim}{j} - 4\underset{\sim}{k}$ and $\underset{\sim}{c} = -\underset{\sim}{i} + 3\underset{\sim}{j}$ where $m \in R$.

a. Find the value of m such that $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}$. 1 mark

b. Consider the vector $\underset{\sim}{d}$ which has a magnitude of 6, is parallel to $\underset{\sim}{a}$, and runs in the same direction as $\underset{\sim}{a}$.

Find $\underset{\sim}{d}$. 2 marks

c. Find a value of m such that $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are **linearly dependent**. 2 marks

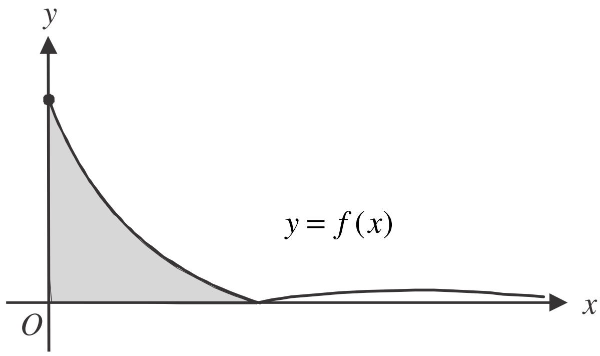
Question 9 (5 marks)

The solution to the differential equation $\frac{dy}{dx}=\frac{x\sqrt{x^2-1}}{e^{2y}}$, where $y=0$ when $x=1$, is given by $y=\log_e\sqrt{\frac{a}{b}(x^2-c)^{\frac{b}{a}}+c}$ where a, b and c are integers. Find the values of a, b and c.

Question 10 (4 marks)

Let $f:[0,\infty)\to R,\ f(x)=\left|\frac{x-1}{(x+1)(x^2+1)}\right|$.

The graph of *f* is shown below.



The region enclosed by the graph of *f* and the *x* and *y* axes is shaded. Find the area of this shaded region.

Specialist Mathematics Formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule	$c^2=a^2+b^2-2ab\cos(C)$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$	
$1+\tan^2(x)=\sec^2(x)$	$\cot^2(x)+1=\operatorname{cosec}^2(x)$
$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$	$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$
$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$	$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$
$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$	
$\sin(2x)=2\sin(x)\cos(x)$	$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

Circular (trigonometric) functions – continued

Function	$\sin^{-1}$(arcsin)	$\cos^{-1}$(arccos)	$\tan^{-1}$(arctan)
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\text{cis}(\theta)$	
$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \text{Arg}(z) \le \pi$
$z_1 z_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$
$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$\text{E}(aX + b) = a\text{E}(x) + b$ $\text{E}(aX + bY) = a\text{E}(x) + b\text{E}(Y)$ $\text{var}(aX + b) = a^2\text{var}(X)$
for independent random variables X and Y	$\text{var}(aX + bY) = a^2\text{var}(X) + b^2\text{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\bar{X}$	mean $\text{E}(\bar{X}) = \mu$ variance $\text{var}(\bar{X}) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n\,dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}\,dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$	$\int \frac{1}{x}\,dx=\log_e\lvert x\rvert+c$
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$	$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$	$\int \cos\ (ax)\,dx=\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=a\sec^2(ax)$	$\int \sec^2(ax)\,dx=\frac{1}{a}\tan(ax)+c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right)=\frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}\,dx=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right)=\frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right)=\frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2}\,dx=\tan^{-1}\left(\frac{x}{a}\right)+c$
	$\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$
	$\int (ax+b)^{-1}\,dx=\frac{1}{a}\log_e\lvert ax+b\rvert+c$
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx}=f(x),\ x_0=a$ and $y_0=b$, then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf(x_n)$
acceleration	$a=\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2}\sqrt{1+(f'(x))^2}\,dx$ or $\int_{t_1}^{t_2}\sqrt{(x'(t))^2+(y'(t))^2}\,dt$

Vectors in two and three dimensions

$\underset{\sim}{r}=x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k}$
$\lvert\underset{\sim}{r}\rvert=\sqrt{x^2+y^2+z^2}=r$
$\dot{\underset{\sim}{r}}=\frac{d\underset{\sim}{r}}{dt}=\frac{dx}{dt}\underset{\sim}{i}+\frac{dy}{dt}\underset{\sim}{j}+\frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r_1}.\underset{\sim}{r_2}=r_1r_2\cos(\theta)=x_1x_2+y_1y_2+z_1z_2$

Mechanics

momentum	$\underset{\sim}{p}=m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R}=m\underset{\sim}{a}$