

# Year 12 *Trial Exam Paper*

## 2016

### SPECIALIST MATHEMATICS

#### Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

**STUDENT NAME:**

### QUESTION AND ANSWER BOOK

#### Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring sheets of paper, notes of any kind or white out liquid/tape into the examination.
- Calculators are not permitted in this examination.

#### Materials provided

- The question and answer book of 15 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

#### Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

**Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination.**

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**Question 1** (4 marks)

- a.** Find the values of  $a$  and  $b$  given that  $2i$  is a solution to the equation  $z^2 + (a + bi)z + bi = 0$ .

2 marks

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- b.** Hence find the other solution to the equation.

2 marks

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**Question 2** (3 marks)

At time  $t$  seconds the velocity of a moving particle is given by  $\underline{v}(t) = \left(\frac{1}{1+t^2}\right)\underline{i} + \underline{j}$ ,  $t \geq 0$ .

- a. Find the position vector  $\underline{r}(t)$  of the particle at time  $t$  given that  $\underline{r}(0) = \underline{j}$ .

2 marks

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- b. Hence find the Cartesian equation of the path of the particle.

1 mark

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**Question 3** (3 marks)

Find the gradient of the tangent to the curve with equation  $(x - y)^2 - \log_e(x) = 1$  at the point  $(1, 2)$ .

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**Question 4** (3 marks)

Find all real solutions of the equation  $\tan(x) = \cot(2x)$  for  $x \in [0, \pi]$ .

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**Question 5** (5 marks)

For students at Academia University, the length of time spent studying at home each week is normally distributed with a standard deviation of 40 minutes.

The length of time the same students spend involved in recreational activities each week is independent of the time spent studying at home each week and is normally distributed with a mean of 190 minutes and a standard deviation of 30 minutes.

- a.** Calculate the standard deviation of the total time that students at Academia University spend studying at home and being involved in recreational activities each week.

2 marks

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A sample of 64 students from Academia University spent a mean time of 719 minutes studying at home each week.

- b.** The university claims that the mean time spent studying at home each week by its students is 730 minutes.
- i.** State appropriate null and alternative hypotheses for the mean time spent studying at home each week by students.

1 mark

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- ii.** Test the university's claim using the 0.05 level of significance.

2 marks

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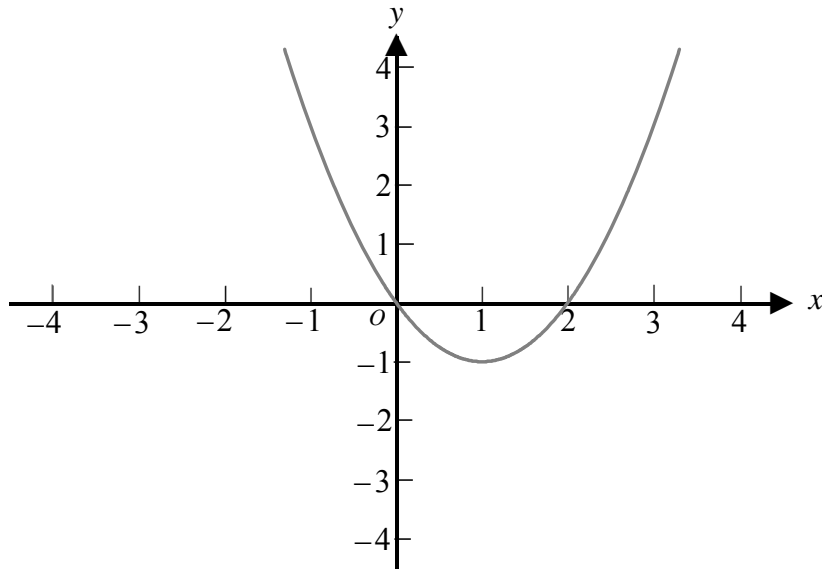
**Question 7** (8 marks)

- a. The graph of  $f(x) = x^2 - 2x$  is shown below.

Sketch the graph of  $y = \frac{1}{|f(x)|}$  on the axes below.

Label all asymptotes with their equations and label all stationary points with their coordinates.

3 marks



- b.** Find an anti-derivative of  $\frac{1}{x^2 - 2x}$ .

4 marks

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- c.** Hence calculate the area bounded by the curve  $y = \frac{1}{|f(x)|}$ , the  $x$ -axis and the lines  $x = 3$  and  $x = 4$ . Express your answer in the form  $\frac{1}{2} \log_e \left( \frac{a}{b} \right)$ , where  $a, b \in Z$ .

1 mark

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**Question 8** (3 marks)

A skydiver releases her parachute when falling at  $36 \text{ ms}^{-1}$  and is immediately subjected to a retardation of  $0.01v^{\frac{3}{2}} \text{ ms}^{-2}$ .

Calculate the time taken for the speed of the parachutist to reduce from  $36 \text{ ms}^{-1}$  to  $9 \text{ ms}^{-1}$ .

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**Question 9** (3 marks)

Consider the three vectors  $\underline{a} = \underline{i} + 2\underline{j} - \underline{k}$ ,  $\underline{b} = 2\underline{i} - \underline{j} + 3\underline{k}$  and  $\underline{c} = p\underline{i} + 2\underline{j} + 2\underline{k}$ , where  $p \in \mathbb{R}$ .

Find the value of  $p$  if  $\underline{a}$ ,  $\underline{b}$  and  $\underline{c}$  are linearly dependent.

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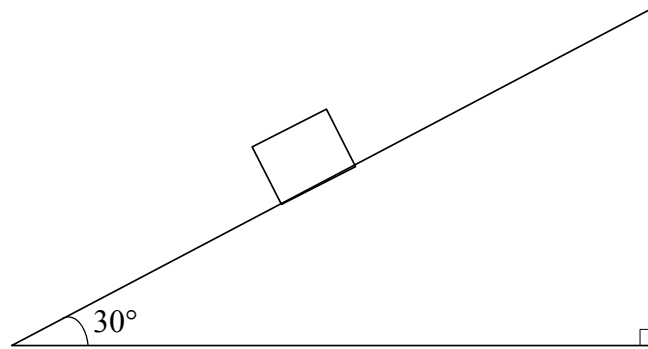
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**Question 10** (4 marks)

A box of mass 10 kg is sliding down a rough plane inclined at an angle of  $30^\circ$  to the horizontal. The friction force between the box and the plane is  $g\sqrt{3}$  and a girl is applying a force of  $S$  newtons on the box in the direction up and parallel to the plane.

- a. On the diagram below, show all the forces acting on the box and label them.

1 mark



- b. Find the acceleration of the box if  $S = 3g$ .

2 marks

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- c. Find the value of  $S$  required for the box to move down the plane at a constant speed.

1 mark

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**END OF QUESTION AND ANSWER BOOKLET**

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