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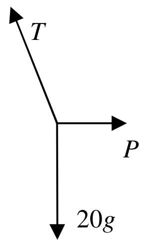
2016 VCAA Specialist Mathematics Exam 1 Solutions

Q1a



Q1b $\sin\theta = \frac{1}{\frac{5}{3}} = \frac{3}{5}$

Q1c $T\cos\theta = 20g$, $T = \frac{20g}{\cos\theta} = \frac{20g}{\frac{4}{5}} = 25g = 245$ N

Q2 $\sigma = \sqrt{16} = 4$, sample size $n = 25$,

sample mean $\bar{x} = \frac{2625}{25} = 105$ grams, .: 95% confidence interval

$\approx \left(\bar{x} - 2\frac{\sigma}{\sqrt{n}}, \ \bar{x} + 2\frac{\sigma}{\sqrt{n}}\right) = \left(105 - \frac{8}{5}, 105 + \frac{8}{5}\right) = (103.4, 106.6)$

Q3 $\cos y + y\sin x = x^2$, $\frac{d}{dx}(\cos y + y\sin x) = \frac{d}{dx}x^2$

$-\sin y\frac{dy}{dx} + y\cos x + \sin x\frac{dy}{dx} = 2x$,

$(-\sin y + \sin x)\frac{dy}{dx} = 2x - y\cos x$, .: $\frac{dy}{dx} = \frac{2x - y\cos x}{\sin x - \sin y}$

At $\left(0, \frac{-\pi}{2}\right)$, $m_T = \frac{dy}{dx} = \frac{\frac{\pi}{2}}{1} = \frac{\pi}{2}$, .: $m_N = -\frac{2}{\pi}$

Equation of the normal at $\left(0, \frac{-\pi}{2}\right)$:

$y + \frac{\pi}{2} = -\frac{2}{\pi}x$, $y = -\frac{2}{\pi}x - \frac{\pi}{2}$

Q4 $x = \tan^{-1} t$, surface area $A(t) = 6(\tan^{-1} t)^2$ mm^2,

$\frac{dA}{dt} = \frac{12\tan^{-1} t}{1 + t^2}$

When $t = 1$, $\frac{dA}{dt} = \frac{12\tan^{-1} 1}{1 + 1^2} = 6 \times \frac{\pi}{4} = \frac{3\pi}{2}$ mm^2/day

Q5a $\hat{b} = \frac{1}{\sqrt{14}}\left(\tilde{i} - 2\tilde{j} + 3\tilde{k}\right)$ Scalar resolute: $\tilde{a}.\hat{b} = \frac{-13}{\sqrt{14}}$,

vector resolute: $(\tilde{a}.\hat{b})\tilde{b} = -\frac{13}{14}\left(\tilde{i} - 2\tilde{j} + 3\tilde{k}\right)$

Q5b Linear dependent, let $\tilde{c} = m\tilde{a} + n\tilde{b}$

.: $3m + n = 1$ …… (1)

$5m - 2n = 0$ …… (2) and $-2m + 3n = d$

(1) − (2) : $-2m + 3n = 1$, .: $d = 1$

Q6 $\frac{(1 - \sqrt{3}i)^4}{1 + \sqrt{3}i} = \frac{\left(2\text{cis}\left(-\frac{\pi}{3}\right)\right)^4}{2\text{cis}\frac{\pi}{3}} = \frac{8\text{cis}\left(-\frac{4\pi}{3}\right)}{\text{cis}\frac{\pi}{3}} = 8\text{cis}\left(-\frac{5\pi}{3}\right) = 8\text{cis}\frac{\pi}{3}$

$= 4 + 4\sqrt{3}i$

Q7 $\frac{dy}{dx} = x(x^2 + 2)^{\frac{1}{2}}$, arc length $= \int_0^2 \sqrt{1 + x^2(x^2 + 2)}\,dx$

$= \int_0^2 \sqrt{(x^2 + 1)^2}\,dx = \int_0^2 (x^2 + 1)dx = \left[\frac{x^3}{3} + x\right]_0^2 = \frac{8}{3} + 2 = \frac{14}{3}$

Q8a $\tilde{r} = (3\sin 2t - 2)\tilde{i} + (3 - 2\cos 2t)\tilde{j}$

$\tilde{v} = \frac{d\tilde{r}}{dt} = (6\cos 2t)\tilde{i} + (4\sin 2t)\tilde{j}$

speed $= |\tilde{v}| = \sqrt{(6\cos 2t)^2 + (4\sin 2t)^2} = \sqrt{36\cos^2 2t + 16\sin^2 2t}$

$= \sqrt{20\cos^2 2t + 16} = 4\sqrt{\frac{5}{4}\cos^2 2t + 1}$ m/s

Q8b When $t = \frac{\pi}{12}$, speed $= 4\sqrt{\frac{5}{4}\left(\frac{\sqrt{3}}{2}\right)^2 + 1} = 4\sqrt{\frac{15}{16} + 1} = \sqrt{31}$ m/s

Q8c $\tilde{v} = (6\cos 2t)\tilde{i} + (4\sin 2t)\tilde{j}$,

$\tilde{a} = \frac{d\tilde{v}}{dt} = (-12\sin 2t)\tilde{i} + (8\cos 2t)\tilde{j}$

Net force $\tilde{F} = m\tilde{a} = (-36\sin 2t)\tilde{i} + (24\cos 2t)\tilde{j}$

$|\tilde{F}| = \sqrt{(-36\sin 2t)^2 + (24\cos 2t)^2} = 4\sqrt{81\sin^2 2t + 36\cos^2 2t}$

$= 4\sqrt{45\sin^2 2t + 36}$, .: maximum magnitude $= 36$ N when $\sin 2t = 1$

Q9 $\cos(x - y) = \cos x\cos y + \sin x\sin y = \frac{3}{5}$

$\tan x\tan y = \frac{\sin x\sin y}{\cos x\cos y} = 2$, .: $\sin x\sin y = 2\cos x\cos y$

.: $3\cos x\cos y = \frac{3}{5}$, .: $\cos x\cos y = \frac{1}{5}$

$\cos(x + y) = \cos x\cos y - \sin x\sin y = -\cos x\cos y = -\frac{1}{5}$

Q10 $\sqrt{2 - x^2}\frac{dy}{dx} = \frac{1}{2 - y}$

$\int (2 - y)dy = \int \frac{1}{\sqrt{2 - x^2}}dx$, $\frac{(2 - y)^2}{-2} = \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) + c$

Given $y(1) = 0$, .: $-2 = \frac{\pi}{4} + c$, $c = -2 - \frac{\pi}{4}$

$\frac{(2 - y)^2}{-2} = \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) - 2 - \frac{\pi}{4}$, $(2 - y)^2 = \frac{\pi}{2} + 4 - 2\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)$

$2 - y = \sqrt{\frac{\pi}{2} + 4 - 2\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)}$ will satisfy $y(1) = 0$

.: $y = 2 - \sqrt{\frac{\pi}{2} + 4 - 2\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)}$.

Please inform mathline@itute.com re conceptual and/or mathematical errors