Neap:

Trial Examination 2016

VCE Specialist Mathematics Units 3&4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 8 pages.

Formula sheet.

Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (2 marks)

Find the value of $\int_0^{\frac{\pi}{4}} \frac{\sec^2(x)}{1 + \tan(x)} dx$.

Question 2 (3 marks)

The position vector of a particle of mass 0.25 kg at time t seconds, $t \geq 0$, is given by $\underset{\sim}{r}(t) = (t^4 - 2t^2)\underset{\sim}{i} + (4t^3 - t^4)\underset{\sim}{j}$.

a. Find an expression for the momentum $\underset{\sim}{p}$ of the particle at time t. 1 mark

b. A force $\underset{\sim}{F}$ acts on the particle at time t.
Find when $\underset{\sim}{F}$ acts in the direction of unit vector $\underset{\sim}{j}$. 2 marks

Question 3 (3 marks)

Solve the equation $\cos\left(\frac{x}{2}\right) = \sin\left(\frac{x}{4}\right)$, $0 \leq x \leq 4\pi$.

Question 4 (6 marks)

Consider $P(z) = z^3 + z^2 + bz + 12, b \in R, z \in C$.

a. Given that $P(1 - \sqrt{3}i) = 0$, solve the equation $P(z) = 0$. 3 marks

b. Find the smallest positive integer, k, such that $(1 - \sqrt{3}i)^k \in R$ and, for this value of k, state the value of $(1 - \sqrt{3}i)^k$. 3 marks

Question 5 (4 marks)

The study scores of a large population of Victorian Specialist Mathematics students have a mean of 30 and a standard deviation of 7. A random sample of 100 students is to be selected from the population. The sample mean is denoted by $\bar{X}$.

a. State the mean of $\bar{X}$. 1 mark

b. Find the standard deviation of $\bar{X}$. 1 mark

c. The sample size is now increased by a factor of 4.
Determine how the mean of $\bar{X}$ and the standard deviation of $\bar{X}$ would change as a result of this increase in sample size. 2 marks

Question 6 (3 marks)

If $y = \cos^{-1}\left(\frac{2}{x}\right)$, $x > 2$, show that $\frac{dy}{dx} = \frac{2}{x\sqrt{x^2 - 4}}$.

Question 7 (9 marks)

The acceleration of a particle moving in the x–y plane is $-g\underset{\sim}{\text{j}}$. At time $t = 0$, the particle leaves the point with position $\underset{\sim}{\text{r}}(0) = h\underset{\sim}{\text{j}}$ with velocity $\dot{\underset{\sim}{\text{r}}}(0) = V\cos(\theta)\underset{\sim}{\text{i}} + V\sin(\theta)\underset{\sim}{\text{j}}$.

a. Show that the particle's position vector at time t is given by

$\underset{\sim}{\text{r}}(t) = V\cos(\theta)t\underset{\sim}{\text{i}} + \left(V\sin(\theta)t - \frac{1}{2}gt^2 + h\right)\underset{\sim}{\text{j}}$. 4 marks

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b. Show that the particle's path is given by $y = h + \tan(\theta)x - \frac{g\sec^2(\theta)}{2V^2}x^2$. 2 marks

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When a projectile is fired horizontally with a speed of U m/s from the top of a cliff of height h metres above sea level, the projectile hits a stationary target in the water. In addition, if the projectile is fired from the same position with a speed of U m/s, but with an angle of elevation of $\tan^{-1}(3)$, the projectile also hits the target.

c. Find, in terms of U and g, an expression for the horizontal distance, x metres, travelled by the projectile. 3 marks

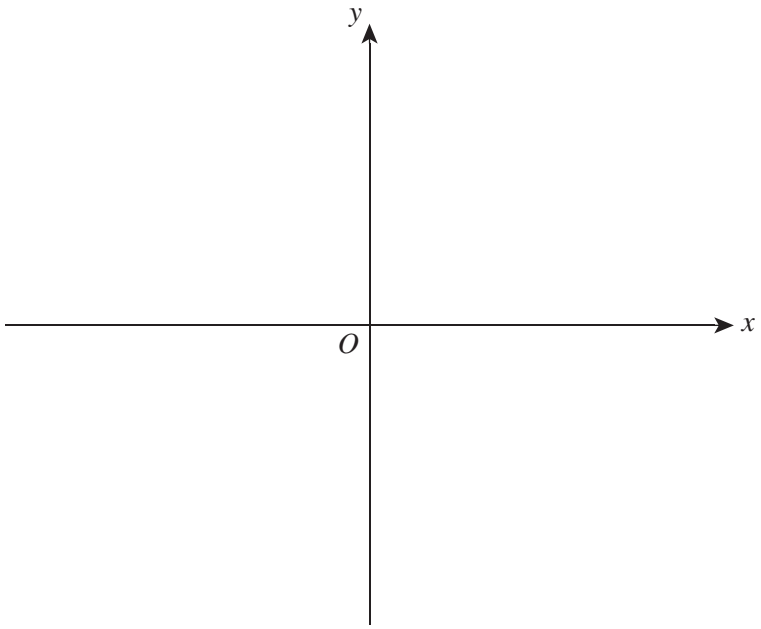
Question 8 (3 marks)

Show that the vectors $\underset{\sim}{a} = \underset{\sim}{i} + 2\underset{\sim}{j}$, $\underset{\sim}{b} = -\underset{\sim}{i} + 5\underset{\sim}{k}$ and $\underset{\sim}{c} = \underset{\sim}{i} + 6\underset{\sim}{j} + 10\underset{\sim}{k}$ are linearly dependent.

Question 9 (7 marks)

The curve C has equation $y = \dfrac{x^2}{x-2}$.

a. On the set of axes below, sketch C, labelling the equations of any asymptotes and the coordinates of any axial intercepts and turning points. 4 marks



b. **Hence**, find the set of values for p such that the equation $x^2 = p(x^2 - 4)$ has no real roots. 3 marks

END OF QUESTION AND ANSWER BOOKLET