Solution Pathway

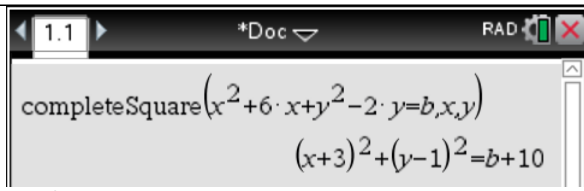
Below are sample answers. Please consider the merit of alternative responses.

Section A: Multiple-choice Answers

1.	**E**	**6.**	**E**	**11.**	**C**	**16.**	**A**
2.	**A**	**7.**	**A**	**12.**	**E**	**17.**	**D**
3.	**D**	**8.**	**B**	**13.**	**D**	**18.**	**D**
4.	**C**	**9.**	**B**	**14.**	**C**	**19.**	**A**
5.	**B**	**10.**	**D**	**15.**	**E**	**20.**	**E**

Section A: Multiple-choice Solutions

MC1

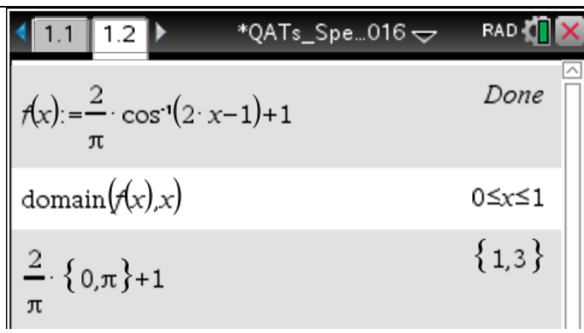


$\Rightarrow r^2 = 4 = b + 10$

$\therefore a = -3$ and $b = -6$

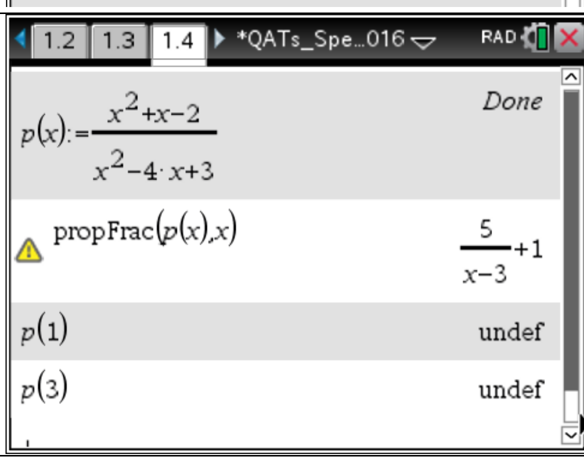
Answer E

MC2

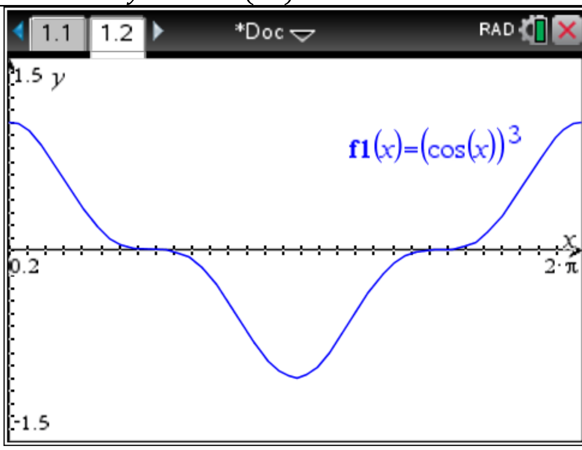


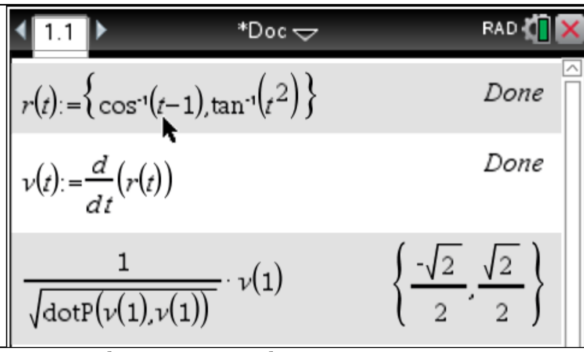
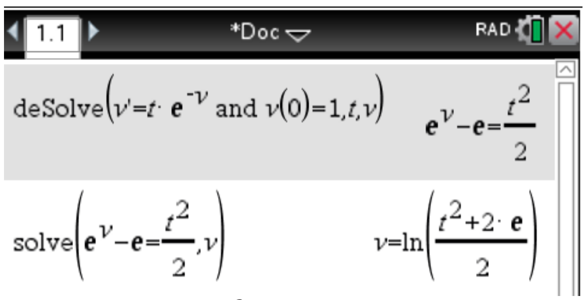
Answer A

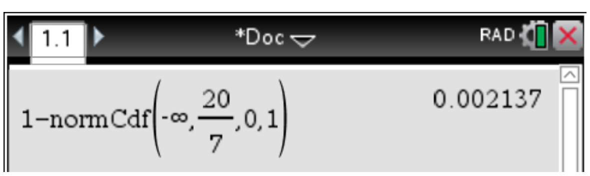
MC3



Answer D

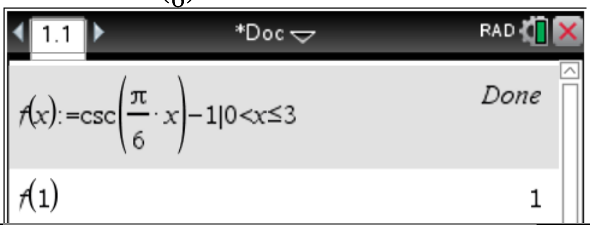
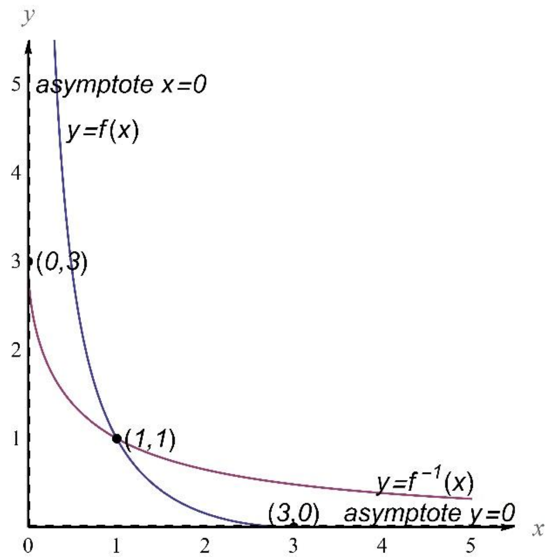
MC4	$\text{expand}\left(\dfrac{3\cdot x+1}{(x+2)^2\cdot(x^2+9)}\right) \quad \dfrac{-19\cdot x}{169\cdot(x^2+9)}+\dfrac{103}{169\cdot(x^2+9)}+\dfrac{19}{169\cdot(x+2)}-\dfrac{5}{13\cdot(x+2)^2}$	**Answer C**
MC5	$\lvert z-c\rvert = r \Rightarrow \lvert z-c\rvert^2 = r^2 \Rightarrow (z-c)\overline{(z-c)} = r^2 \Rightarrow (z-c)(\bar{z}-\bar{c}) = r^2$	**Answer B**
MC6	For a polynomial with real coefficients, it follows by the conjugate root theorem, that complex roots must occurs in conjugate pairs, so will always be even in number.	**Answer E**
MC7	The solution (i.e. integral curve) of the differential equation is independent of the y −coordinate, and look like a cosine with period π. Therefore $y = A\cos(2x)$ with $A > 0$.	**Answer A**
MC8	 $4\int_0^{\frac{\pi}{2}} \cos^3(x)\ dx = 4\int_0^{\frac{\pi}{2}} (1-\sin^2(x))\cos(x)\,dx$ $= 4\int_{\sin(0)}^{\sin\left(\frac{\pi}{2}\right)} (1-u^2)du\ = 4\int_0^1 (1-u^2)du$	**Answer B**
MC9	$V = \pi\int_0^{\cos^{-1}(1)} (x(y))^2 dy = \pi\int_0^{\cos^{-1}(1)} \left(\dfrac{\cos(y)}{3}\right)^2 dy = \dfrac{\pi}{18}\int_0^{\frac{\pi}{2}} (1+\cos(2y))dy$	**Answer B**
MC10	$y_1 = y_0 + hf(x_0) = 1 + h(\sec(0)) = 1 + h$	**Answer D**
MC11	$\left(\hat{\underset{\sim}{b}}\cdot\hat{\underset{\sim}{a}}\right)\hat{\underset{\sim}{a}}$ is not necessarily parallel to $\left(\hat{\underset{\sim}{a}}\cdot\hat{\underset{\sim}{b}}\right)\hat{\underset{\sim}{b}}$ so they would not necessarily be equal.	**Answer C**
MC12	1.4 1.5 1.6 *QATs_Spe…016 RAD $a:=[1\ \ 1\ \ 2] \quad [1\ \ 1\ \ 2]$ $b:=[2\ \ 1\ \ -2] \quad [2\ \ 1\ \ -2]$ $\dfrac{\text{dotP}(a,b)}{\sqrt{\text{dotP}(b,b)}} \quad \dfrac{-1}{3}$	**Answer E**
MC13	$\lvert\underset{\sim}{a}+\underset{\sim}{b}\rvert^2 - \lvert\underset{\sim}{a}-\underset{\sim}{b}\rvert^2 = (\underset{\sim}{a}+\underset{\sim}{b})\cdot(\underset{\sim}{a}+\underset{\sim}{b}) - (\underset{\sim}{a}-\underset{\sim}{b})\cdot(\underset{\sim}{a}-\underset{\sim}{b})$ $= (\underset{\sim}{a}\cdot\underset{\sim}{a} + 2\underset{\sim}{a}\cdot\underset{\sim}{b} + \underset{\sim}{b}\cdot\underset{\sim}{b}) - (\underset{\sim}{a}\cdot\underset{\sim}{a} - 2\underset{\sim}{a}\cdot\underset{\sim}{b} + \underset{\sim}{b}\cdot\underset{\sim}{b})$ $= 4\underset{\sim}{a}\cdot\underset{\sim}{b}$	**Answer D**
MC14	Direction is $\dfrac{1}{\lvert\dot{\underset{\sim}{r}}(1)\rvert}\dot{\underset{\sim}{r}}(1)$	**Answer C**

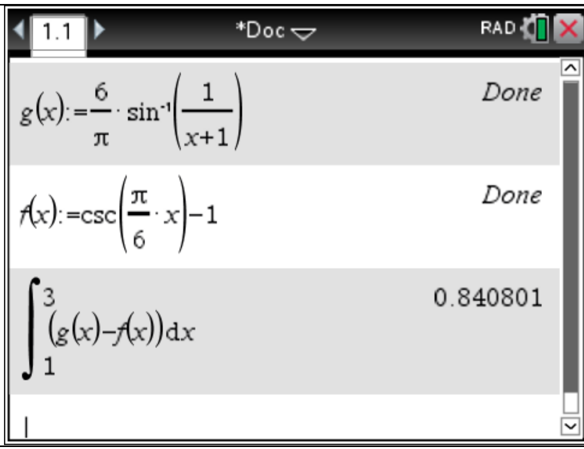
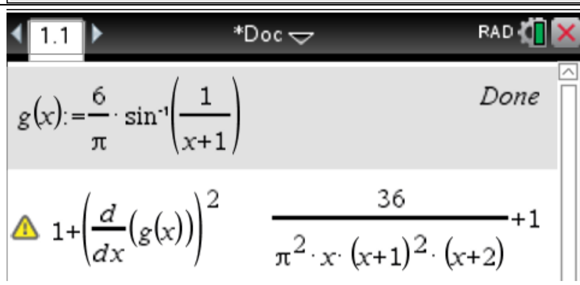
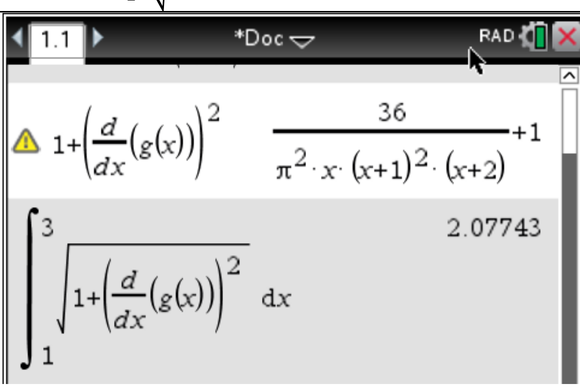
		
MC15	$F = 10\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}v^2\right) = 5\frac{\mathrm{d}}{\mathrm{d}x}(1+2x^2-x^3) = 5(4x-3x^2)$	**Answer E**
MC16	$\frac{\mathrm{d}v}{\mathrm{d}t} = te^{-v}, v(0) = 1.$  $\therefore v(t) = \log_e\left(e + \frac{t^2}{2}\right)$	**Answer A**

MC17	$2T\sin(60°) = 6g \Rightarrow T = 2\sqrt{3}g$	**Answer D**
MC18	$E(5X-3Y) = 5E(X) - 3E(Y) = 5\times 12 - 3\times 4 = 48$ $\mathrm{Var}(5X-3Y) = 5^2\mathrm{Var}(X) + (-3)^2\mathrm{Var}(Y) = 25\times 9 + 9\times 4 = 261$ $\sigma = \sqrt{\mathrm{Var}(5X-3Y)} = 3\sqrt{29}$	**Answer D**
MC19	$\bar{X} \sim N\left(\mu = 30, \sigma = \frac{7}{5}\right)$ $\Pr(\bar{X} > 34) = \Pr\left(Z > \frac{20}{7}\right) = 1 - \Pr\left(Z < \frac{20}{7}\right) \approx 0.0021$ 	**Answer A**
MC20	An error of the second type is a “false negative”, accepting the null hypothesis when this hypothesis is incorrect.	**Answer E**

Section B: Extended Answer Solutions

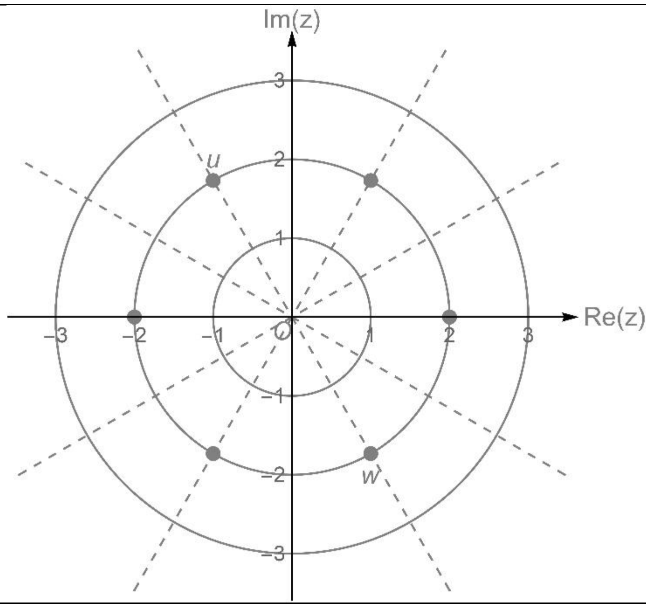
Question 1 (11 marks)

a.	$f(1) = \text{cosec}\left(\frac{\pi}{6}\right) - 1 = 1$ 	(A1)
b.		(A1) Axial intercepts. (A1) Point of intersection. (A1) Graphs with correct shape and labels. (A1) Asymptotes correct with labels.
c.	$x = f^{-1}(f(x)) \Rightarrow x = k\sin^{-1}\left(\frac{1}{\text{cosec}\left(\frac{\pi}{6}x\right) - 1 + 1}\right)$ $x = k\sin^{-1}\left(\sin\left(\frac{\pi}{6}x\right)\right)$ $\therefore k = \frac{6}{\pi}$	(M1) r.a. to develop equation for k starting from $x = f^{-1}(f(x))$
d.	$A = \int_1^3 \left(\frac{6}{\pi}\sin^{-1}\left(\frac{1}{x+1}\right) - \text{cosec}\left(\frac{\pi}{6}x\right) + 1\right) dx$ ≈ 0.841 (3 d.p.)	(A1) (A1) correct to the required precision.

	 	
e.i.	 $L = \int_1^3 \sqrt{1+\left(\frac{6}{\pi}\frac{\mathrm{d}}{\mathrm{d}x}\sin^{-1}\left(\frac{1}{x+1}\right)\right)^2}\,\mathrm{d}x$ $= \int_1^3 \sqrt{1+\frac{36}{\pi^2 x(x+1)^2(x+2)}}\,\mathrm{d}x$	(M1) (A1)
e.ii	 $L \approx 2.077$ (3 $d.p.$)	(A1) correct to the required precision.

Question 2 (9 marks)

<table>
<tr><td>a.i</td><td>

$w=2\text{ cis}\left(-\frac{\pi}{3}\right)$</td><td>(A1) correct modulus.
(A1) correct argument.</td></tr>
<tr><td>a.ii</td><td>$w^6=2^6\text{ cis}\left(-6\times\frac{\pi}{3}\right)=32$</td><td>(M1) must use de Moivre’s Theorem.</td></tr>
<tr><td>a.iii</td><td>
</td><td>(M1) 6 points evenly spaced around circle of radius two.

(A1) correctly labelled u, w.</td></tr>
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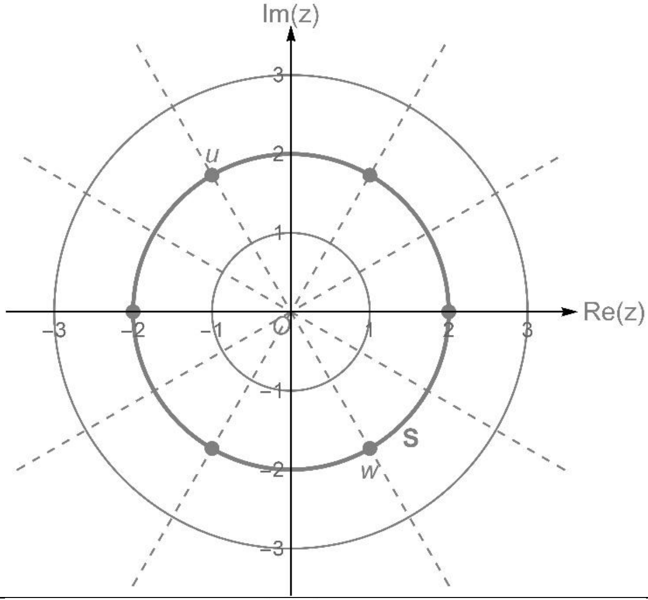
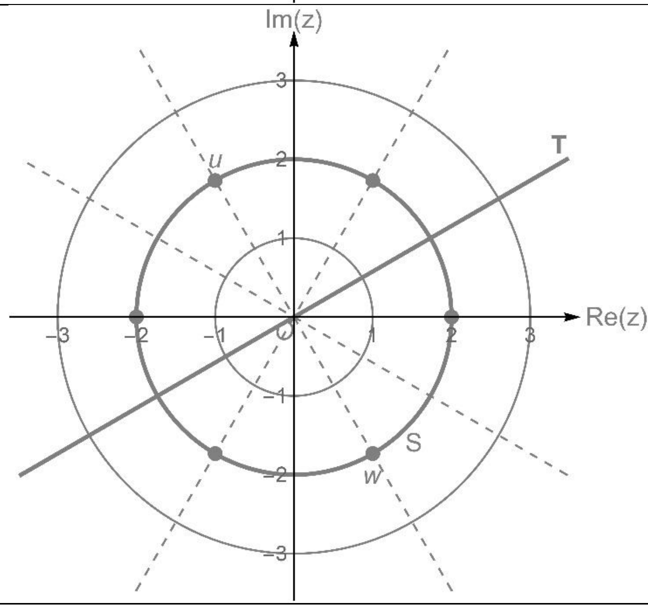
<table>
<tr><td>b.i.</td><td>
</td><td>(A1)</td></tr>
<tr><td>b.ii</td><td>
</td><td>(A1)</td></tr>
<tr><td>b.iii</td><td>$R = \left\{2\ \text{cis}\left(\frac{\pi}{6}\right), 2\ \text{cis}\left(\frac{7\pi}{6}\right)\right\}$
$R = \left\{\sqrt{3} + i, -\sqrt{3} - i\right\}$</td><td>(M1)
(A1)</td></tr>
</table>

Question 3 (11 marks)

<table>
<tr><td>a.</td><td>$$\begin{aligned}\int \frac{-1}{y(m-\log_e(y))}\,dy &= \int \frac{1}{(m-\log_e(y))}\frac{d}{dy}(m-\log_e(y))\,dy \\ &= \int \frac{1}{u}\,du \text{ with } u = m-\log_e(y) \\ &= \log_e(|m-\log_e(y)|)\end{aligned}$$</td><td>(M1) identify du.
(A1) integral w.r.t. u.</td></tr>
</table>

b.	$\int \frac{-1}{N\,(8-\log_e(N))}\,dN = -\int 2\,dt$ (using $m = 8$ in result from part **a.**) $\log_e(\lvert 8-\log_e(N)\rvert) = -2t + C$ $8-\log_e(N) = e^{C}e^{-2t}$ (because $8-\log_e(N) > 0$) $(t, N) = (0, e^2) \Rightarrow 6 = e^{C}$ $\therefore \log_e(N) = 8 - 6e^{-2t}$	(M1) separate variables. (M1) use result from part a. (M1) impose I.C.
c.	$\lim_{t\to\infty}(8-6e^{-2t}) = 8$ $\therefore N \to e^{8}$	(A1)
d.	$\frac{d}{dt}\left(\frac{dN}{dt}\right) = \frac{d}{dN}\left(3\,N\,(8-\log_e(N))\right)\frac{dN}{dt}$ $\frac{d^2N}{dt^2} = 3(7-\log_e(N))\frac{dN}{dt}$ $\frac{d^2N}{dt^2} = 9\,N(7-\log_e(N))(8-\log_e(N))$	(M1) r.a. chain rule. (M1) substitute for $\frac{dN}{dt}$
e.	$\frac{dN}{dt}$ is max $\Rightarrow \frac{d^2N}{dt^2} = 0\ and\ \frac{dN}{dt} \neq 0$ $(7-\log_e(N)) = 0 \Rightarrow N = e^{7}$	(M1) (A1)
f.	$8-6e^{-2t} = 7 \Rightarrow t = \frac{\log_e(6)}{2}$ days	(A1)

Question 4 (10 marks)

a.	$\dot{\underset{\sim}{r}}(t) = -gt\,\underset{\sim}{j} + \underset{\sim}{c}$ $\dot{\underset{\sim}{r}}(0) = 45\underset{\sim}{i} + 0\underset{\sim}{j} \Rightarrow \underset{\sim}{c} = 45\underset{\sim}{i} + 0\underset{\sim}{j}$ $\underset{\sim}{r}(t) = -\frac{1}{2}gt^2\underset{\sim}{j} + 45t\underset{\sim}{i} + \underset{\sim}{d}$ $\underset{\sim}{r}(t) = 0\underset{\sim}{i} + 0\underset{\sim}{j} \Rightarrow \underset{\sim}{d} = 0\underset{\sim}{i} + 0\underset{\sim}{j}$ $\therefore \underset{\sim}{r}(t) = -\frac{1}{2}gt^2\underset{\sim}{j} + 45t\underset{\sim}{i}$	(M1) r.a to antidifferentiate twice. (M1) r.a to employ initial conditions to determine constants of antidifferentiation. (A1)
b.	$\underset{\sim}{j}\cdot\underset{\sim}{r}(T) = -h \Rightarrow -\frac{1}{2}gT^2 = -490$ $T = \sqrt{\frac{980}{g}} = 10$ seconds since $T > 0$.	(M1) (M1)
c.	$\underset{\sim}{r}(T) = 450\underset{\sim}{i} - 490\underset{\sim}{j}$ $\theta = \arctan\left(\frac{49}{45}\right)$	(M1) (A1)
d.	$\dot{\underset{\sim}{r}}(10) = 45\underset{\sim}{i} - 10g\underset{\sim}{j}$ speed $= \left\lvert 45\underset{\sim}{i} - 10g\underset{\sim}{j}\right\rvert$ $= \sqrt{45^2 + 980^2}$ ≈ 981 m/s Ignoring air resistance results in unrealistically large speed at splash down.	(M1) seen or used. (A1) (R1)

Question 5 (10 marks)

a.	$Mg - mg = (M + m)a$ $a = \frac{M - m}{M + m}g$	(M1) or equivalent force balance. (A1)
b.	$Mg - T_1 = Ma$ $T_1 = \frac{2gmM}{m + M}$	(M1) or equivalent force balance. (A1)
c.	$-Mg\sin(c°) + mg = (M + m)b$ $b = \frac{m - M\sin(c°)}{M + m}g$	(M1) or equivalent force balance. (A1)
d	$Mg\sin(d°) - mg = (M + m)b$ $b = \frac{M\sin(d°) - m}{M + m}g$	(M1) or equivalent force balance. (A1)
e	$\frac{m - M\sin(c°)}{M + m}g = \frac{M\sin(d°) - m}{M + m}g$ $\frac{m}{M} = \frac{\sin(c°) + \sin(d°)}{2}$	(M1) equating expressions for b. (A1)

Question 6 (9 marks)

a.	$H_0 : \mu = 30$ $H_1 : \mu > 30$	(A1) (A1)
b.	$\bar{X} \sim N(\mu = 30, \sigma = 2)$ $p = \Pr(\bar{X} \geq 32 \mid \mu = 30) = \Pr(Z \geq 1)$ $\approx 0.159 \text{ (3 d.p.)}$	(M1) (A1)
c.	$p > 0.05 \Rightarrow$ do not reject H_0	(A1)
d.	$\bar{X} \sim N(\mu = 30, \sigma = 2)$ $\Pr\left(Z < 1.6449 = \dfrac{C^* - 30}{2}\right) = 0.95$ $C^* = 30 + 2 \times 1.6449 = 33.290 \text{ (3 } d.p.)$	(M1) (A1)
e.i.	$\bar{X} \sim N(\mu = 32, \sigma = 2)$ $\Pr(\bar{X} \leq 33.290 \mid \mu = 32) = \Pr\left(Z \leq \dfrac{33.290 - 32}{2} \mid \mu = 32\right)$ $\approx 0.741 \text{ (3 d.p.)}$	(A1)
e.ii.	$\Pr(\bar{X} \leq 33.290 \mid \mu = 32)$ is the probability of rejecting H_1 when it is indeed correct, so it is a Type II error.	(A1) must have reason.

