Free Exam
For 2016-18 VCE Study Design

Engage Education Foundation

Units 3 and 4 Specialist Maths: Exam 1

Practice Exam Question and Answer Booklet

Duration: 15 minutes reading time, 1 hour writing time

Structure of book:

Section	Number of questions	Number of questions to be answered	Number of marks
A	8	8	40
		Total	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers and rulers.
- Students are not permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- No calculator is allowed in this examination.

Materials supplied:
- This question and answer booklet of 13 pages including a formula sheet.

Instructions:
- You must complete all questions of the examination.
- Write all your answers in the spaces provided in this booklet.

Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagram in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Questions

Question 1

A box sits on a scale in a lift accelerating downwards at 2.8 m/s^2. The scale reads 35 kg. The mass of the box is m kg.

a. Find m.

3 marks

b. If the lift now is accelerating upwards at 4.9 m/s^2, what would be the output given by the scale?

2 marks

Total: 5 marks

Question 2

Find all solutions to the equation:

$z^4 + 9z^2 + 64 = 0, z \in \mathbb{C}$

Express your solutions in Cartesian form.

5 marks

Question 3

a. Find the derivative with respect to x of the following relation, expressing your answer in terms of x:

$$y = x^x \text{ where } x, y \in \mathbb{R}^+$$

2 marks

b.

i. Write down the range of the function $f(x) = -2x\tan^{-1}(x)$.

1 mark

ii. Find $\frac{d}{dx}(x\cos^{-1}(x))$.

1 mark

iii. Hence, find the area enclosed by the x-axis and the graph of $y = \cos^{-1}(x)$ between the values of $x = \frac{1}{2}$ and $x = 1$

3 marks

Total: 7 marks

Question 4

Consider the parallelogram ABCD below:

Let $\overrightarrow{AB} = \boldsymbol{a}$ and $\overrightarrow{AD} = \boldsymbol{d}$.

a. Find $\overrightarrow{AC}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{d}$.

1 mark

b. Let M be the mid-point of AC Using vector methods, show that B, M and D are collinear.

2 marks

Total: 3 marks

Question 5

Solve $3\sin^2(\theta) + \cos^2(\theta) + \sqrt{3}\sin(2\theta) = 4, \theta \epsilon [-\pi, \pi]$.

5 marks

Question 6

The acceleration a m/s^2 of a body moving in a straight line in terms of its displacement x m is given by $a = 2x^2 + 1$.

Given that $v = 2$ when $x = 1$, where v m/s is the velocity of the body, find the possible velocities of the body when the displacement of the body is $x = 0$.

4 marks

Question 7

a. Consider two independent random variables X and Y, where $E(X) = 12$ and $Var(X) = 4$; $E(Y) = 5$ and $Var(Y) = 9$.

i. Find $E(4X - 2Y + 6)$.

1 mark

ii. Find $Var(4X - 2Y + 6)$. Hence, find the standard deviation of this distribution.

2 marks

b. A normally distributed population is predicted to have a mean height of 176 cm. The height variance is **known** to be 256 cm. To test this prediction, a random sample of 64 individuals from the population is obtained. Their sample mean is found to be 173 cm.

By calculating the 95% confidence interval for the population mean, state whether the prediction of 176 cm should be rejected under the significance level of 0.05.

2 marks

Total: 5 marks

Question 8

The position vector of a particle moving relative to an origin *O* at time *t* seconds is given by

$r(t) = (k\tan(t) - 1)\boldsymbol{i} + (3\sec(t) - 8)\boldsymbol{j}, \mathrm{t} \in [0, \pi]$

Where the components are measured in metres and *k* is a positive real number.

a. Show that the Cartesian equation of the path of the particle is $\frac{(y+8)^2}{9} - \frac{(x+1)^2}{k^2} = 1.$

1 mark

b. Find an expression for the gradient at any point on the path in terms of x, y and k.

2 marks

c. The line $3y + 2x + 2 = 0$ is the normal to the path $x = 2$. Find the value of k in the simplest form.

3 marks

Total: 6 marks

Formula Sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

Function	$\sin^{-1}$(arcsin)	$\cos^{-1}$(arccos)	$\tan^{-1}$(arctan)
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\,\text{cis}(\theta)$	
$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \text{Arg}(z) \le \pi$
$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$
$z^n = r^n\,\text{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$\text{E}(aX + b) = a\text{E}(X) + b$ $\text{E}(aX + bY) = a\text{E}(X) + b\text{E}(Y)$ $\text{var}(aX + b) = a^2\text{var}(X)$
for independent random variables X and Y	$\text{var}(aX + bY) = a^2\text{var}(X) + b^2\text{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\overline{X}$	mean $\text{E}(\overline{X}) = \mu$ variance $\text{var}(\overline{X}) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$	$\int \frac{1}{x}dx=\log_e\lvert x\rvert+c$
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$	$\int \sin(ax)dx=-\frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$	$\int \cos(ax)dx=\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=a\sec^2(ax)$	$\int \sec^2(ax)dx=\frac{1}{a}\tan(ax)+c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right)=\frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+c,a>0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right)=\frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c,a>0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right)=\frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$
	$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$
	$\int (ax+b)^{-1}dx=\frac{1}{a}\log_e\lvert ax+b\rvert+c$
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx}=f(x)$, $x_0=a$ and $y_0=b$, then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf(x_n)$
acceleration	$a=\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2}\sqrt{1+(f'(x))^2}\,dx$ or $\int_{t_1}^{t_2}\sqrt{(x'(t))^2+(y'(t))^2}\,dt$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$
$\lvert \underset{\sim}{r} \rvert = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

Mechanics

momentum	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R} = m\underset{\sim}{a}$

End of Booklet