Year 12 *Trial Exam Paper*
2017

SPECIALIST MATHEMATICS
Written examination 2

Reading time: 15 minutes
Writing time: 2 hours

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring sheets of paper, notes of any kind or white out liquid/tape into the examination.

Materials provided
- Question and answer book of 23 pages.
- Formula sheet.
- Answer sheet for multiple-choice questions.

Instructions
- Write your **name** in the space provided above, **and** on the multiple-choice answer sheet.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- You must answer the questions in English.

At the end of the examination
- Place the multiple-choice answer sheet inside the front cover of this book.
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination.

SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1

The implied domain and range of the function $f(x) = a\sin^{-1}\left(bx - \frac{\pi}{2}\right) + 1$, where $a, b > 0$, are respectively

A. $\left[\frac{\pi - 1}{2b}, \frac{1+\pi}{2b}\right]$ and $\left[\frac{2 - a\pi}{2}, \frac{a\pi + 2}{2}\right]$

B. $\left[\frac{\pi - 2}{2b}, \frac{2+\pi}{2b}\right]$ and $\left[\frac{2 - a\pi}{2}, \frac{a\pi + 2}{2}\right]$

C. $\left[-\frac{\pi}{2}(b-1), \frac{\pi}{2}(b+1)\right]$ and $[-a, a]$

D. $\left[\frac{\pi - 2}{2b}, \frac{2+\pi}{2b}\right]$ and $\left[\frac{1 - a\pi}{2}, \frac{a\pi + 1}{2}\right]$

E. $\left[\frac{\pi - 2}{b}, \frac{2+\pi}{b}\right]$ and $\left[\frac{2 - a\pi}{2}, \frac{a\pi + 2}{2}\right]$

Question 2

For the graph $f(x) = \frac{x^4 + 5}{x^2}$, which of the following statements is **not** true?

A. There is a vertical asymptote at $x = 0$.

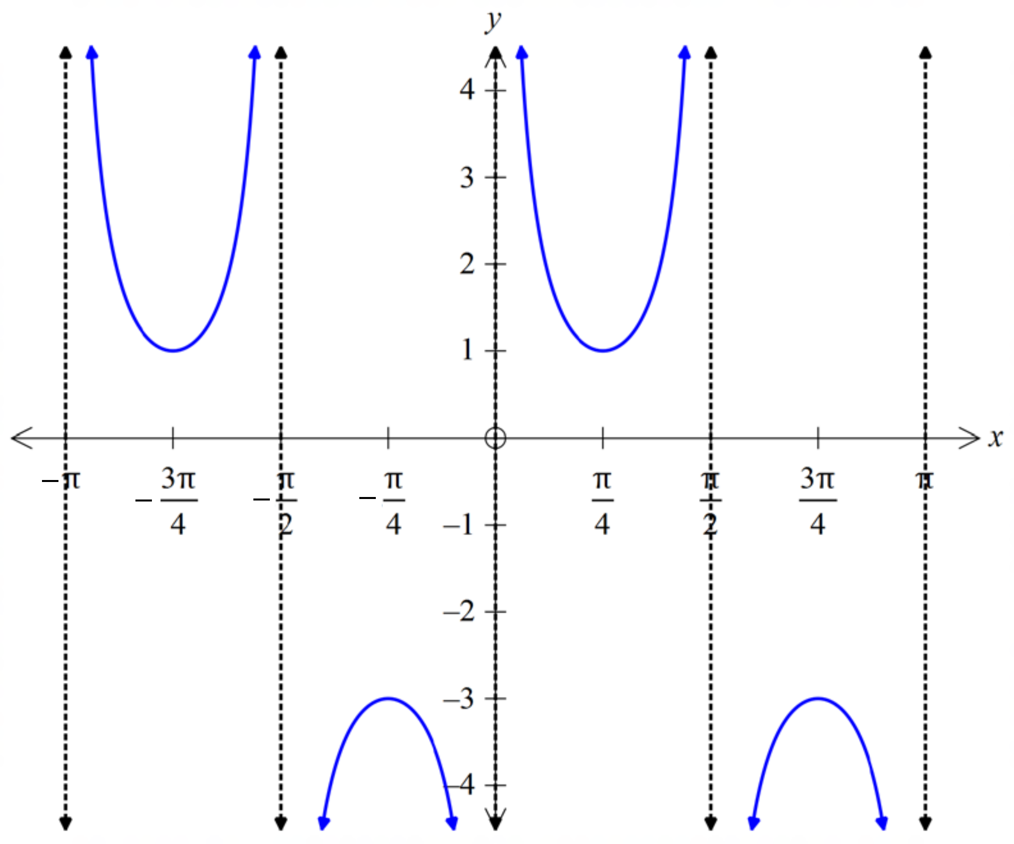
B. The graph has a stationary point of inflection.

C. The graph has two stationary points.

D. The graph has no axes intercepts.

E. The graph has a non-straight line asymptote of $y = x^2$.

Question 3



For the graph $y = f(x)$ shown above, $f(x)$ could be given by

A. $y = 2\sec\left(2x - \frac{\pi}{4}\right) - 1$

B. $y = 2\operatorname{cosec}\left(2x - \frac{\pi}{2}\right) - 1$

C. $y = 2\sec\left(\frac{x}{2} - \frac{\pi}{2}\right) - 1$

D. $y = 2\sec\left(2x - \frac{\pi}{2}\right) - 1$

E. $y = 2\operatorname{cosec}\left(2x - \frac{\pi}{4}\right) - 1$

Question 4

The rule of the relation determined by the parametric equations $x = 5\sec(t) + 1$ and $y = 4\tan(t) - 2$, where $t \geq 0$, is

A. $\dfrac{(x-1)^2}{25} + \dfrac{(y+2)^2}{16} = 1$

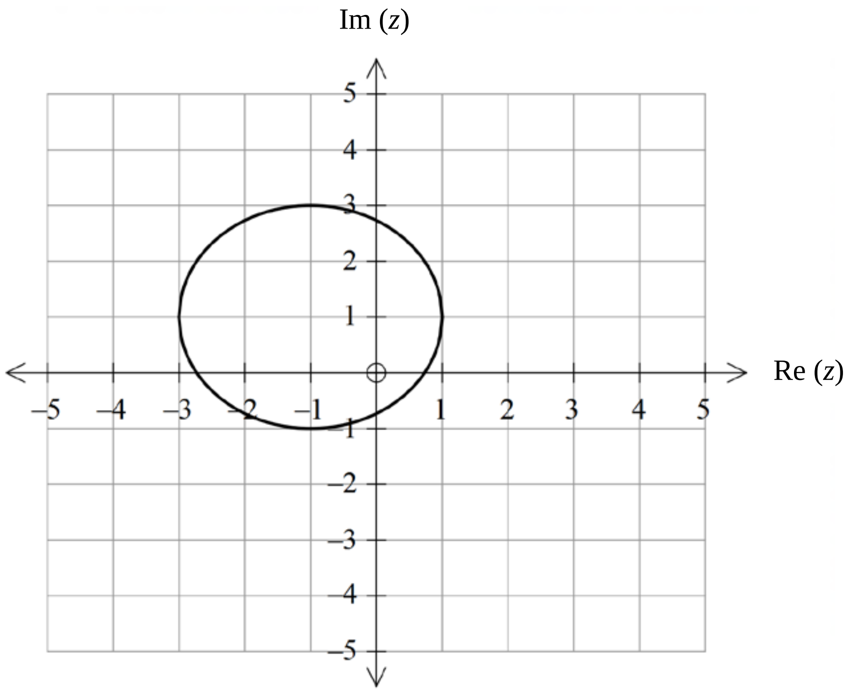
B. $\dfrac{(y+2)^2}{16} - \dfrac{(x-1)^2}{25} = 1$

C. $\dfrac{(x-1)^2}{25} - \dfrac{(y+2)^2}{16} = 1$

D. $\dfrac{(x-1)^2}{25} + 1 = \dfrac{(y+2)^2}{16}$

E. $\dfrac{(x+1)^2}{5} - \dfrac{(y-2)^2}{4} = 1$

Question 5



Given that $z \in C$, the graph shown above can be represented by

A. $\text{Arg}(z + 1 + i) = 2\pi$

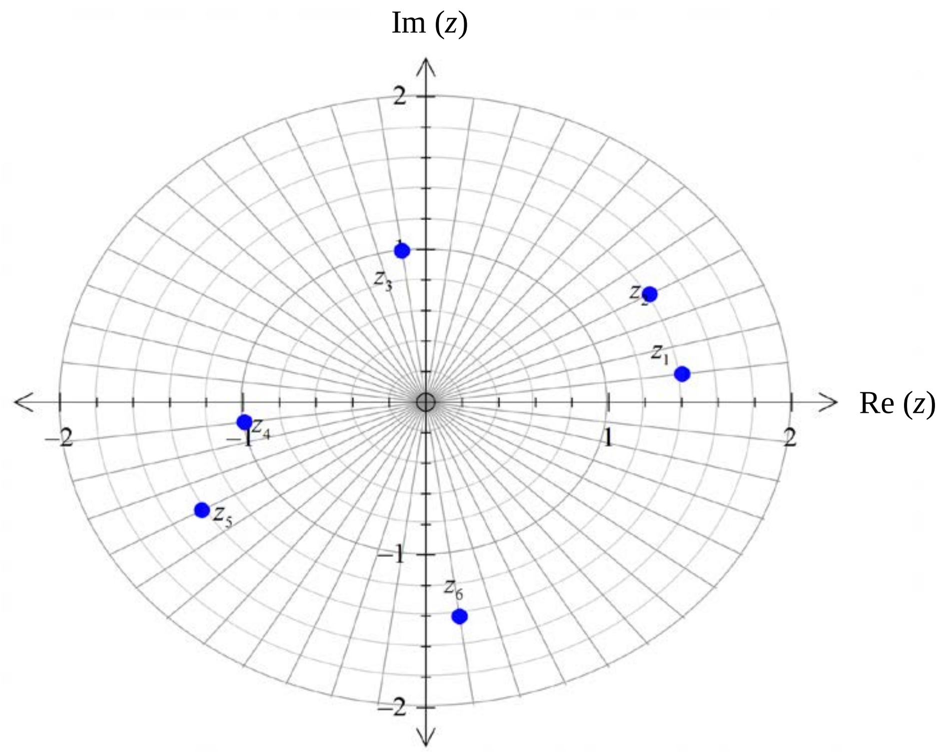
B. $\{z : |z + 1 - i| = 4\}$

C. $\{z : |z - (i - 1)| = 2\}$

D. $\text{Arg}(z + 1 - i) = \pi$

E. $\{z : |z + 1| = |z + 2 + i|\}$

Question 6



Which of the points shown on the complex plane above are solutions to $z^4 = 2\sqrt{3} + 2i$?

A. z_2 and z_5 only

B. z_3 and z_4 only

C. z_1, z_3, z_4 and z_6 only

D. z_1 and z_6 only

E. z_1 and z_4 only

Question 7

For the polynomial $P(z) = z^3 + 2iz^2 + 9z + 18i$, which of the following statements is true?

A. $P(z) = 0$ has only one real root

B. $P(z) = 0$ has two complex roots

C. $P(-3) = 0$

D. $P(z) = 0$ has no real roots

E. $P(2i) = 0$

Question 8

Using a suitable substitution, $\int_{e^a}^{e^{5b}} \frac{[\log_e(x)]^4}{5x}\,dx,$ where *a* and *b* are real constants, can be written as

A. $\int_a^{5b} 5u^4\,du$

B. $\int_{\log_e(a)}^{\log_e(5b)} \frac{u^4}{5}\,du$

C. $\int_a^{5b} \frac{u^4}{5}\,du$

D. $\int_{e^a}^{e^{5b}} \frac{u^4}{5}\,du$

E. $\int_a^{5b} u^4\,du$

Question 9

Given that $y_0 = y(2) = 0$ for $\frac{dy}{dx} = x^3 - 2x,$ using Euler's method with increments of 0.1, an approximation for the value of y_3 is closest to

A. 1.5039

B. 0.9686

C. 0.12303

D. 1.8876

E. 1.5309

Question 10

A particle travelling in a straight line with position *x* and velocity *v* at time *t* is given by $v = \sin(x)$. The acceleration of this particle can be given by:

A. $2\sin(x)$

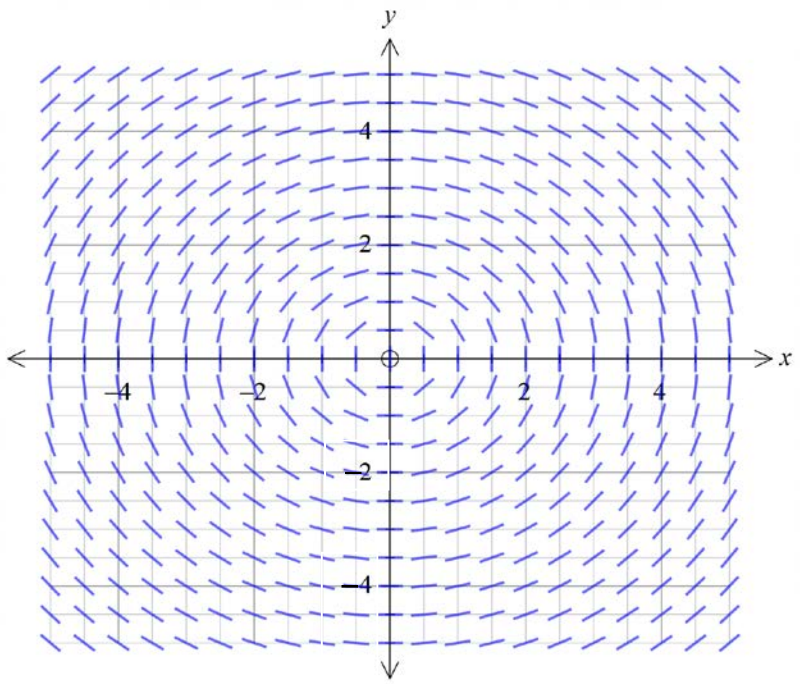
B. $\sin(2x)$

C. $\frac{1}{2}\sin(x)\cos(x)$

D. $\frac{1}{2}\sin(2x)$

E. $2\sin(x)\cos(x)$

Question 11



The differential equation that best represents the slope field shown above could be

A. $\frac{dy}{dx} = -\frac{y}{x}$

B. $\frac{dy}{dx} = -\frac{x}{y}$

C. $\frac{dy}{dx} = x^2$

D. $\frac{dy}{dx} = \frac{1}{y}$

E. $\frac{dy}{dx} = \frac{x}{y}$

Question 12

If $\underset{\sim}{a} = -6\underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}$ and $\underset{\sim}{b} = -4\underset{\sim}{i} + 5\underset{\sim}{j} - 3\underset{\sim}{k}$, then $|4\underset{\sim}{a} - 2\underset{\sim}{b}|$ is

A. $\frac{1}{\sqrt{264}}\left(-16\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}\right)$

B. $\sqrt{246}$

C. $2\sqrt{66}$

D. $2\sqrt{258}$

E. $-16\underset{\sim}{i} - 7\underset{\sim}{j} - 2\underset{\sim}{k}$

Question 13

Let $\underset{\sim}{a} = 3\underset{\sim}{i} + m\underset{\sim}{j} - 3\underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$, where m is a real constant.

If the scalar resolute of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$ is 5, then the value of m is

A. 1

B. 6

C. 15

D. 4

E. 12

Question 14

The position vectors of two moving particles is given by $\underset{\sim}{r} = (2t+1)\underset{\sim}{i} + (6-4t)\underset{\sim}{j}$ and $\underset{\sim}{s} = (3t^2+2)\underset{\sim}{i} + (3t+2)\underset{\sim}{j}$, where $t > 0$. At what time, t, will the velocity vectors of the two particles be perpendicular?

A. 12

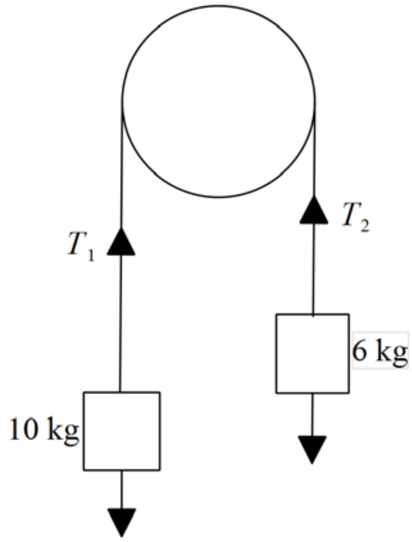
B. 1

C. 0

D. 6

E. The velocity vectors are never perpendicular

Question 15



The diagram above shows objects of mass 10 kg and 6 kg attached to the ends of a light inextensible string that passes over a smooth pulley. If the system is released from rest, assuming the system remains connected, the magnitude of the acceleration of the system is

A. $\frac{g}{4}$ ms^{-2}

B. $4g$ ms^{-2}

C. 0.25 ms^{-2}

D. $0.6g$ ms^{-2}

E. 0.6 ms^{-2}

Question 16

A tennis ball with a mass of 200 grams is moving to the right in a straight line with a velocity of 40 ms^{-1}. It is hit by a tennis racquet and experiences a force for a period of time, after which it is then moving with a velocity to the left of 35 ms^{-1}. The change in the momentum of the tennis ball, in kg ms^{-1}, is

A. 0

B. 15

C. –1

D. 1

E. 3000

Question 17

X and Y are both independent random variables. The random variable X has a mean of 9 and a standard deviation of 5, and the random variable Y has a mean of 6 and a standard deviation of 2. If the random variable Z is defined by $Z = 2X - 3Y$, then the mean, μ, and standard deviation, σ, of Z is given by

A. $\mu = 0,\ \sigma = 4$

B. $\mu = 0,\ \sigma = 136$

C. $\mu = 0,\ \sigma = 2\sqrt{34}$

D. $\mu = 0,\ \sigma = 8$

E. $\mu = 0,\ \sigma = 2$

Question 18

A fitness company claims that after 6 weeks of their fitness program their clients have a mean weight loss of 4 kg. A prospective client wishes to test this claim at a significance level of 0.05, using the null and alternative hypothesis of $\mathrm{H}_0 : \mu = 4$ and $\mathrm{H}_1 : \mu \neq 4$.

If the p value of the client's test is 0.002, which of the following statements is true?

A. A type I error would occur if the null hypothesis is true and not accepted.

B. The p value is less than the significance level, so the null hypothesis is accepted.

C. The p value is greater than the significance level, so the null hypothesis is not rejected.

D. A one-tailed test should be carried out to find the p value.

E. A type II error would occur if the null hypothesis is false and not rejected.

Question 19

A random sample of 150 students from a given school has a mean height of 168 cm and a standard deviation of 12 cm.

An approximate 90% confidence interval for the mean height of the students at this school is closest to

A. (166.080, 169.920)

B. (166.388, 169.612)

C. (165.472, 170.528)

D. (167.794, 168.206)

E. (167.869, 168.131)

Question 20

The weight of a certain brand of chocolate bar is normally distributed with a mean weight of 65 grams and a standard deviation of 2.5 grams. A random sample of 10 chocolate bars is selected.

The probability that the mean weight of this sample of chocolate bars is less than 63.5 grams is approximately

A. 0.0289

B. 0.9711

C. 0.2743

D. 0.7257

E. 0.2341

END OF SECTION A
TURN OVER

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where g = 9.8.

Question 1 (8 marks)

Consider the function $f(x) = \dfrac{x^2 - 2x + 5}{x - 1}$, $x \in R \setminus \{1\}$.

a. **i.** Find the coordinates of any stationary points of *f(x)*.

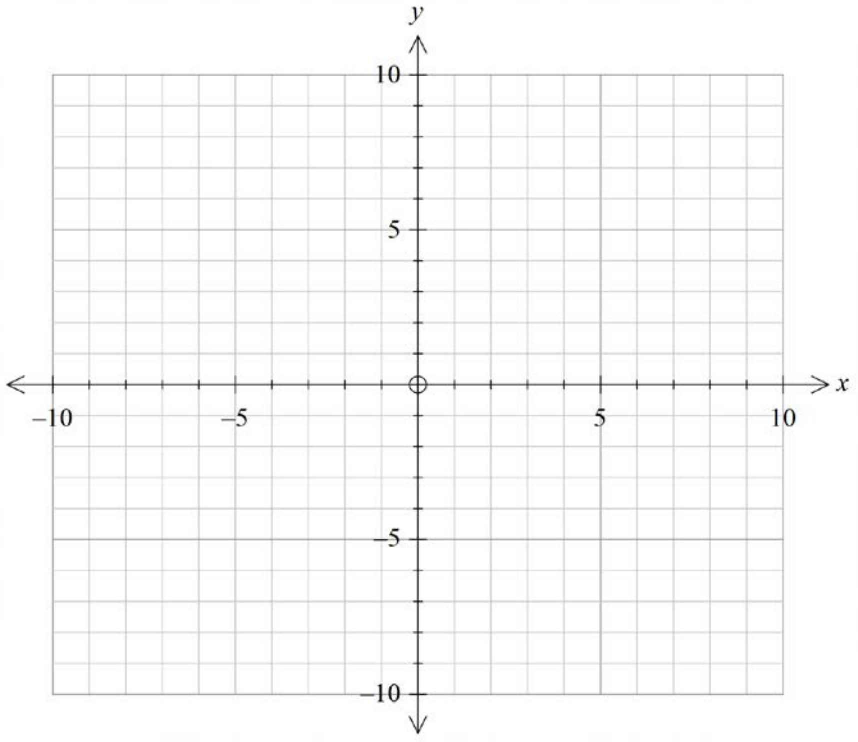
2 marks

ii. Show that the stationary point with a positive *x* coordinate is a minimum.

2 marks

b. Sketch the graph $y = f(x)$ on the axes below. Label any turning points with their coordinates and any asymptotes with their equation.

3 marks



c. Find the length of the curve over the interval $x \in [3, 6]$.

1 mark

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Question 2 (12 marks)

a. Let the complex number $u = 2 + 2i$. Express u in polar form.

2 marks

b. Find the cartesian equation of the line given by $v = \{z : |\overline{z} + z| = 4\}$, where $z \in C$ and $x > 0$.

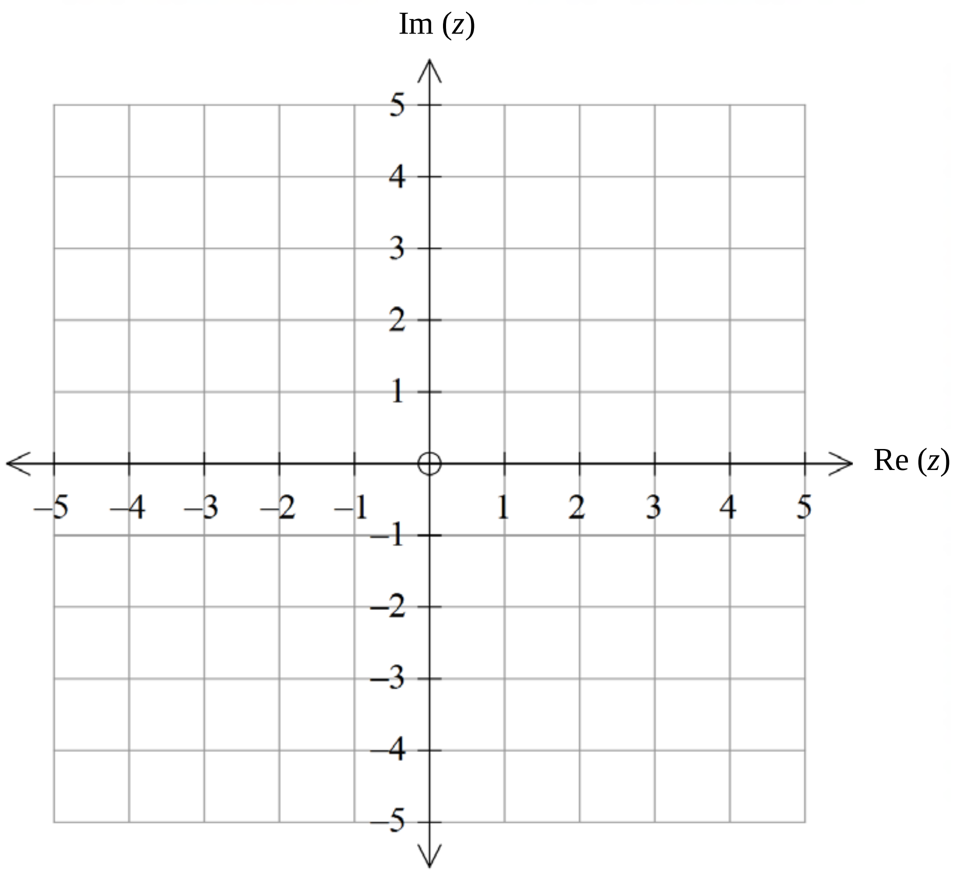
2 marks

c. Find the cartesian equation of the circle $w = \{z : (z - 2 - 2i)(\overline{z} - 2 + 2i) - 4 = 0\}$, where $z \in C$.

3 marks

d. Sketch the line segment from $z = 0$ to $z = u$ from **part a.**, the line v from **part b.**, and the circle w from **part c.** on the Argand diagram below.

3 marks



e. The line from (0, 0) to u and the line v form a sector of the circle. Calculate the area of the sector formed in **part d.**

2 marks

Question 3 (12 marks)

A glass company is investigating the shape of the wine glasses it manufactures. The company uses an inverse sine curve to model the shape of the glasses manufactured.

a. **i.** Differentiate $\sqrt{1-x^2}+x\sin^{-1}(x)$ and state the domain for which the derivative exists.

2 marks

ii. Hence find an antiderivative of $\int 2\sin^{-1}\left(\frac{x}{2}\right)dx$.

2 marks

The company has decided to model the shape of the wine glass using the equation $y = \sin^{-1}\left(\frac{x}{2} - 2\right) + \frac{\pi}{2}$, where x and y are the respective radius and height of the glass, in centimetres.

b. A solid can be generated by rotating the area bounded by the section of the curve from $x = 2$ to $x = 6$ and the y-axis, about the y-axis.

i. Write an integral that represents the volume of the glass.

1 mark

ii. Hence find the volume of the glass, correct to two decimal places.

1 mark

Water is added to the empty glass such that the height of the water in the glass changes at a rate of $3t$, where $t \geq 0$.

c. Write a differential equation relating the radius of water in the glass, x cm, to the length of time, t seconds, that water is added to the glass.

3 marks

d. Solve the differential equation found in **part c.** if the radius of water in the glass is initially 2 cm.

3 marks

Question 4 (11 marks)

A walker, W, is observed from an origin, O, walking laps around a park. The position vector, in metres, of the walker relative to the origin at any time t seconds after the walk began is given by

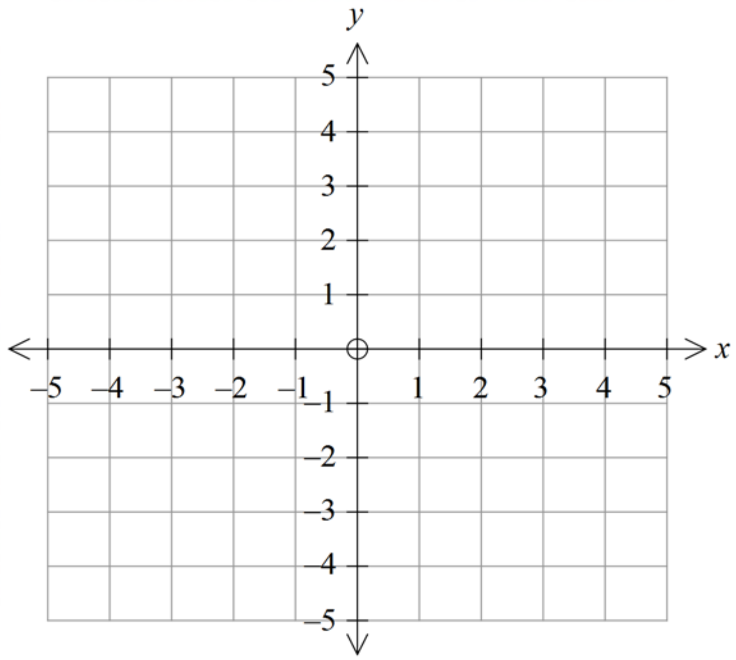
$$\underset{\sim}{r}_W = (2 - 3\cos(t))\underset{\sim}{i} + (1 - 2\sin(t))\underset{\sim}{j}.$$

a. Determine the cartesian equation of the walker's path.

2 marks

b. Sketch the path of the walker on the axis provided below. Indicate the walker's direction of motion with an arrow.

2 marks



At the same time that the walker starts walking, a dog, *D*, escapes control of its owner and runs with a position vector, relative to the same origin as the walker, which is given by

$$\underset{\sim}{r}_D = (2 - 3\cos(t))\,\underset{\sim}{i} + \left(\frac{24t}{5\pi} - 4\right)\underset{\sim}{j}$$

c. Show that the dog will collide with the walker when $t = \frac{5\pi}{6}$ seconds.

2 marks

d. At the time of the collision, find the angle, in degrees, between the velocity vectors of the walker and the dog.

Give your answer correct to one decimal place.

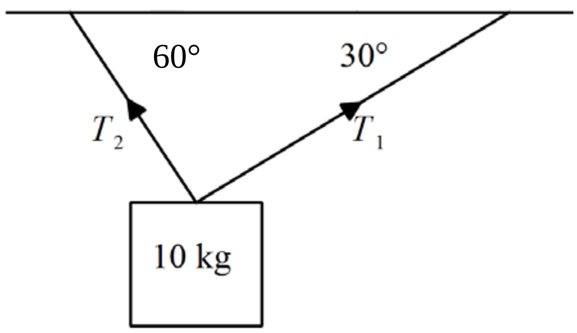
3 marks

e. Determine the speed of the dog, correct to two decimal places, at the time of the collision.

2 marks

Question 5 (11 marks)

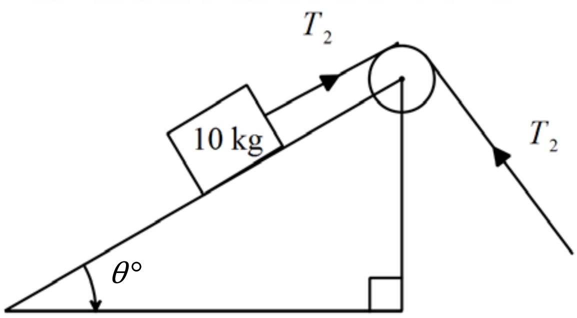
A 10 kg box is suspended by two strings, as shown in the diagram below.



a. Calculate the tension, in newtons, in each rope if the system is in equilibrium.

2 marks

The string with tension T_2 is connected to a second mass of 10 kg positioned on a smooth incline, over a smooth pulley. The system can be represented by the diagram below.



b. Calculate the value of θ if the 10 kg mass is to remain stationary.

2 marks

c. **i.** Hence for what values of θ will the mass accelerate up the plane?

1 mark

ii For what values of θ will the mass accelerate down the plane?

1 mark

Let $\theta = 10°$ and let T_2 remain the same as in **part a.** The system is now no longer in equilibrium and the mass accelerates from rest, up the inclined plane.

d. If the distance the mass travels to the top of the inclined plane is 2.5 m, calculate the velocity of the mass at the top of the plane, correct to the nearest integer.

2 marks

e. If the string snaps when the mass is at the top of the inclined plane and the mass then follows a projectile path, calculate the time for the mass to hit the ground if the top of the inclined plane is 2 metres above the ground.

Give your answer in seconds, correct to two decimal places.

3 marks

Question 6 (6 marks)

A particular shoe manufacturer knows that the mean shoe size of the adult population is 9.5 with a standard deviation of 1.5.

A sports shoe store believes that the mean shoe size of the adult population is not 9.5, so it selects 60 customers at random to test this claim.

a. Let the random variable $\bar{X}$ represent the mean shoe size from the random sample.

Determine the standard deviation of $\bar{X}$.

1 mark

From the sample of 60 customers selected, the sports shoe store found that the mean adult shoe size of its customers is 9.

b. Give a suitable hypothesis, H_0 and H_1, to test the shoe store's belief about the mean adult shoe size.

2 marks

c. **i.** Find the *p* value for the sports shoe store's test, correct to four decimal places.

2 marks

ii. Determine whether the sample selected supports the sports shoe store's belief that the mean shoe size of the adult population is not 9.5, for a 5% level of significance.

1 mark

END OF QUESTION AND ANSWER BOOK