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Specialist Mathematics

2017

Trial Examination 2
(2 hours)

SECTION A Multiple-choice questions

Instructions for Section A

Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

Question 1 The graph of $y = \dfrac{x^3 - 201x^2 + 10100x + 10100}{x^2 - 201x + 10100}$ has n straight line asymptotes. The value of n is

A. 0

B. 1

C. 2

D. 3

E. 4

Question 2 The range of function f with the rule $f(x) = \arcsin(ax) + \dfrac{\pi}{2}$ is $\left[\dfrac{\pi}{2} - b, \pi\right]$, where $0 < b < \dfrac{\pi}{2}$.
For $a > 0$, the domain of f is

A. $\left[-\dfrac{b}{a}, \dfrac{1}{a}\right]$

B. $\left[\dfrac{b}{a}, \dfrac{1}{a}\right]$

C. $\left[-\dfrac{a}{b}, a\right]$

D. $\left[\dfrac{\sin a}{b}, a\right]$

E. $\left[-\dfrac{\sin b}{a}, \dfrac{1}{a}\right]$

Question 3 The Cartesian equation of the relation given by $x = \cot(u) - 1$ and $y = \sec^2(u) - 2$ is

A. $(x+1)^2(y+1) = 1$

B. $(x+1)^2(y-1) = 1$

C. $(x+1)^2 + y = 1$

D. $(x+1)^2 - y = 1$

E. $(x+1)^2 + y = -1$

Question 4 On the Argand diagram $z = -i\,\text{cis}\left(\frac{4\pi}{9}\right)$ is closest to

A. $z = -\cos\left(\frac{4\pi}{9}\right)$

B. $z = \cos\left(\frac{4\pi}{9}\right)$

C. $z = \sin\left(\frac{4\pi}{9}\right)$

D. $z = |-i|$

E. $z = i$

Question 5 Given $z = \sqrt{2}(a+b)\text{cis}\left(\frac{3\pi}{4}\right) + 2a + 2b$ where $a, b \in R^+$, $z^n \in R$ when n is a whole number multiple of

A. 1

B. 2

C. 3

D. 4

E. 5

Question 6 Given $f:(a\pi, 2a\pi)\to R,\ f(x)=\text{cosec}\left(\dfrac{x}{a}\right),\ g=f$ when

A. $g:(a\pi, 2a\pi)\to R,\ g(x)=-\text{cosec}\left(\dfrac{x}{a}+\pi\right)$

B. $g:(a\pi, 2a\pi)\to R,\ g(x)=-\text{cosec}\left(\dfrac{x}{a}-2\pi\right)$

C. $g:(a\pi, 2a\pi)\to R,\ g(x)=\sec\left(\dfrac{x}{a}-a\pi\right)$

D. $g:(a\pi, 2a\pi)\to R,\ g(x)=-\sec\left(\dfrac{x}{a}-a\pi\right)$

E. $g:(a\pi, 2a\pi)\to R,\ g(x)=\sec\left(\dfrac{x-\pi}{a}\right)$

Question 7 $\left|\dfrac{z-i}{z-1}\right|>1$ when

A. $\text{Arg}(z)\in\left(\dfrac{\pi}{4},\pi\right)$

B. $\text{Arg}(z)\in\left(\dfrac{\pi}{4},\dfrac{3\pi}{4}\right)$

C. $\text{Arg}(z)\in\left(0,\dfrac{\pi}{2}\right)$

D. $\text{Arg}(z)\in\left(-\dfrac{3\pi}{4},\dfrac{\pi}{4}\right)$

E. $\text{Arg}(z)\in\left(-\dfrac{\pi}{2},0\right)$

Question 8 The solution to the equation $\left(\sin^{-1}x-\dfrac{1}{2}\right)\left(\tan^{-1}x-\dfrac{\pi}{3}\right)=0$ is

A. $\sin\left(\dfrac{1}{2}\right)$ or $\sqrt{3}$

B. $\dfrac{\pi}{6}$ or $\sqrt{3}$

C. $\sin\left(\dfrac{1}{2}\right)$ or $\dfrac{\pi}{6}$

D. $\sin\left(\dfrac{1}{2}\right)$ only

E. $\dfrac{\pi}{6}$ only

Question 9 A particle moves in the x-y plane. Its position vector is given by $\tilde{r} = (\cos t - \sin t)\tilde{i} - \left(\frac{1}{2}\cos 2t\right)\tilde{j}$ where $\tilde{i}$ and $\tilde{j}$ are unit vectors in the positive x and y directions respectively.

When $t = \frac{5\pi}{4}$, $\frac{dy}{dx} =$

A. $-\frac{\sqrt{2}}{4}$

B. $-\frac{\sqrt{2}}{2}$

C. $\frac{1}{2}$

D. $\frac{1}{\sqrt{2}}$

E. $\frac{3}{\sqrt{2}}$

Question 10 Given $y = f(x)$, $f'(x) = \frac{1}{2x}$ and $y_0 = f(1) = 0$, then y_2 using Euler's formula with step size 0.1 is closest to

A. 0.095

B. 0.096

C. 0.10

D. 0.11

E. 0.12

Question 11 $v^2 = u^2 + 2as$ is one of the equations for linear motion with non-zero constant acceleration.

A. The four quantities in the equation are all vector quantities.

B. Three out of the four quantities in the equation are vector quantities.

C. Two out of the four quantities in the equation are vector quantities.

D. Only one out of the four quantities in the equation is a vector quantity.

E. The four quantities in the equation are all scalar resolutes in the line of motion.

Question 12 A vector which ***cannot*** be perpendicular to $3\underset{\sim}{i} - \underset{\sim}{j}$ is

A. $\alpha\underset{\sim}{i} + 3\alpha\underset{\sim}{j} + \underset{\sim}{k}$ where $\alpha \in R\backslash\{0\}$

B. $-\alpha\underset{\sim}{i} - 3\alpha\underset{\sim}{j} + (\alpha+1)\underset{\sim}{k}$ where $\alpha \in R$

C. $\alpha\underset{\sim}{i} + 3\alpha\underset{\sim}{j} - \alpha\underset{\sim}{k}$ where $\alpha \in R\backslash\{0\}$

D. $-\alpha\underset{\sim}{i} + 3\alpha\underset{\sim}{j} - (\alpha-1)\underset{\sim}{k}$ where $\alpha \in R$

E. $-\alpha\underset{\sim}{i} + 3\alpha\underset{\sim}{j} - \underset{\sim}{k}$ where $\alpha \in R\backslash\{0\}$

Question 13 The vector resolute of $3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ perpendicular to $\underset{\sim}{b}$ is $\underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k}$.
A vector parallel to $\underset{\sim}{b}$ is

A. $\frac{1}{\sqrt{14}}\underset{\sim}{j}$

B. $\frac{1}{\sqrt{17}}\underset{\sim}{i} - \frac{1}{\sqrt{17}}\underset{\sim}{k}$

C. $\frac{1}{\sqrt{14}}\underset{\sim}{i} + \frac{1}{\sqrt{14}}\underset{\sim}{k}$

D. $\frac{1}{\sqrt{17}}\underset{\sim}{i} - \frac{1}{\sqrt{17}}\underset{\sim}{j} + \frac{1}{\sqrt{17}}\underset{\sim}{k}$

E. $\frac{1}{\sqrt{14}}\underset{\sim}{i} + \frac{1}{\sqrt{14}}\underset{\sim}{j} - \frac{1}{\sqrt{14}}\underset{\sim}{k}$

Question 14 A particle moves in a horizontal plane. $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors pointing to the east and the north respectively. The acceleration (m s^{-2}) of the particle is given by $\underset{\sim}{a} = \underset{\sim}{i} - \underset{\sim}{j}$.
The direction of motion of the particle is

A. north-east (NE)

B. south-east (SE)

C. north-west (NW)

D. south-west (SW)

E. indeterminable without further information

Question 15 Given $\int_{-1}^{0} f(x)dx = 2$ and $\int_{0}^{1} f(x)dx = -\frac{1}{2}$, the value of the definite integral $\int_{-1}^{1} f(|x|)dx$ is

A. -1

B. 1

C. -1.5

D. 1.5

E. -2.5

Question 16 Given $f'(x) = \cot^2\left(\frac{\pi}{4} + x\right)$ and $f\left(\frac{\pi}{4}\right) = 2$, the value of $f(0)$ is closest to

A. 0.22

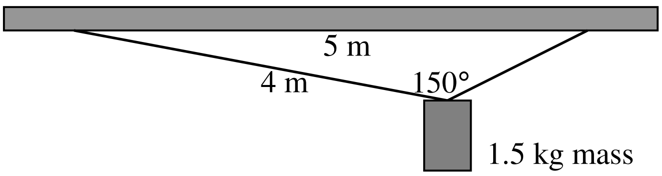
B. 1.8

C. 2.214

D. 2.215

E. 2.216

Question 17 A 1.5 kg mass is suspended by two light strings fastened to the ceiling. The longer string is 4 m long. It is 5 m from the shorter string along the ceiling. The two strings make an angle of 150° at the mass as shown in the diagram.



Given the tension in the longer string is T_L and the tension in the shorter string is T_S, the value of the ratio $\frac{T_S}{T_L}$ is closest to

A. 1.3

B. 1.2

C. 1.1

D. 1.0

E. 0.90

Question 18 Nuts and bolts are manufactured in a factory in large quantities.
The mass of a nut is a random variable denoted by X, and the mass of a bolt is also a random variable and denoted by Y, where $Y = 3X$. Random variable X has a standard deviation of 0.050 grams.
A random sample of two nuts and two bolts are taken from the factory without replacement.
The standard deviation of the total mass (in grams) of the sample is closest to

A. 0.15

B. 0.16

C. 0.19

D. 0.22

E. 1.60

Question 19 The heights of a large population of 15 year old students have an approximate normal distribution with mean of 168.0 cm and standard deviation of 9.0 cm. The probability that a random sample of nine students has an average height of 175.0 cm is closest to

A. 0.01

B. 0.02

C. 0.12

D. 0.21

E. 0.22

Question 20 A random sample of 36 of a brand of long-life batteries was measured to last for an average period $\bar{x} = 29.52$ hours. Assuming the standard deviation of the battery life is 0.45 hours, the 95% confidence interval for the population mean μ is calculated to be $(29.37, 29.67)$, corrected to two decimal places.
Which one of the following statement is true?

A. $\Pr(29.37 < \mu < 29.67) \approx 0.95$

B. $\Pr\left((29.37 < \mu < 29.67)'\right) \approx 0.05$

C. $\mu \in (29.37, 29.67)$ always

D. $\bar{x} \in (29.37, 29.67)$ always

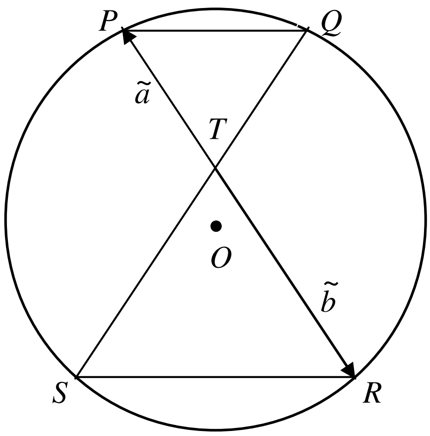
E. $|\bar{x} - \mu| < 0.15$ always

SECTION B Extended-answer questions

Instructions for Section B

Answer **all** questions.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this examination are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

Question 1 Two ***equilateral*** triangles are drawn inside a circle by four chords as shown in the following diagram. The radius of the circle is r.
The magnitudes of vectors $\tilde{a}$ and $\tilde{b}$ are a and b respectively, where $\tilde{a} = \overrightarrow{TP}$ and $\tilde{b} = \overrightarrow{TR}$.
O is the centre of the circle. T is the intersection of chords PR and QS.



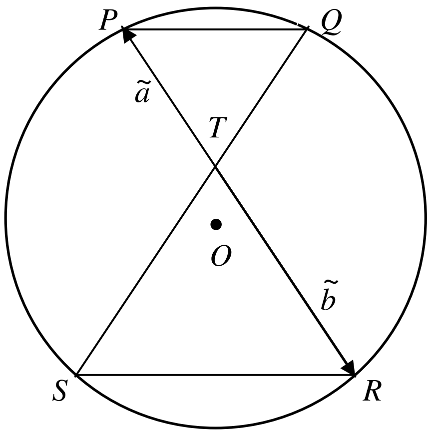
a. Determine the size of $\angle PTS$ in degrees. 1 mark

b. Determine the size of $\angle POS$ in degrees. 1 mark

c. Let $\overrightarrow{TS}$ be $\tilde{c}$. Express $\overrightarrow{PS}$ in terms of $\tilde{a}$ and $\tilde{c}$. 1 mark

d. Hence find $|\overrightarrow{PS}|$ in terms of a and b. 2 marks

e. Use vector methods to show that $a^2 + ab + b^2 = 3r^2$. 3 marks
The diagram is redrawn for your convenience.



Question 2 Equation $|z-1|+|z-i|=2$ defines an ellipse on an Argand diagram.

a. Write down the complex number at the centre, *C*, of the ellipse. 1 mark

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b. Determine the length of the longest chord of the ellipse. 1 mark

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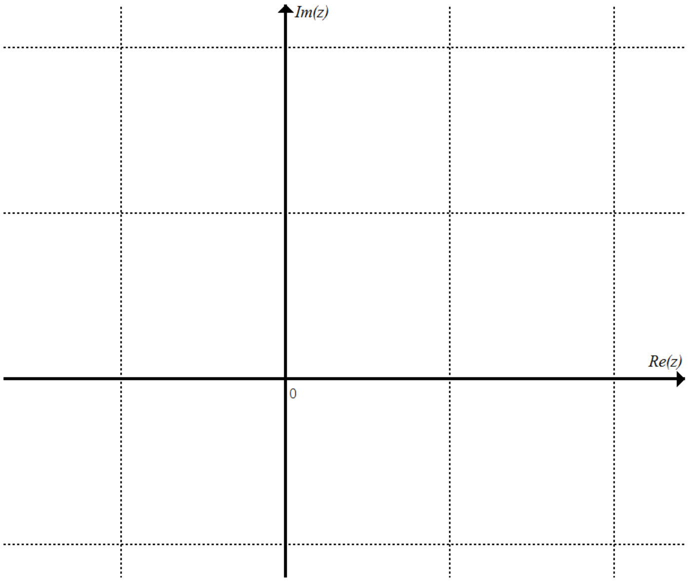
c. Sketch the ellipse accurately on the Argand diagram below. Mark and label the two complex numbers on the ellipse closest to C, and the two complex numbers on the ellipse furthest from C.

4 marks



d. Let $z=x+yi$. Show that the cartesian equation for $|z-1|+|z-i|=2$ is $3(x+y)^2=4(x+xy+y)$.

3 marks

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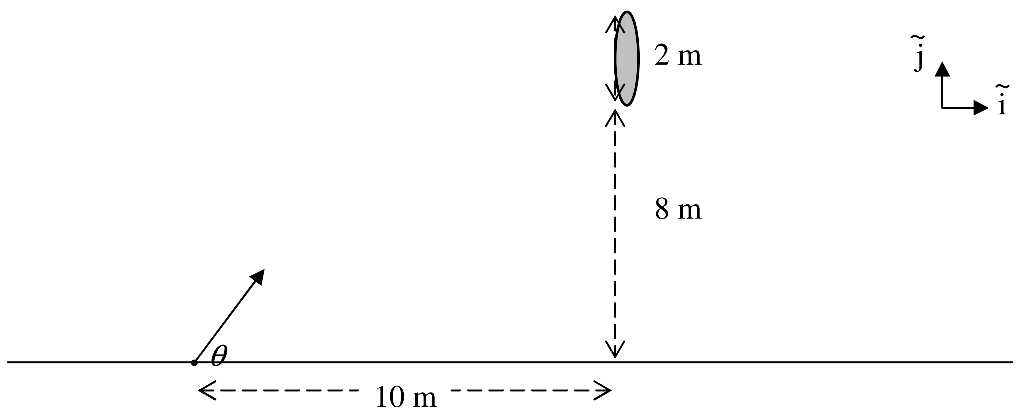
e. When $\frac{dy}{dx} = 0$, show that $y = 2 - 3x$. 2 marks

f. Hence find the exact value of $\text{Re}(z)$ when $\text{Im}(z)$ is a minimum value. 2 marks

Question 3 A vertical circular ring is positioned 8 m above the ground measured from the bottom of the ring. The diameter of the ring is 2 m. A particle is projected at ground level at an angle of θ with the horizontal. Initially ($t = 0$), at the point of projection, the particle is 10 m horizontally from the ring
The speed of projection is 20 m s^{-1}. The particle moves towards the ring under gravity only.
$\tilde{i}$ and $\tilde{j}$ are unit vectors pointing in the directions as shown in the following diagram.



The acceleration of the particle at time t (in seconds) is given by $\tilde{a} = -9.8\,\tilde{j}$.

a. By anti-differentiation show that the position of the particle at time t is given by
$\tilde{r}(t) = (20\cos\theta)t\,\tilde{i} + \left((20\sin\theta)t - 4.9t^2\right)\tilde{j}$. 2 marks

b. Set up x (in the $\tilde{i}$ direction) and y (in the $\tilde{j}$ direction) axes with the origin O at the point of projection.
Show that the path of the particle is given by $y = (\tan\theta)\,x - \dfrac{0.01225x^2}{\cos^2\theta}$. 2 marks

c. Find the values of θ and express your answers in degrees to 1 decimal place, that allow the particle to pass through the region enclosed by the ring (shaded). Write your answers as two inclusive intervals.

3 marks

d. The particle hits the highest point of the ring. Determine the longest distance, in metres correct to 1 decimal place, travelled by the particle before it hits the ring.

2 marks

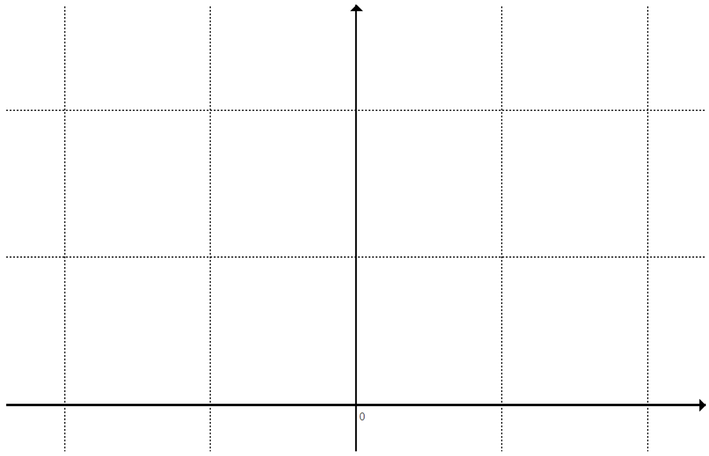
e. Determine the average speed, in m s^{-1} correct to 1 decimal place, of the particle along the path described in part d.

2 marks

Question 4 A container is formed by rotating about the y-axis the curve $y = 2\tan^{-1}(x-1)$ for $1 \le x \le 1+\sqrt{3}$. In this question length is measured in metres.

a. Sketch the curve on the diagram below. Show clear scales on the axes and coordinates of the end points.

2 marks



b. Write a definite integral giving the maximum volume of the container. 2 marks

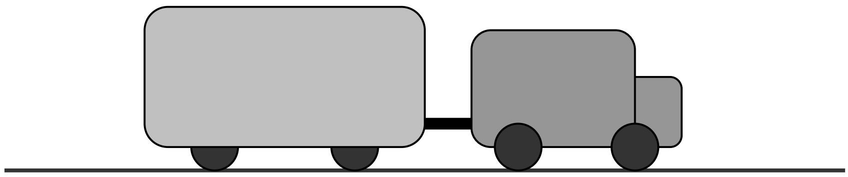
c. Find the volume of water in the container when it is filled to a depth of h metres.
Express your answer in terms of h. 2 marks

d. The flow of water into the container is controlled so that the depth is rising at a constant rate of 0.05 m per minute. Determine the rate of flow of water into the container when it is filled to half of its full capacity. Write your answer in m^3 per minute, correct to 2 decimal places.

4 marks

e. Give the exact time taken (in minutes) to completely fill the container. 1 mark

Question 5 A caravan (2000 kg) is towed by a large car (2000 kg) on a horizontal road. While they are in motion the resistive forces are $240v$ newtons on the car and $160v$ newtons on the caravan. The maximum driving force of the car is 12000 newtons. The car is at rest initially.



a. Calculate the maximum acceleration of the caravan when the car travels at 10 m s^{-1}. 1 mark

b. Calculate the tension in the tow bar when the car travels at 10 m s^{-1} at maximum acceleration. 1 mark

c. Calculate the maximum speed of the car while towing the caravan. 1 mark

d. Calculate the maximum total forward momentum of the car and caravan. 1 mark

e. Assume that maximum driving force is maintained since the start of the motion.

(i) Show that $\frac{dv}{dx} = \frac{3}{v} - \frac{1}{10}$. 2 marks

(ii) Hence find the *exact* distance travelled by the car when the speed of 10 m s^{-1} is reached. 3 marks

Question 6

A factory has a machine which makes thick floor tiles. The thickness measurements of these tiles are normally distributed with the mean $\mu = 25.0$ millimetres and the standard deviation $\sigma = 0.2$ millimetres.

Many random samples of 5 tiles are taken, and the mean thickness $\overline{X}$ of each sample is calculated.

a. Estimate the mean of $\overline{X}$. 1 mark

b. Calculate the standard deviation of $\overline{X}$, correct to 3 decimal places. 1 mark

c. Find the probability that $24.8 < \overline{X} < 25.2$, correct to 3 decimal places. 2 marks

The factory has a second machine which also makes thick floor tiles.
A random sample of 5 tiles made by the second machine is taken. The thickness of each tile from the sample is measured, and the mean thickness of the tiles is calculated to be 25.1 millimetres.

d. Estimate the mean thickness of the tiles made by the second machine. 1 mark

e. The thickness measurements of the tiles from the second machine are also normally distributed with the same standard deviation $\sigma = 0.2$ millimetres as the first machine.

i) Calculate the approximate 95% confidence interval for the mean thickness of the tiles made by the second machine to 3 decimal places.

2 marks

ii) 20 random samples of 5 tiles made by the second machine are taken. The approximate 95% confidence interval for the mean thickness of the tiles made by the second machine is calculated for each sample.
How many are expected to contain the mean value of the thickness of the tiles made by the second machine?

1 mark

End of Exam 2