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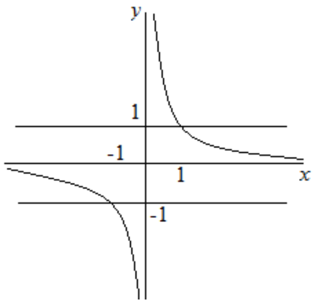
2017 VCAA Specialist Mathematics Exam 2 Solutions

SECTION A – Multiple-choice questions

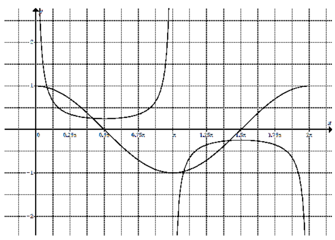
1	2	3	4	5	6	7	8	9	10
C	E	D	E	A	B	D	B	B	E

11	12	13	14	15	16	17	18	19	20
C	A	C	A	B	D	A	E	D	C

Q1 $-1 \le \frac{1}{x} \le 1$, .: $x \le 1$ or $x \ge 1$ **C**



Q2 **E**



Q3 $(z+i)^2(z-i)(z+2i)(z+1)(z-1)=0$ **D**

Q4 $z^n=\sqrt{2}\text{cis}\left(\frac{\pi}{4}+2\pi k\right)$, $z=2^{\frac{1}{2n}}\text{cis}\left(\frac{\pi}{4n}+\frac{2\pi k}{n}\right)$ **E**

Q5 $|z-2+i|=|z-4|$ defines a perpendicular bisector of a line segment joining $z=2-i$ and $z=4$. $z=3-\frac{1}{2}i$ lies at the midpoint of the line segment. .: it also lies on the perpendicular bisector. **A**

Q6 $\frac{dy}{dx}=e^x\tan^{-1}y$, $\frac{d^2y}{dx^2}=\frac{e^x}{1+y^2}\frac{dy}{dx}+e^x\tan^{-1}y=\frac{3\pi}{8}$ at $(0,1)$ **B**

Q7 $u=2-x$, $x=2-u$, $\frac{du}{dx}=-1$

$\int_1^2 x^2\sqrt{2-x}\,dx=-\int_1^2(2-u)^2u^{\frac{1}{2}}\frac{du}{dx}dx=-\int_1^0\left(4u^{\frac{1}{2}}-4u^{\frac{3}{2}}+u^{\frac{5}{2}}\right)du$ **D**

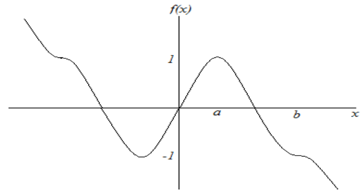
Q8 $f'(x)=3x^2-2mx$, $f''(x)=6x-2m\ge 0$, $x\ge\frac{m}{3}$ **B**

Q9 $x=1$, $y=2$, $\frac{dy}{dx}=4$

$x=0.9$, $y\approx 2-0.1\times 4=1.6$, $\frac{dy}{dx}=3.52$

$x=0.8$, $y\approx 1.6-0.1\times 3.52=1.248$ **B**

Q10 The following graph shows a possible $f(x)$. **E**



Q11 Let $\tilde{a}=m\tilde{b}+n\tilde{c}$

.: $2i+3j+dk=(m+2n)\tilde{i}+(m+n)\tilde{j}-(4m+2n)\tilde{k}$

.: $m+2n=2$, $m+n=3$ and $d=-(4m+2n)$.: $d=-14$ **C**

Q12 $\sin t=\frac{1-x}{\sqrt{a}}$, $\cos t=b(1-y)$

$\left(\frac{1-x}{\sqrt{a}}\right)^2+(b(1-y))^2=1$, $\frac{1}{a}=b^2$ to be a circle, .: $ab^2=1$ **A**

Q13 $\hat{b}=\frac{1}{3}\left(2\tilde{i}+2\tilde{j}-\tilde{k}\right)$, $(\tilde{a}.\hat{b})\hat{b}=-\frac{14}{9}\left(2\tilde{i}+2\tilde{j}-\tilde{k}\right)$ **C**

Q14 $m_1g\sin 2\theta=m_2g\sin\theta$, $\frac{m_1}{m_2}=\frac{\sin\theta}{\sin 2\theta}=\frac{\sin\theta}{2\sin\theta\cos\theta}=\frac{\sec\theta}{2}$ **A**

Q15 $\tilde{v}=\frac{\left(-\tilde{i}+5\tilde{j}\right)-\left(3\tilde{i}+\tilde{j}\right)}{2}=-2\tilde{i}+2\tilde{j}$ **B**

Q16 Resultant force $R=80\cos 40°-20=41.3\,\text{N}$

$R=\frac{m\Delta v}{\Delta t}=\frac{m(v-0)}{\Delta t}=\frac{p}{\Delta t}$, $p=R\Delta t=41.3\times 5\approx 207$ **D**

Q17 Resultant force $=\sqrt{10^2+8^2-2(10)(8)\cos 120°}\approx 15.62$ N

$\frac{\sin\theta}{8}=\frac{\sin 120°}{15.62}$, $\theta=26.3°$ **A**

Q18 $E(W)=E(4U-3V)=4\times 5-3\times 8=-4$

$Var(4U-3V)=Var(4U+{}^-3V)=4^2Var(U)+\left({}^-3\right)^2V=25$

.: $\sigma=\sqrt{25}=5$.: $\Pr(W>5)=\Pr\left(Z>\frac{5-(-4)}{5}\right)=\Pr(Z>1.8)$ **E**

Q19 Original width: $2\times z\frac{s}{\sqrt{n}}$, reduced width: $2\times z\frac{s}{\sqrt{m}}$

$2\times z\frac{s}{\sqrt{m}}=25\%\times 2\times z\frac{s}{\sqrt{n}}$, .: $\frac{1}{\sqrt{m}}=\frac{1}{4\sqrt{n}}$, .: $m=16n$ **D**

Q20 **C**

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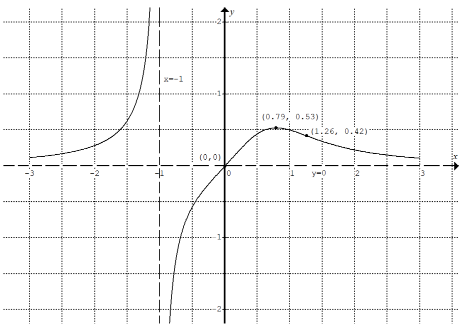
SECTION B

Q1a $x=-1$, $y=0$

Q1aii $f'(x)=\dfrac{1-2x^3}{(1+x^3)^2}$. Let $f'(x)=0$, $x=\dfrac{1}{\sqrt[3]{2}}\approx 0.79$, $y\approx 0.53$

Q1aiii Let $f''(x)=0$, $x\approx 1.26$, $y\approx 0.42$

Q1b



Q1ci $\displaystyle\int_0^a \pi\left(\frac{x}{1+x^3}\right)^2 dx=\int_a^3 \pi\left(\frac{x}{1+x^3}\right)^2 dx$

Q1cii $a=0.98$

Q2a $s=\frac{1}{2}gt^2=\frac{1}{2}(9.8)2^2=19.6$, distance $=19.6$ m

Q2b $v=gt=9.8\times 2=19.6$ m s^{-1}

Q2c $a=g-0.01v^2=0$, $v=\sqrt{980}=14\sqrt{5}$ m s^{-1}

Q2di $\dfrac{dv}{dt}=g-0.01v^2$, $\dfrac{dt}{dv}=\dfrac{1}{9.8-0.01v^2}$

From the start of the fall, time taken is $t=\displaystyle\int_{19.6}^{30}\frac{1}{9.8-0.01v^2}dv+2$

Q2dii Time taken ≈ 5.8 s

Q2e $v\dfrac{dv}{dx}=9.8-0.01v^2$, $\dfrac{dx}{dv}=\dfrac{v}{9.8-0.01v^2}$

$x=\displaystyle\int_{19.6}^{30}\frac{v}{9.8-0.01v^2}dv+19.6$, total distance fallen ≈ 120 m

Q3a $x=\sqrt{2}$, $y=3\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)=\frac{3\pi}{4}$, $\left(\sqrt{2},\frac{3\pi}{4}\right)$

Q3b $\begin{cases} -3\cos^{-1}\left(\frac{-x}{2}\right) & -2\le x<-\sqrt{2} \\ 3\sin^{-1}\left(\frac{x}{2}\right) & -\sqrt{2}\le x\le 0\end{cases}$

Q3c Area $=4\times\displaystyle\int_0^{\frac{3\pi}{4}}\left(2\cos\left(\frac{y}{3}\right)-2\sin\left(\frac{y}{3}\right)\right)dy=24\left(\sqrt{2}-1\right)\approx 9.9\text{ cm}^2$

Q3d First quadrant: $y=3\sin^{-1}\left(\frac{x}{2}\right)$, $\dfrac{dy}{dx}=\dfrac{3}{\sqrt{4-x^2}}=\dfrac{3}{2}$ at $x=0$

Angle with y-axis $=\tan^{-1}\left(\frac{2}{3}\right)\approx 33.7°$

Acute angle between the edges $\approx 2\times 33.7=67.4°$

Q3e Length $=4\times\displaystyle\int_0^{\sqrt{2}}\sqrt{1+\left(\frac{3}{\sqrt{4-x^2}}\right)^2}\,dx+4\times\int_{\sqrt{2}}^{2}\sqrt{1+\left(\frac{-3}{\sqrt{4-x^2}}\right)^2}\,dx$

$=\displaystyle\int_0^2 4\sqrt{1+\frac{9}{4-x^2}}\,dx=\int_0^2\sqrt{16+\frac{144}{4-x^2}}\,dx$, .: $a=16$ and $b=144$

Q4a $-2-2\sqrt{3}\,i=4\text{cis}\left(-\frac{2\pi}{3}\right)$

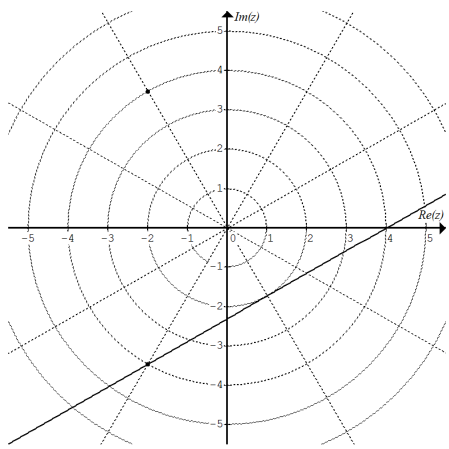
Q4b $z=\dfrac{-4\pm\sqrt{16-64}}{2}=-2\pm 2\sqrt{3}\,i$

Q4c $-2+2\sqrt{3}\,i=-\left(2-2\sqrt{3}\,i\right)$, i.e. anticlockwise (or clockwise) rotation of $2-2\sqrt{3}\,i$ by 180° about the origin.

$-2-2\sqrt{3}\,i=-4+\left(2-2\sqrt{3}\,i\right)$, i.e. horizontal translation of $2-2\sqrt{3}\,i$ to the left by 4 units.

Q4d $\sqrt{x^2+y^2}=\sqrt{(x-2)^2+\left(y+2\sqrt{3}\right)^2}$, $x-\sqrt{3}\,y-4=0$

Q4e



Q4f b can be obtained by applying the following sequence of transformations to a.

$a \to \overline{a} \to i^2\,\overline{a} \to i^2\,\overline{a} - 4 = b$, .: $b = i^2\,\overline{a} - 4 = -\overline{a} - 4$

Alternatively,
let $a = \mathrm{Re}(a) + \mathrm{Im}(a)i$ and $b = \mathrm{Re}(b) + \mathrm{Im}(b)i$ where

$\dfrac{\mathrm{Re}(a) + \mathrm{Re}(b)}{2} = -2$ and $\mathrm{Im}(b) = \mathrm{Im}(a)$, .: $\dfrac{a+b}{2} = -2 + \mathrm{Im}(a)i$

.: $b = -4 - a + 2\,\mathrm{Im}(a)i = -4 - (\mathrm{Re}(a) - \mathrm{Im}(a)i)$, .: $b = -4 - \overline{a}$

Q4g Area $= \dfrac{1}{2}\left(4\sqrt{3}\right)(2) + \dfrac{2}{3}\pi\, 4^2 = 4\sqrt{3} + \dfrac{32\pi}{3}$

Q5a $\tilde{r}_{\mathrm{B}}(0) = -\tilde{i} + 3\tilde{j}$, $\mathrm{B}(-1, 3)$; $\tilde{r}_{\mathrm{J}}(0) = \tilde{i} + \tilde{j}$, $\mathrm{J}(1, 1)$

As t increases from 0, $1 - 2\cos t$ increases, $1 - \sin t$ decreases
Both move clockwise (viewing the diagram).

Q5bi $\tilde{\dot{r}}_{\mathrm{B}} = (2\sin t)\tilde{i} + (\cos t)\tilde{j}$, $\tilde{\dot{r}}_{\mathrm{J}} = (-\cos t)\tilde{i} + (\sin t)\tilde{j}$, $t > 0$

Same speed: $4\sin^2 t + \cos^2 t = \cos^2 t + \sin^2 t$, .: $\sin t = 0$, $t = \pi$

Q5bii $\tilde{r}_{\mathrm{B}}(\pi) = 3\tilde{i} + 3\tilde{j}$, $\mathrm{B}(3, 3)$

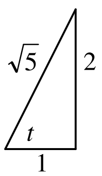
Q5ci $|\tilde{r}_{\mathrm{B}} - \tilde{r}_{\mathrm{J}}| = |(\sin t - 2\cos t)\tilde{i} + (1 + \sin t + \cos t)\tilde{j}\,|$

$= \sqrt{(\sin t - 2\cos t)^2 + (1 + \sin t + \cos t)^2}$

Q5cii Min. distance ≈ 0.33 km

Q5d At the same place and at the same time:
Let $x = 1 - 2\cos t = 1 - \sin t$ and $y = 3 + \sin t = a - \cos t$

.: $\sin t = 2\cos t$, .: $\tan t = 2$, $\sin t = \dfrac{2}{\sqrt{5}}$, $\cos t = \dfrac{1}{\sqrt{5}}$



.: $3 + \dfrac{2}{\sqrt{5}} = a - \dfrac{1}{\sqrt{5}}$, .: $a = 3\left(1 + \dfrac{1}{\sqrt{5}}\right)$ when $t = \tan^{-1}(2)$

Q6a $\Pr(X \ge 2000) \approx 0.798 = 79.8\%$

Q6b mean $= n\mu = 10 \times 2005 = 20050$ mL

standard deviation $= n\dfrac{\sigma}{\sqrt{n}} = \sqrt{n}\,\sigma = 6\sqrt{10}$ mL

Q6c Using the figures calculated in Q6b,
$\Pr(X \ge 20000) = 0.996 = 99.6\%$

Q6d $\Pr\left(Z \ge \dfrac{20000 - 20050}{\sqrt{10}\,\sigma}\right) \ge 0.999$ or $\Pr\left(Z < \dfrac{-50}{\sqrt{10}\,\sigma}\right) \le 0.001$

.: $\dfrac{-50}{\sqrt{10}\,\sigma} = -3.09$, $\sigma \approx 5.1$

Q6e $n = 10$, mean of $\overline{X} = \mu = 2005$, $\mathrm{sd}(\overline{X}) = \dfrac{\sigma}{\sqrt{n}} = \dfrac{2}{\sqrt{10}}$

$p = \Pr(\overline{X} \le 2004) \approx 0.0569$, .: $p > 0.05$, .: insufficient evidence against the claim, hence the claim should be accepted.

Please inform mathline@itute.com re conceptual and/or mathematical errors