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2017 Trial Examination

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STUDENT NUMBER | | | | | | | | | Letter | |

SPECIALIST

Units 3 & 4 – Written Examination 2

Reading time: 15 minutes
Writing time: 2 hours

QUESTION & ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 23 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

SECTION A - Multiple Choice Questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where g = 9.8.

Question 1

The Cartesian equation of the relation given by $x = \frac{1}{3}\cos(2t)$ and $y = \frac{1}{2}\sin(t)$ is:

A. $9x^2 + 4y^2 = 1$

B. $\frac{x^2}{9} + \frac{y^2}{4} = 1$

C. $3x^2 + 4y^2 - 1 = 0$

D. $3x + 8y^2 - 1 = 0$

E. $3x + 4y^2 - 1 = 0$

Question 2

The range of the function $y = 2\arctan(\sqrt{3}) - 3\arctan(3x + 1)$ is:

A. $[-\frac{5\pi}{6}, \frac{13\pi}{6}]$

B. $(-\frac{5\pi}{6}, \frac{13\pi}{6})$

C. $[-\frac{2+\pi}{6}, \frac{\pi-2}{6}]$

D. $(-\frac{2+\pi}{6}, \frac{\pi-2}{6})$

E. $(-\frac{2+\pi}{2}, \frac{\pi-2}{2})$

SECTION A - continued

Question 3

The straight line asymptotes of the graph of the function with rule $f(x)=\frac{2x^3+5x^2+2x-1}{x^2-1}$ are given by:

A. $x=-1$ and $y=2x+5$

B. $x=-1,\ x=1$ and $y=2x+5$

C. $x=1$ and $x=-\frac{5}{2}$

D. $x=-1, x=1$ and $y=-\frac{5}{2}$

E. $x=1$ and $y=2x+5$

Question 4

One of the roots of the polynomial $z^3+bz^2+cz-39,\ b,c\in R$ is $2+3i$. Then the values of b and c are respectively:

A. 1 and 1

B. -7 and -1

C. -1 and 1

D. -7 and 25

E. 7 and -25

Question 5

If $rg(a-2bi)=\frac{5\pi}{6}$, then the real values of a and b could be:

A. -6 and $-\sqrt{3}$ respectively.

B. $-\sqrt{3}$ and -6 respectively.

C. 6 and $\sqrt{3}$ respectively.

D. $\sqrt{3}$ and 6 respectively.

E. -6 and $\sqrt{3}$ respectively.

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Question 6

z_1, z_2, z_3 and z_4 are the roots of the equation $z^4 = -ri$, where r is a positive real number.
Which one of the following statements is **not** true?

A. $z_1 + z_2 + z_3 + z_4 = 0$

B. $z_1z_2 + z_2z_3 + z_3z_4 + z_4z_1 = 0$

C. $z_1z_2z_3 + z_2z_3z_4 + z_3z_4z_1 + z_4z_1z_2 = 0$

D. $z_1z_2z_3z_4 = ri$

E. $z_1 = \overline{z_2},\ \ z_3 = \overline{z_4}$

Question 7

Given that $x = \tan(t) - 2t$ and $= 2\log_e(\sec(t)),\ t \in (0, \frac{\pi}{2})$, then $\frac{dy}{dx}$ in terms of t is:

A. $\cot(2t)$

B. $\frac{2}{\sec^3(t) - 2\sec(t)}$

C. $-\tan(2t)$

D. $2\cos^3(t) - 4\cos(t)$

E. $2\tan(2t)$

Question 8

Using a suitable substitution, the definite integral

$$\int_0^1 ((3x^2 - 2x + 19)\sqrt[3]{x^3 - x^2 + 19x + 8})dx$$

is equivalent to:

A. $\int_2^3 u^{\frac{1}{3}}\, du$

B. $\int_2^3 u^3 du$

C. $\frac{1}{3}\int_2^3 u^3 du$

D. $\int_8^{27} u^{\frac{1}{3}}\, du$

E. $\int_8^{27} u^3 du$

SECTION A - continued

Question 9

If $f(x) = \frac{dy}{dx} = \sin(x) + \cos(x)$, where $y_0 = y(0) = 1$, then y_3 using Euler's formula with step size 0.1 is closest to:

A. $1.1 + 0.1f(0.1)$

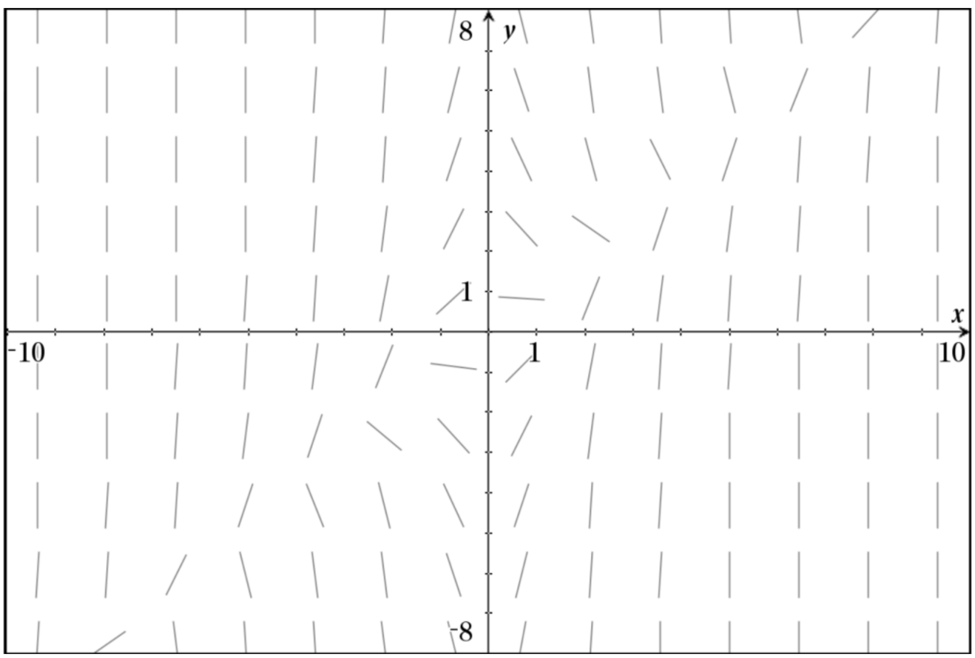
B. $1 + 0.1f(0.1)$

C. $1.21 + 0.1f(0.2)$

D. $1.21 + 0.2f(0.2)$

E. $1.21 + 0.2f(0.1)$

Question 10



The direction field for the differential equation $\frac{dy}{dx} - x^2 + xy = 0$ is shown above. A solution of this different equation that includes $(-1, -1)$ could also include:

A. $(4,\ 2.1)$

B. $(-2, -3.2)$

C. $(3,\ 1)$

D. $(-4, -2)$

E. $(5,\ 4.6)$

SECTION A – continued
TURN OVER

Question 11

A type of chemical solution with concentration 200g/L flows into a cylindrical tank at a rate of 8L per minute. This tank initially has 300L of the same type of solution. At the same time the solution flows out of the tank at a rate of 6L per minute. Let Q g be the amount of chemical in the tank after t minutes. Then a correct differential equation regarding Q is:

A. $\frac{dQ}{dt}+\frac{Q}{300+2t}=1600$

B. $\frac{dQ}{dt}+\frac{3Q}{150+t}=200$

C. $\frac{dQ}{dt}+\frac{3Q}{150-t}=1600$

D. $\frac{dQ}{dt}+\frac{3Q}{150+t}=1600$

E. $\frac{dQ}{dt}-\frac{3Q}{150+t}=1600$

Question 12

The magnitude of the vector component of $\underset{\sim}{u}=-\underset{\sim}{i}+a\underset{\sim}{j}+\ \underset{\sim}{k}$ in the perpendicular direction of the vector $\underset{\sim}{v}=\underset{\sim}{i}+3\underset{\sim}{j}+2\ \underset{\sim}{k}$ is $\frac{\sqrt{10}}{2}$. The values of a are:

A. 2 or $-\frac{4}{5}$.

B. $\frac{-1+\sqrt{35}}{3}$ or $\frac{-1-\sqrt{35}}{3}$

C. $\frac{-72+\sqrt{170}}{218}$ or $\frac{-72-\sqrt{170}}{218}$

D. -0.55 or -0.01

E. $\frac{-13+\sqrt{35}}{3}$ or $\frac{-13-\sqrt{35}}{3}$

Question 13

If the vectors $\underset{\sim}{a}=3\underset{\sim}{i}+2\underset{\sim}{j}+4\ \underset{\sim}{k}$, $\underset{\sim}{b}=m\underset{\sim}{i}+\underset{\sim}{j}-2\ \underset{\sim}{k}$ and $\underset{\sim}{c}=\underset{\sim}{i}+m\underset{\sim}{j}+\ 2\underset{\sim}{k}$ are linearly independent then the values of m must be:

A. 2 or 1.

B. $m\neq 1,2$

C. $m\in R\backslash\{-1,\frac{1}{2}\}$.

D. -1 or $\frac{1}{2}$.

E. $m\in R\backslash\{-1,-2\}$

SECTION A - continued

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Question 14

A particle of mass 12 kg is subject to forces $\underset{\sim}{F}_1 = 3\underset{\sim}{i} - 4\underset{\sim}{j}$ newtons and $\underset{\sim}{F}_2 = -8\underset{\sim}{i} + 6\underset{\sim}{j}$ newtons. If no other forces act on the particle, the magnitude of the particle's acceleration, in ms^{-2}, is:

A. $\sqrt{29}$

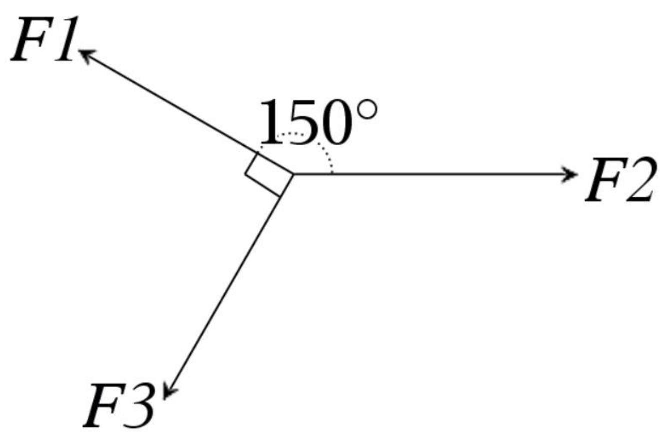
B. $-5\underset{\sim}{i} + 2\underset{\sim}{j}$

C. $-\frac{5}{12}\underset{\sim}{i} + \frac{1}{6}\underset{\sim}{j}$

D. $\frac{5}{4}$

E. $\frac{\sqrt{29}}{12}$.

Question 15



A block is acted on by three forces with magnitude F_1, F_2 and F_3, as shown above. Which one of the following statements is true?

A. $\frac{2\sqrt{3}}{3}F_1 = F_2 = 2F_3$

B. $2F_1 = \sqrt{3}F_2 = F_3$

C. $2F_1 = F_2 = \frac{2\sqrt{3}F_3}{3}$

D. $2\sqrt{3}F_1 = \sqrt{3}F_2 = 2F_3$

E. $\frac{2F_1}{\sqrt{3}} = 2F_2 = F_3$

SECTION A - continued
TURN OVER

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Question 16

A netball is hit at a height of 2.5 m from the ground at a depression angle of 15° to the horizontal. The initial speed of the ball is $30\,ms^{-1}$. Neglecting air resistance, the distance travelled horizontally by the ball before hitting the ground is closest to:

A. 9.01 m

B. 7.95 m

C. 8.1 m

D. 8.3 m

E. 6.2m

Question 17

The acceleration, in ms^{-2}, of a particle is given by $a = \frac{v(2x+1)}{(x+1)(3x+2)}$, where v and x are the velocity and displacement t seconds after $t = 0$. When $x = 2$, $v = 5\ m/s$. The velocity when $x = 10$ is equal to

A. $\int_2^{10}(\frac{1}{x+1} - \frac{1}{3x+2})dx$

B. $\int_2^{10}(\frac{1}{x+1} - \frac{1}{3x+2})dx + 5$

C. $\int_5^{10}(\frac{1}{x+1} - \frac{1}{3x+2})dx + 2$

D. $\int_2^{10}(\frac{1}{3x+1} - \frac{1}{x+1})dx + 5$

E. $\int_2^{10} v(\frac{1}{x+1} - \frac{1}{3x+1})dx + 5$

SECTION A - continued

Question 18
In a farm the weight of large apples is normally distributed with mean 430 g and standard deviation 25 g. The weight of small apples is normally distributed with mean 250 g and standard deviation 10 g. The mean, μ g, and standard deviation, σ g, of the total weight of 5 large apples and 10 small apples are

A. $\mu = 680,\ \sigma = 35$
B. $\mu = 4650,\ \sigma = 5\sqrt{165}$
C. $\mu = 4650,\ \sigma = 5\sqrt{665}$
D. $\mu = 4650,\ \sigma = 15$
E. $\mu = 4650,\ \sigma = 5\sqrt{65}$

Question 19
A random variable X is normally distributed with an unknown mean, μ, and a known standard deviation 5.2. A random sample of size 64 was selected from this population, and the average of this sample was determined to be 20.5. A 95% confidence interval for μ is approximately

A. (19.43, 21.57)

B. (18.83, 22.17)

C. (19.56, 21.43)

D. (19.67, 21.33)

E. (19.23, 21.77)

Question 20
In a market the mean of the weight of a type of oranges is 200g and the standard deviation is 30g. The weights of the oranges are normally distributed. A random sample of 36 oranges are selected from the population. It's found that the sample mean is 211.6g. If these data are used to test the hypotheses H_0: $\mu = 200$ and $H_1 : \mu > 200$, with $\alpha = 0.05$, then the p-value for this class and the conclusion are

A. p = 0.0102 and do not reject H_0

B. p = 0. 0102 and reject H_0

C. p = 0.1718 and reject H_0

D. p = 0.1718 and reject H_1

E. p = 0.0102 and reject H_1

END OF SECTION A
TURN OVER

SECTION B – Extended Response questions

Instructions for Section B
Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where g = 9.8.

Question 1 (10 marks)

Let $f(x) = \frac{2x^3 - 4x^2 + x - 5}{x-2}$, $x \in R\backslash\{2\}$.

a. Find the stationary point of the graph of $f(x)$. Express your answer in coordinate form, giving values correct to two decimal places.

1 mark

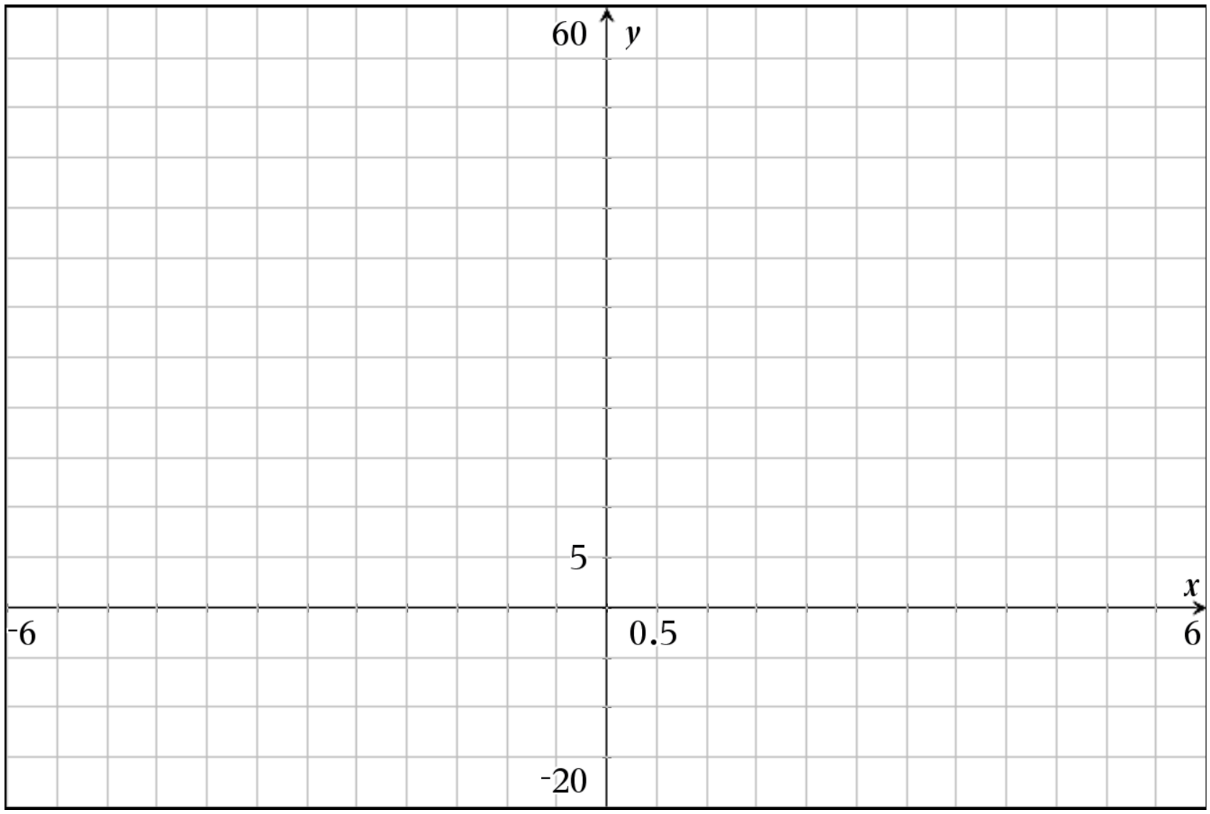
b. Find the point of inflection of the graph given in Part a. Express your answer in coordinate form, giving values correct to two decimal places.

2 marks

SECTION B- Question 1- continued

c. Sketch the graph of $f(x) = \frac{2x^3-4x^2+x-5}{x-2}$ for $x \in [-3, 2) \cup (2, 5]$ on the axes below, labelling the turning point and the point of inflection with their coordinates, correct to two decimal places. Intercepts should also be labelled.

3 marks



A glass is to be modelled by rotating the curve that is part of the graph where $x \in [-3, 1]$ about the x-axis, to form a solid revolution.

d. **i**. Write down a definite integral, in terms of x, which gives the length of the curve to be rotated.

1 mark

ii. Find the length of this curve, correct to two decimal places.

1 mark

SECTION B- Question 1- continued
TURN OVER

e. Find the volume of the solid, giving values correct to the nearest integer.

2 mark

Question 2 (11 marks)

A line in the complex plane is given by $\left|z+\frac{9}{2}+\left(\frac{7\sqrt{3}}{2}-1\right)i\right|=\left|z-\left(\sqrt{3}+1\right)i\right|,\ z\in C$.

a. Find the equation of this line in the form $y=mx+c$.

2 marks

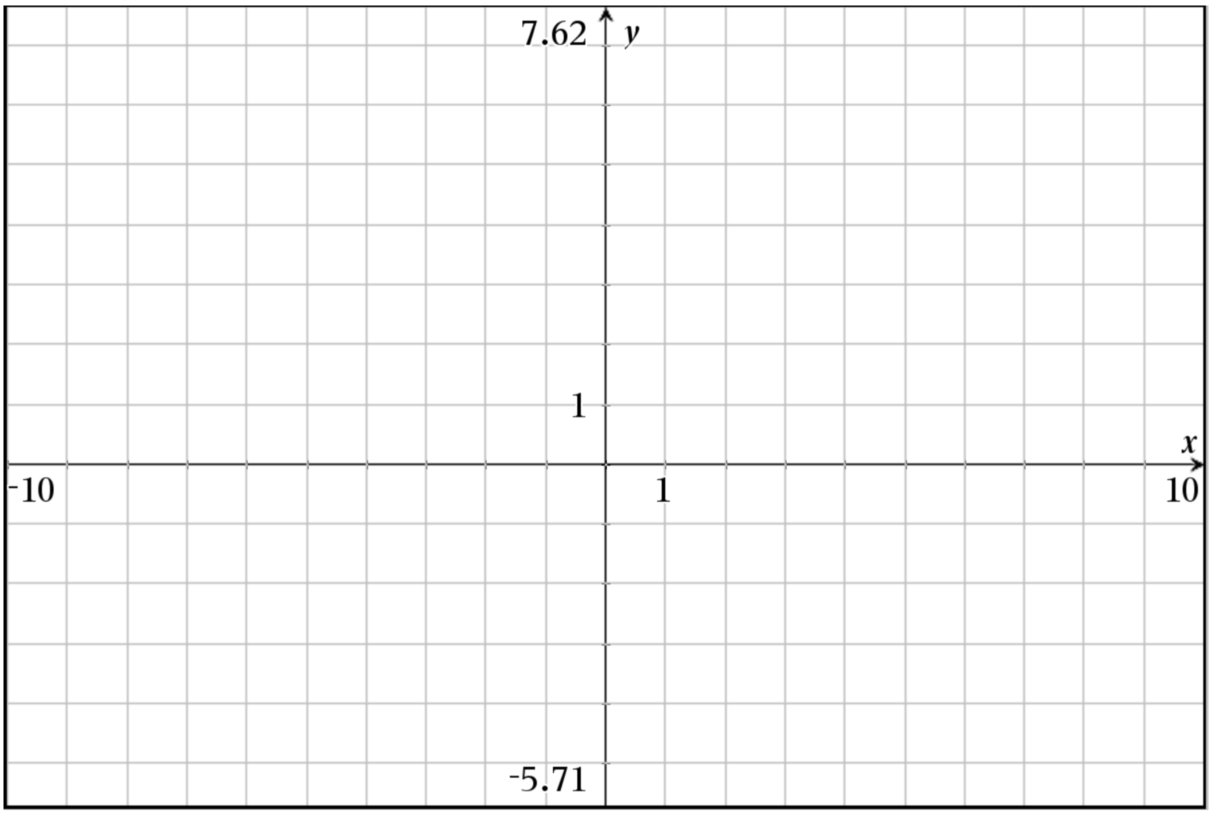
b. Find the intersecting points B and C, where B is in the second quadrant, between the line $\left|z+\frac{9}{2}+\left(\frac{7\sqrt{3}}{2}-1\right)i\right|=\left|z-\left(\sqrt{3}+1\right)i\right|$ and the circle $|z+1-i|=5$ with centre A.

2 marks

SECTION B- Question 2- continued

c. Sketch the line $\left|z+\frac{9}{2}+\left(\frac{7\sqrt{3}}{2}-1\right)i\right|=\left|z-\left(\sqrt{3}+1\right)i\right|$ and the circle $|z+1-i|=5$, labelling the intersections, on the Argand diagram below.

2 marks



d. Find the angle $\angle BAC$.

2 marks

SECTION B- Question 2- continued
TURN OVER

e. The line $\left|z+\frac{9}{2}+\left(\frac{7\sqrt{3}}{2}-1\right)i\right|=\left|z-\left(\sqrt{3}+1\right)i\right|$ cuts the circle $|z+1-i|=5$ into two segments. Find the area of the minor segment.

3 marks

SECTION B- continued

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Question 3 (9 marks)

The population P of kangaroos in a specific region is modelled by the differential equation

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{k}\right),$$

where t is the number of years after the start of Year 2000, r and k are real positive numbers.

a. **i**. Find the general solution of the differential equation, expressing P in terms of t.

2 marks

ii. If the limiting number of kangaroos in this region is 5000, find the value of k.

1 mark

SECTION B- Question 3- continued

TURN OVER

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b. Show that $P = \frac{5000\times\left(\frac{13}{12}\right)^{\frac{t}{10}}}{1+\left(\frac{13}{12}\right)^{\frac{t}{10}}}$ is a solution of the differential equation satisfying $t = 0, P = 2500$ and $t = 10, P = 2600$.

3 marks

c. Find an expression of $\frac{d^2P}{dt^2}$ in terms of P.

3 marks

SECTION B- continued

Question 4 (11 marks)

Relative to the origin O, the positions of two particles at time t seconds are given by

$$\underset{\sim}{r}_A = 5\cos(2t)\,\underset{\sim}{i} + 3\sin(2t)\,\underset{\sim}{j},\quad 0 \le t \le \frac{5\pi}{12}$$

$$\underset{\sim}{r}_B = (-3 + 3\sec(t))\,\underset{\sim}{i} + \tan(t)\,\underset{\sim}{j},\quad 0 \le t \le \frac{5\pi}{12}$$

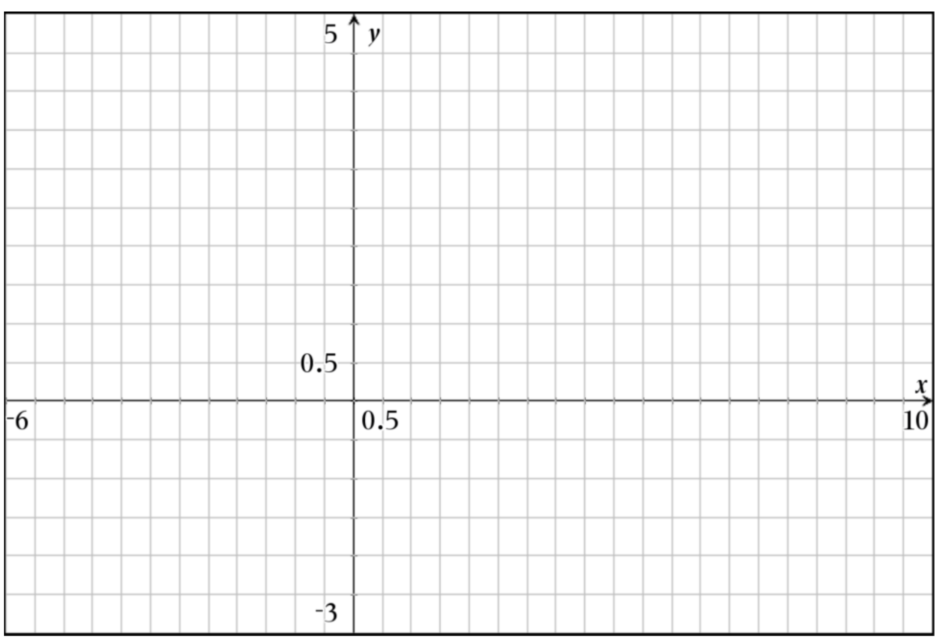
where displacements are measured in metres.

a. Find the times in seconds and the coordinates of the point when the particles cross the same point, giving your answer to two decimal places.

3 marks

b. Sketch and label the path of each particle on the axes below. Show the direction of motion of each particle with arrow and label the end points, correct to 2 decimal places.

3 marks



SECTION B - Question 4 – continued
TURN OVER

c. Find the obtuse angle between the moving directions when they cross the same point. Give your answer correct to the nearest degree.

2 marks

d. **i.** Find the value of t in seconds, correct to three decimal places, when the particles are closest.

2 marks

ii. Find the minimum distance between the two particles, in metres, correct your answer to two decimal places.

1 mark

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Question 5 (11 marks)
A particle moving through air experiences a resistance of magnitude N= $0.025mv^2$ where m kg is the mass of the particle and v m/s is its velocity.

The particle is projected vertically upward from a point O, and t seconds after the instant of projection its height above O is x metres.

a. By choosing an appropriate derivative form for acceleration, show that a differential equation relating v to x is $\frac{dv}{dx} = -\frac{40g+v^2}{40v}$

2 marks

b. Given that the initial velocity is $80\ ms^{-1}$, find an expression of x in terms of v and g.

2 marks

SECTION B- Question 5- continued
TURN OVER

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c. Find the greatest height, in metres, reached by the particle, correct your answer to two decimal places.

2 marks

The same particle is dropped from rest from a point 120 metres above *O*, and t seconds after the instant of dropping is y metres from the point at which it was dropped, and is travelling at a velocity v m/s. The particle experiences a resistance of magnitude $0.001mv^2$ N where m kg is the mass of the particle. A differential equation between v and y is given by

$$\frac{dv}{dy} = \frac{1000g - v^2}{1000v}.$$

d. Find the speed in ms^{-1} with which the particle reaches point *O*, giving your answer correct to three decimal places.

3 marks

SECTION B- Question 5- continued

e. Find the time when the particle reaches point *O*, giving your answer correct to two decimal places.

2 marks

SECTION B - continued
TURN OVER

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Question 6 (8 marks)
A manufacturer of orange juice uses machines to dispense liquid ingredients into bottles that move along a filling line. The machine that dispenses liquid ingredients is working properly when an average of 200 ml is dispensed. The standard deviation of the process is 5 ml. The amount of juice dispensed each time can be assumed to be normally distributed.

A sample of 25 bottles is selected periodically and the filling line is stopped if there is evidence that the average amount dispensed is actually less than 200 ml. Suppose that the average amount dispensed in a particular sample of 25 bottles is 197.5 ml.

a. Find the probability that the total amount of liquid ingredients in 25 bottles is more than 5050 ml, correct to 4 decimal places.

2 marks

b. Find the probability that the average amount of liquid ingredients in 25 bottles is more than 201 ml, correct to 4 decimal places.

2 marks

SECTION B - Question 6- continued

Suppose that the average amount dispensed in a particular sample of 25 bottles is 197.5 ml.

c. Find a 95% confidence interval for μ, the mean amount dispensed machine based on this sample data, correct to 2 decimal places.

1 mark

d. Write down suitable hypotheses to test whether the average amount dispensed by the machine is less than 200 ml.

1 mark

e. Carry out this hypothesis test by finding the p value and give your conclusion. Use $\alpha = 0.05$.

2 marks

END OF QUESTION AND ANSWER BOOK

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