VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2017

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER									Letter

SPECIALIST MATHEMATICS

Written examination 1

Friday 10 November 2017

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 11 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1 (3 marks)

Find the equation of the tangent to the curve given by $3xy^2 + 2y = x$ at the point $(1, -1)$.

Question 2 (4 marks)

Find $\int_1^{\sqrt{3}} \frac{1}{x\left(1+x^2\right)}\,dx$, expressing your answer in the form $\log_e\left(\sqrt{\frac{a}{b}}\right)$, where a and b are positive integers.

Question 3 (3 marks)

Let $z^3 + az^2 + 6z + a = 0$, $z \in C$, where a is a real constant.

Given that $z = 1 - i$ is a solution to the equation, find all other solutions.

Question 4 (3 marks)

The volume of soft drink dispensed by a machine into bottles varies normally with a mean of 298 mL and a standard deviation of 3 mL. The soft drink is sold in packs of four bottles.

Find the approximate probability that the mean volume of soft drink per bottle in a randomly selected four-bottle pack is less than 295 mL. Give your answer correct to three decimal places.

Question 5 (4 marks)

Relative to a fixed origin, the points B, C and D are defined respectively by the position vectors $\underset{\sim}{b} = \underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$, $\underset{\sim}{c} = 2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{d} = a\underset{\sim}{i} - 2\underset{\sim}{j}$, where a is a real constant.

Given that the magnitude of angle BCD is $\frac{\pi}{3}$, find a.

Question 6 (3 marks)

Let $f(x)=\dfrac{1}{\arcsin(x)}$.

Find $f'(x)$ and state the largest set of values of x for which $f'(x)$ is defined.

Question 7 (4 marks)

The position vector of a particle moving along a curve at time t is given by $\underset{\sim}{r}(t)=\cos^3(t)\underset{\sim}{i}+\sin^3(t)\underset{\sim}{j},\ 0\le t\le\dfrac{\pi}{4}$.

Find the length of the path that the particle travels along the curve from $t=0$ to $t=\dfrac{\pi}{4}$.

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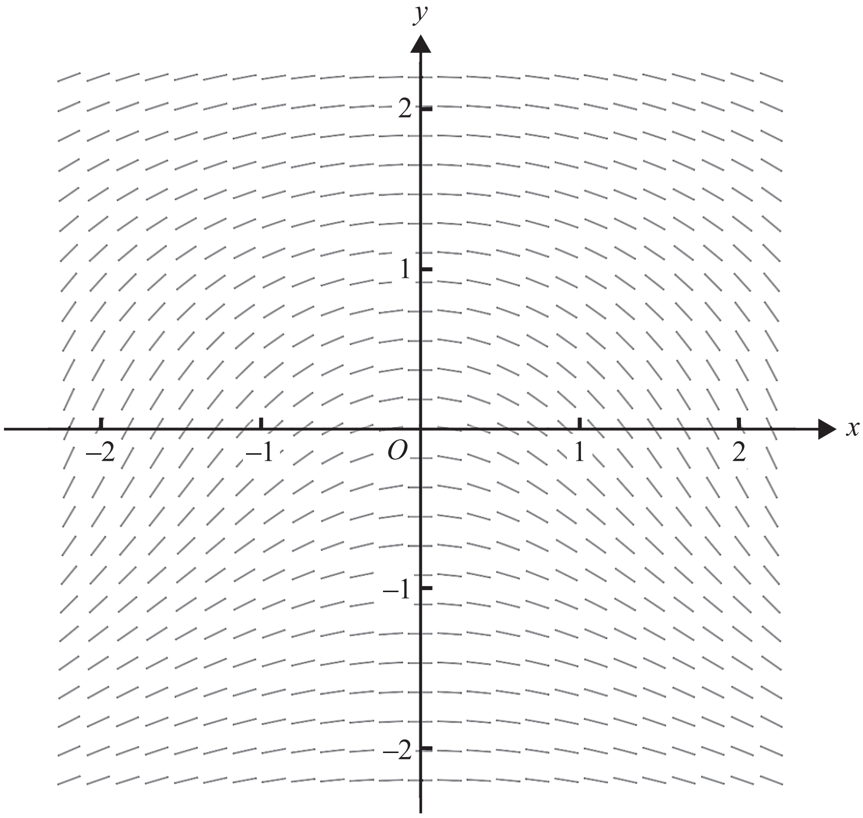
Question 8 (4 marks)

A slope field representing the differential equation $\frac{dy}{dx} = \frac{-x}{1+y^2}$ is shown below.



a. Sketch the solution curve of the differential equation corresponding to the condition $y(-1) = 1$ on the slope field above and, hence, estimate the positive value of x when $y = 0$. Give your answer correct to one decimal place. 2 marks

b. Solve the differential equation $\frac{dy}{dx} = \frac{-x}{1+y^2}$ with the condition $y(-1) = 1$. Express your answer in the form $ay^3 + by + cx^2 + d = 0$, where a, b, c and d are integers. 2 marks

Question 9 (5 marks)

A particle of mass 2 kg with initial velocity $3\underset{\sim}{i} + 2\underset{\sim}{j}$ ms^{-1} experiences a constant force for 10 seconds.

The particle's velocity at the end of the 10-second period is $43\underset{\sim}{i} - 18\underset{\sim}{j}$ ms^{-1}.

a. Find the magnitude of the constant force in newtons. 2 marks

b. Find the displacement of the particle from its initial position after 10 seconds. 3 marks

Question 10 (7 marks)

a. Show that $\frac{d}{dx}\left(x\arccos\left(\frac{x}{a}\right)\right) = \arccos\left(\frac{x}{a}\right) - \frac{x}{\sqrt{a^2 - x^2}}$, where $a > 0$. 1 mark

b. State the maximal domain and the range of $f(x) = \sqrt{\arccos\left(\frac{x}{2}\right)}$. 2 marks

c. Find the volume of the solid of revolution generated when the region bounded by the graph of $y = f(x)$, and the lines $x = -2$ and $y = 0$, is rotated about the x-axis. 4 marks

END OF QUESTION AND ANSWER BOOK

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2017

SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

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Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

Circular functions – continued

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r\left(\cos(\theta) + i\sin(\theta)\right) = r\operatorname{cis}(\theta)$	
$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \operatorname{Arg}(z) \leq \pi$
$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\operatorname{cis}\left(\theta_1 - \theta_2\right)$
$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$ $\mathrm{E}(aX + bY) = a\mathrm{E}(X) + b\mathrm{E}(Y)$ $\operatorname{var}(aX + b) = a^2\operatorname{var}(X)$
for independent random variables X and Y	$\operatorname{var}(aX + bY) = a^2\operatorname{var}(X) + b^2\operatorname{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\overline{X}$	mean $\mathrm{E}\left(\overline{X}\right) = \mu$ variance $\operatorname{var}\left(\overline{X}\right) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n\,dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}\,dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$	$\int \frac{1}{x}\,dx=\log_e\lvert x\rvert+c$
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$	$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$	$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=a\sec^2(ax)$	$\int \sec^2(ax)\,dx=\frac{1}{a}\tan(ax)+c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right)=\frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}\,dx=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right)=\frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right)=\frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2}\,dx=\tan^{-1}\left(\frac{x}{a}\right)+c$
	$\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$
	$\int (ax+b)^{-1}\,dx=\frac{1}{a}\log_e\lvert ax+b\rvert+c$
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx}=f(x)$, $x_0=a$ and $y_0=b$, then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf(x_n)$
acceleration	$a=\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2}\sqrt{1+\left(f'(x)\right)^2}\,dx$ or $\int_{t_1}^{t_2}\sqrt{\left(x'(t)\right)^2+\left(y'(t)\right)^2}\,dt$

Vectors in two and three dimensions

$\underset{\sim}{r}=x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k}$
$\lvert \underset{\sim}{r}\rvert=\sqrt{x^2+y^2+z^2}=r$
$\dot{\underset{\sim}{r}}=\frac{d\underset{\sim}{r}}{dt}=\frac{dx}{dt}\underset{\sim}{i}+\frac{dy}{dt}\underset{\sim}{j}+\frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1\cdot\underset{\sim}{r}_2=r_1r_2\cos(\theta)=x_1x_2+y_1y_2+z_1z_2$

Mechanics

momentum	$\underset{\sim}{p}=m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R}=m\underset{\sim}{a}$

END OF FORMULA SHEET