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Students Name:………………………………………

SPECIALIST MATHEMATICS UNITS 3 & 4

TRIAL EXAMINATION 1

2018

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 10 questions.
All questions should be answered in the spaces provided.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may **not** bring any notes or calculators into the exam.
Where more than one mark is allocated to a question, appropriate working must be shown.
An exact answer is required to a question unless otherwise specified.
Unless otherwise indicated, diagrams in this exam are not drawn to scale.
The acceleration due to gravity should be taken to have magnitude g m/s^2 where $g = 9.8$
Formula sheets can be found on pages 11 – 13 of this exam.

Question 1 (2 marks)

Evaluate $\int_0^{\frac{\pi}{6}} \cos^2(3x)\,dx$.

Question 2 (3 marks)

Given the relation $5y - 2x^2y + x = 7$, find the equation of the tangent to the graph of this relation at the point $(1, 2)$.

Question 3 (3 marks)

The weight of eggs produced at a free range farm varies normally with a mean of 68 g and a standard deviation of 4 g.
These eggs are sold in boxes which contain 16 eggs.
Find, correct to four decimal places, the approximate probability that the mean weight of an egg in a randomly selected box is less than 65 g.

Question 4 (3 marks)

Find the value of a given that $z = 2 - i$ is a solution to the equation $z^3 - 7z^2 + (a^2 + 1)z - (4a - 1) = 0$, where a is a real constant.

Question 5 (5 marks)

Consider the three vectors $\underset{\sim}{a} = \underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$, $\underset{\sim}{b} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + d\underset{\sim}{k}$ and $\underset{\sim}{c} = \underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}$ where d is a real constant.

a. Find a unit vector in the direction of $\underset{\sim}{c}$. 1 mark

b. Find the vector resolute of $\underset{\sim}{a}$ perpendicular to $\underset{\sim}{c}$. 2 marks

c. Find the value of d if the three vectors are **linearly dependent**. 2 marks

Question 6 (4 marks)

Solve the differential equation

$$-\frac{1}{x}\frac{dy}{dx}=\sqrt{\frac{4-y^2}{4-x^2}}, \quad \text{given that } y(2)=\sqrt{3}.$$

Express your answer in the form $y=a\cos\left(b-\sqrt{c-x^2}\right)$ where a, b and $c \in R$.

Question 7 (3 marks)

The velocity *v*, in ms^{-1}, of a 5 kg mass is given by $v = 4\arccos(2x^2 - 1)$ where *x*, in metres, is its displacement from the origin and $x > 0$.
Find the net force *F*, in newtons, acting on the mass in terms of *x*.

Question 8 (5 marks)

Let $f:(-\infty, 4] \rightarrow R, \quad f(x)=(x-1)\sqrt{4-x}$.

S represents the region enclosed by the graph of f and the x-axis.

a. Find the area of S. 3 marks

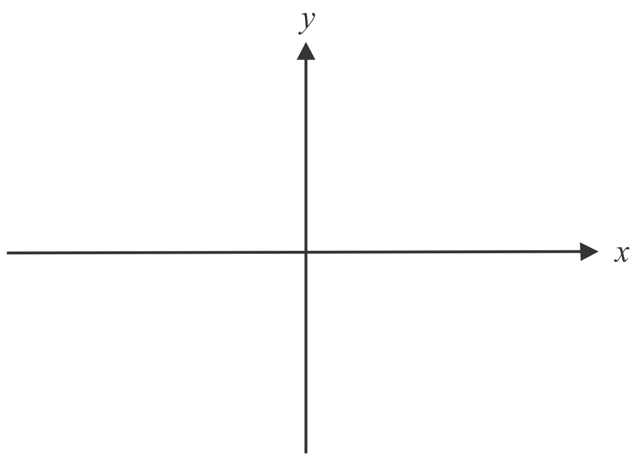
b. The region S is rotated around the x-axis to form a solid of revolution. Find the volume of this solid of revolution. 2 marks

Question 9 (4 marks)

A curve is defined parametrically by $x = 2\sqrt{4-t}$ and $y = 2\sqrt{t+4}$.
Find the length of this curve from $t = 2$ to $t = 4$.

Question 10 (8 marks)

a. Sketch the graph of $f(x) = 2\arctan(3x)$ on the set of axes below.
Label any asymptotes with their equations. 2 marks



b. Find the rule and the domain of the inverse function f^{-1}. 2 marks

c. Find $\int \tan\left(\frac{x}{2}\right)dx$. 1 mark

d. Hence find the area enclosed by the graph of f, the x-axis and the line $x = \frac{1}{3}$. 3 marks

Specialist Mathematics Formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule	$c^2=a^2+b^2-2ab\cos(C)$

Circular functions

$\cos^2(x)+\sin^2(x)=1$	
$1+\tan^2(x)=\sec^2(x)$	$\cot^2(x)+1=\operatorname{cosec}^2(x)$
$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$	$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$
$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$	$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$
$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$	
$\sin(2x)=2\sin(x)\cos(x)$	$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

Circular functions – continued

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\text{cis}(\theta)$	
$\lvert z \rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \text{Arg}(z) \le \pi$
$z_1 z_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$
$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$\text{E}(aX + b) = a\text{E}(x) + b$ $\text{E}(aX + bY) = a\text{E}(x) + b\text{E}(Y)$ $\text{var}(aX + b) = a^2\text{var}(X)$
for independent random variables X and Y	$\text{var}(aX + bY) = a^2\text{var}(X) + b^2\text{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\bar{X}$	mean $\text{E}(\bar{X}) = \mu$ variance $\text{var}(\bar{X}) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n\,dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}\,dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$	$\int \frac{1}{x}\,dx=\log_e\lvert x\rvert+c$
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$	$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$	$\int \cos\,(ax)\,dx=\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=a\sec^2(ax)$	$\int \sec^2(ax)\,dx=\frac{1}{a}\tan(ax)+c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right)=\frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}\,dx=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right)=\frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right)=\frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2}\,dx=\tan^{-1}\left(\frac{x}{a}\right)+c$
	$\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$
	$\int (ax+b)^{-1}\,dx=\frac{1}{a}\log_e\lvert ax+b\rvert+c$
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx}=f(x),\ x_0=a$ and $y_0=b$, then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf(x_n)$
acceleration	$a=\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2}\sqrt{1+(f'(x))^2}\,dx$ or $\int_{t_1}^{t_2}\sqrt{(x'(t))^2+(y'(t))^2}\,dt$

Vectors in two and three dimensions

$\underset{\sim}{r}=x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k}$
$\lvert\underset{\sim}{r}\rvert=\sqrt{x^2+y^2+z^2}=r$
$\dot{\underset{\sim}{r}}=\frac{d\underset{\sim}{r}}{dt}=\frac{dx}{dt}\underset{\sim}{i}+\frac{dy}{dt}\underset{\sim}{j}+\frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1.\underset{\sim}{r}_2=r_1r_2\cos(\theta)=x_1x_2+y_1y_2+z_1z_2$

Mechanics

momentum	$\underset{\sim}{p}=m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R}=m\underset{\sim}{a}$