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Specialist Mathematics

2018

Trial Examination 2
(2 hours)

SECTION A Multiple-choice questions

Instructions for Section A

Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

Question 1 The graph of $y = \frac{1}{x-1} + \frac{1}{(x+1)^2}$ has m straight line asymptotes and n points of inflection.

The values of m and n are respectively

A. $2, 2$

B. $3, 3$

C. $3, 2$

D. $2, 3$

E. $2, 1$

Question 2 For $a > 0$, the range of $f : \left[-\frac{1}{a}, 0\right) \to R, f(x) = \arccos(ax) - \frac{\pi}{2}$ is

A. $\left[\frac{\pi}{2}, 0\right)$

B. $\left[0, \frac{\pi}{2}\right)$

C. $\left(\frac{\pi}{2}, 0\right]$

D. $\left(0, \frac{\pi}{2}\right]$

E. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Question 3 The Cartesian equation of the relation given by $x = \sin\left(\cos^{-1} t\right)$ and $y = \cos\left(\sin^{-1} t\right)$ is

A. $y = x$ for $0 \le x \le 1$

B. $y = -x$ for $-1 \le x \le 0$

C. $y = |x|$ for $-1 \le x \le 1$

D. $x = |y|$ for $-1 \le y \le 1$

E. $x = y$ for $-1 \le y \le 1$

Question 4 If $w \in \left\{ z : \left| z + i\,\text{cis}\left(\frac{\pi}{4}\right) \right| = 1 \right\}$, w **cannot** be

A. 0

B. $\sqrt{2}$

C. $-\sqrt{2}\,i$

D. $\sqrt{2}\,i$

E. $-2i\,\text{cis}\left(\frac{\pi}{4}\right)$

Question 5 If $z = a\,\text{cis}\left(\frac{3\pi}{4}\right)$ where $a \in R^+$, then $\frac{z + a\sqrt{2}}{z - a\sqrt{2}\,i} =$

A. -1

B. $-i$

C. i

D. $a\,i$

E. $2a$

Question 6 If $z_1 = a\,\text{cis}\left(n + \frac{3\pi}{22}\right)$, $z_2 = b\,\text{cis}\left(n - \frac{4\pi}{11}\right)$ and $w = z_1 + z_2$, where $a, b, n \in R^+$, then

A. $|w| = a + b$

B. $|w| = a - b$

C. $|w| = \sqrt{a^2 + b^2}$

D. $|w| = \sqrt{a + b}$

E. $|w| = \sqrt{a^2 - b^2}$

Question 7 Given $f(x) = \left| a\,\text{cosec}\left(\frac{x + \alpha}{2}\right) \right|$ and $g(x) = \left| b\sec\left(\frac{x + \beta}{2}\right) \right|$, $f(x) - g(x) = 0$ when

A. $a + b = 0$ and $|\alpha - \beta| = \pi$

B. $a - b = 0$ and $|\alpha - \beta| = 2\pi$

C. $a + b = 1$ and $|\alpha - \beta| = \pi$

D. $a - b = 1$ and $|\alpha - \beta| = 2\pi$

E. $a + b = 1$ and $|\alpha - \beta| = 3\pi$

Question 8 A solution (x, y) to the simultaneous equations $x + y = \frac{1}{2}$ and $\sin\left(\cos^{-1} x - \sin^{-1} y\right) = \frac{1}{2}$ is

A. $\left(-\frac{2}{12}, \frac{8}{12}\right)$

B. $\left(\frac{9}{12}, -\frac{3}{12}\right)$

C. $\left(\frac{3}{12}, \frac{3}{12}\right)$

D. $\left(-\frac{3}{12}, \frac{9}{12}\right)$

E. $\left(\frac{12}{12}, -\frac{6}{12}\right)$

Question 9 A particle moves in the *x*-*y* plane. Its position vector at $t > 0$ is given by $\underset{\sim}{r} = \left(\tan^{-1} t\right)\underset{\sim}{i} + \left(\log_e t\right)\underset{\sim}{j}$ where $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors in the positive *x* (to the east) and *y* (to the north) directions respectively.
The speed of the particle when it is east of the origin *O* is closest to

A. 3.0

B. 2.5

C. 2.0

D. 1.5

E. 1.0

Question 10 Consider $y = f(x)$ where $\frac{dx}{dy} = \frac{x}{1 - x^2}$ and $f(1) = -0.5$.
If $f(\alpha) = -1$, the value of α is closest to

A. -1.80

B. -0.40

C. 0.39

D. 0.41

E. 1.80

Question 11 $s = ut + \frac{1}{2}at^2$ is one of the constant acceleration formulas for particles in rectilinear motion.

s is the displacement from the particle's initial position (position at time $t = 0$), u is the initial velocity (velocity at $t = 0$), and a is the constant acceleration.

Given $s = -2$ at time $t = 2$ and $s = 4$ at $t = 4$, the time when the particle is at its initial position **again** is

A. 0

B. 1

C. 2

D. 3

E. 4

Question 12 The scalar resolute of $3\tilde{i} - \tilde{j}$ perpendicular to $\tilde{i} - 3\tilde{j} + \tilde{k}$ is closest to

A. 1.5

B. 2.6

C. 3.1

D. 3.3

E. 3.7

Question 13 The scale reading is 50 kg when a student stands on a scale in the bathroom. The scale reading is 45 kg when the student stands on the scale in an accelerating lift. The motion of the lift is

A. upward with a downward acceleration of about 1 m s^{-2}

B. upward with an upward acceleration of about 1 m s^{-2}

C. downward with an upward acceleration of about 1 m s^{-2}

D. upward with an upward acceleration of about 0.1 m s^{-2}

E. downward with no acceleration

Question 14 A particle moves in a vertical plane. $\tilde{i}$ and $\tilde{j}$ are horizontal and vertically upward unit vectors respectively. The particle has an acceleration (m s^{-2}) of $-9.8\tilde{j}$, and an initial velocity (m s^{-1}) of $10\tilde{i}$.

After t second(s) the particle travels at $45°$ to the horizontal. The value of t is

A. closest to 0.5

B. closest to 1.0

C. closest to 1.5

D. closest to 2.0

E. indeterminable without further information

Question 15 Given $\int_{-1}^{0} f(x)dx = 2$ and $\int_{0}^{1} f(x)dx = -\frac{1}{2}$, the value of the definite integral $\int_{-1}^{1} |f(x)|dx$ is

A. $\geq \frac{5}{2}$

B. $= \frac{5}{2}$

C. $\geq \frac{3}{2}$

D. $= \frac{3}{2}$

E. $\leq \frac{3}{2}$

Question 16 A particle starts from rest at $x = 0$. It travels in a straight line in the positive x-direction. Its motion is represented by $2v\frac{dv}{dx} = \frac{1}{\sqrt{9 - x^2}}$ where v is its velocity at time t.

The time required to travel from $x = 1$ to $x = 2$ is closest to

A. 0.8

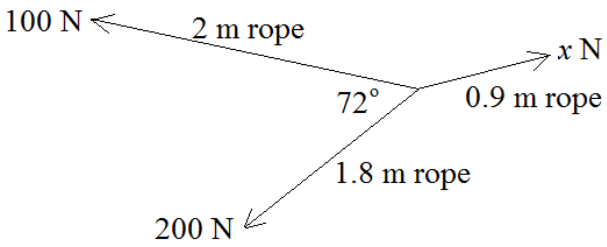
B. 1.1

C. 1.4

D. 1.7

E. 3.0

Question 17 Three horizontal ropes are fastened together at the ends. Forces are applied to the ropes to keep them in equilibrium as shown in the following diagram. The 1.8 m and 2 m ropes make a $72°$ angle.



The value of x (refer to the diagram) is closest to

A. 160

B. 190

C. 195

D. 250

E. 290

Question 18 The age X (years) of members in a large senior citizen club has a normal distribution with mean of 75 and standard deviation of 12. A random sample of 25 members is taken from the club.

$\Pr(\overline{X} = 80)$ is closest to

A. 0.019

B. 0.029

C. 0.059

D. 0.105

E. 0.115

Question 19 The weight W (kg) of a person in a large population had a normal distribution with mean of 75 and standard deviation of 12 five years ago. To find out whether the population has gained weight a random sample of 100 people was taken from the population and the mean weight of the sample was 78. Assuming the distribution and standard deviation remained the same, the p-value is closest to

A. 0.05

B. 0.02

C. 0.01

D. 0.009

E. 0.006

Question 20 A random sample of 36 long-life batteries of a particular brand was measured to have an average life 29.52 hours and standard deviation of 0.45 hours.

The 80% confidence interval for the mean life of the long-life batteries of the brand is closest to

A. $(29.32, 29.69)$

B. $(29.32, 29.70)$

C. $(29.33, 29.71)$

D. $(29.37, 29.67)$

E. $(29.42, 29.62)$

SECTION B Extended-answer questions

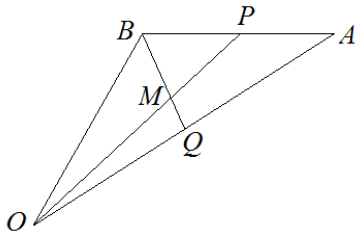
Instructions for Section B

Answer **all** questions.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this examination are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

Question 1 P and Q are midpoints of the sides of ΔOAB as shown in the diagrams.

OP and BQ intersect at M. $\frac{OM}{MP} = x$ and $\frac{BM}{MQ} = y$.

Use only vector addition/subtraction to answer parts a, b and c.



a. Show that $\overrightarrow{OP} = \frac{\overrightarrow{OA} + \overrightarrow{OB}}{2}$. 2 marks

b. Hence show that $\overrightarrow{OM} = \frac{x}{x+1}\left(\frac{2\overrightarrow{OQ} + \overrightarrow{OB}}{2}\right)$. 2 marks

c. Show that $\overrightarrow{OM}$ can also be expressed as $\frac{y\,\overrightarrow{OQ} + \overrightarrow{OB}}{y+1}$. 3 marks

d. Hence find the values of x and y. 2 marks

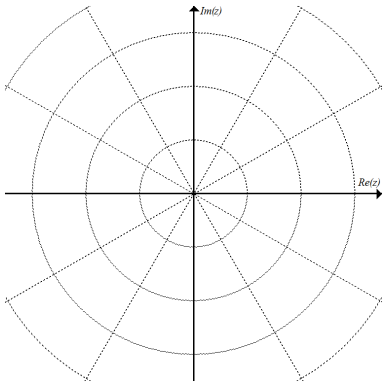
Question 2 Consider $(-ai)^{\frac{1}{m}} = (-b)^{\frac{1}{n}}$ where a, b, m and n are natural numbers.

a. Given $(-i)^{\frac{1}{m}} = (-1)^{\frac{1}{n}}$ is possible when $m = 3$ and $n = 2$. Find the next higher values of *m* and *n* to make $(-i)^{\frac{1}{m}} = (-1)^{\frac{1}{n}}$ possible. 3 marks

b. Verify that $m = 3$ and $n = 6$ make $(-8i)^{\frac{1}{m}} = (-64)^{\frac{1}{n}}$ possible. 1 mark

c. Find one z such that $z = (-8i)^{\frac{1}{3}} = (-64)^{\frac{1}{6}}$. 1 mark

d. Plot all sixth roots of -64 on the following diagram. Show clear scales on the axes. 2 marks



e. Write all $z = i(-8i)^{\frac{1}{3}}$ in polar form. 3 marks

Question 3 In this question metre (m) and second (s) are used for distance and time.

$\tilde{i}$ and $\tilde{j}$ are horizontal orthogonal unit vectors in the positive x and y directions respectively, and $\tilde{k}$ is a unit vector pointing upwards in the positive z direction. The origin O is at ground level.

At time $t \geq 0$ a particle moves with velocity $\tilde{v} = -10\sin t\,\tilde{i} + 10\cos t\,\tilde{j} - 9.8t\,\tilde{k}$

At time $t = 0$ the particle's position vector is $\tilde{r} = 10\tilde{i} + 44.1\tilde{k}$.

a. Determine the magnitudes of the vertical and horizontal accelerations at time t. 2 marks

b. Determine the position vector of the particle at time t. 2 marks

c. Find the time taken for the particle to reach the ground, correct to 1 decimal place. 1 mark

d. Determine the coordinates of the landing point of the particle, correct to 1 decimal place. 2 marks

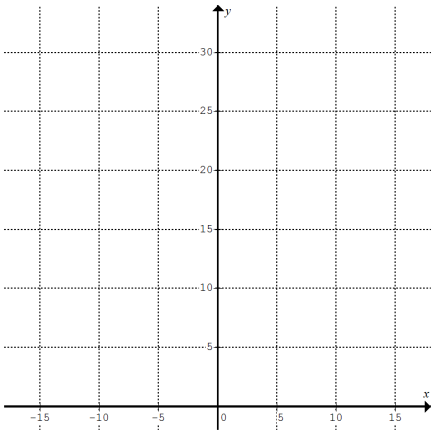
e. Calculate the speed of the particle just before it hits the ground. 2 marks

f. Find the angle (in decimal degrees) with the horizontal at which the particle hits the ground, correct to 1 decimal place.

1 mark

Question 4 A relation is rotated about the *y*-axis. The solid of revolution is to model an hourglass to be made. The lower half of the relation is given by equation $y = 5\cos^{-1}\left(\frac{x-5}{4.85}\right)$. Length is measured in cm.

a. Sketch $y = 5\cos^{-1}\left(\frac{x-5}{4.85}\right)$. Show the coordinates of the end points. 3 marks



b. Write down the transformations to the lower half of the relation to obtain the upper half of the relation. Find the equation of the upper half of the relation. 2 marks

c. Consider such an hourglass. At the start of timing the whole content of the hourglass is in the upper half and the height of the content is 15 cm. Show that the total content volume in the hourglass is 1599 cm^3.

3 marks

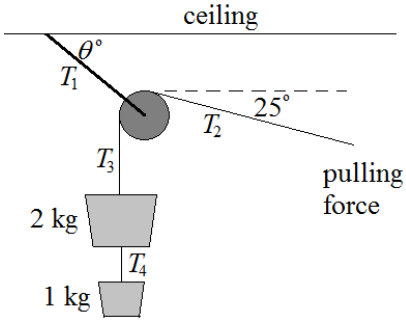
d. Calculate the constant flow rate (cm^3 per min) of the content to the lower half, correct to 2 decimal places.

1 mark

e. Find the depth (correct to 2 decimal places) of the content in the lower half of the hourglass 45 min after it begins to flow. Assume a uniform surface of the content.

2 marks

Question 5 Two buckets of sand are lifted upwards with a cord through a frictionless pulley of negligible mass. The pulley is suspended by another cord fastened to the ceiling. The tension in Cord 1 is T_1 N, in Cord 2 is T_2, in Cord 3 is T_3, in Cord 4 is T_4. Refer to the following diagram for other measurements.
Note: Cord 2 and Cord 3 are two sections of the same cord.



A pulling force is applied to Cord 2.
The rate of increase in the length of Cord 2 increases at a rate of 0.1 m s^{-1} in a second.

a. Find the acceleration of the 2-kg bucket. 1 mark

b. Calculate the pulling force required, correct to 1 decimal place. 2 marks

c. Determine the values of T_2, T_3 and T_4, correct to 1 decimal place. 3 marks

d. Find the value of θ. 2 marks

e. Determine the value of T_1, correct to 1 decimal place. 2 marks

Question 6 The height X cm of a person in a large population has a normal distribution. The mean and standard deviation are 170 cm and 20 cm respectively.

a. Consider the total height H cm of 10 randomly chosen people.

i. Determine $\text{E}(H)$. 1 mark

ii. Determine $\text{sd}(H)$, correct to 1 decimal place. 2 marks

b. A random sample of 10 people is to be taken from the population.

i. Find the expectation and the standard deviation of the sample mean $\overline{X}$, correct to 1 decimal place. 2 marks

ii. Determine $\Pr(\overline{X} > 175)$, correct to 1 decimal place. 1 mark

c. A sample of 10 people is taken from another large population. The mean and standard deviation of X are different from those in the previous population.
The heights (cm) are: 162, 178, 177, 180, 159, 138, 182, 179, 149 and 185

i. Determine the sample mean and standard deviation of X, correct to 1 decimal place. 2 marks

ii. Determine the approximate 95% confidence interval for the population mean, correct to 1 decimal place. 1 mark

iii. Determine the sample size required to reduce the width of interval in part ii by half. 1 mark

End of Exam 2