Year 2018

VCE

Specialist Mathematics

Trial Examination 2

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Victorian Certificate of Education **2018**

STUDENT NUMBER

Figures									Letter
Words									

SPECIALIST MATHEMATICS

Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 35 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple-choice questions and sign your name in the space provided.
- All written responses must be in English.
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The domain and range of the function with rule $f(x) = \tan^{-1}\left(\frac{1}{\sqrt{x}}\right)$ are respectively

A. R and $(0, \infty)$

B. $R \backslash \{0\}$ and $[-\pi, \pi]$

C. $(0, \infty)$ and $\left(0, \frac{\pi}{2}\right)$

D. $(0, \infty)$ and $\left(0, \frac{\pi}{2}\right]$

E. $(0, \infty)$ and $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Question 2

If the complex number $\frac{2-ki}{3+ki}$ where $k \in R$, has a zero real part, then

A. $k = \pm\sqrt{6}$ or $k = 0$.

B. $k = 0$ only.

C. $k = \pm 3$ only.

D. $k = \pm 2$ only.

E. $k = \pm\sqrt{6}$ only.

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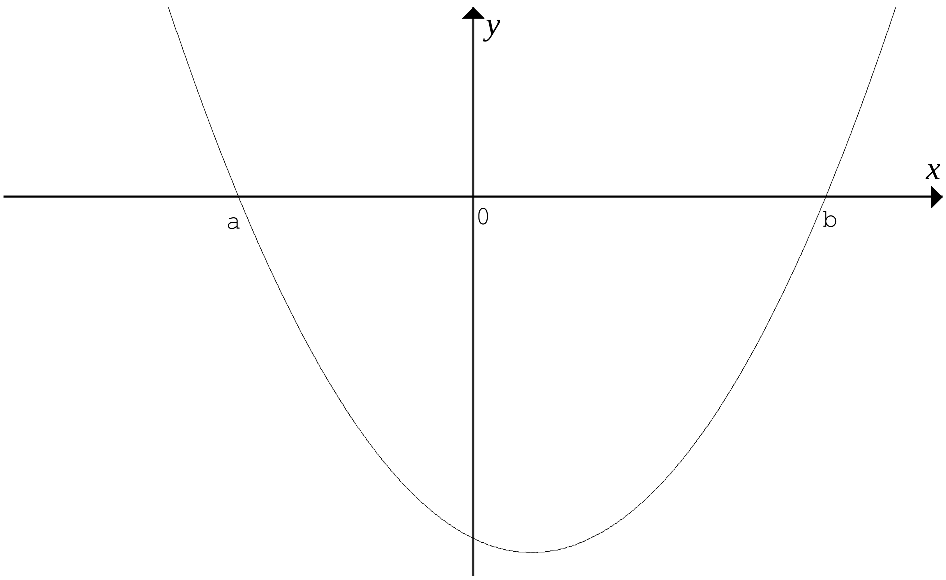
Question 3

The graph of the quadratic function $y = f(x)$ is shown.

The graph crosses the *x*-axis at $x = a$ and $x = b$.



Then the graph of $y = \dfrac{1}{f(x)}$ has

A. vertical asymptotes at $x = a$ and $x = b$ only and a maximum turning point at $x = \dfrac{a+b}{2}$

B. vertical asymptotes at $x = a$ and $x = b$ only and a minimum turning point at $x = \dfrac{a+b}{2}$

C. a horizontal asymptote at $y = 0$ only and a maximum turning point at $x = \dfrac{a+b}{2}$

D. vertical asymptotes at $x = a$ and $x = b$ and a horizontal asymptote at $y = 0$ and a maximum turning point at $x = \dfrac{a+b}{2}$

E. vertical asymptotes at $x = a$ and $x = b$ and a horizontal asymptote at $y = 0$ and a minimum turning point at $x = \dfrac{a+b}{2}$

Question 4

A cubic polynomial with real coefficients has $z = -2a$ and $z = 3a - ai$ as two of its roots, where a is a non-zero real number. The polynomial could be

A. $z^3 - 4az^2 - 2a^2z + 20a^3$

B. $z^3 - 8az^2 - 2a^2z + 20a^3$

C. $z^3 + 4az^2 - 2a^2z - 20a^3$

D. $z^3 - 8az^2 + 22a^2z - 20a^3$

E. $z^3 + 8az^2 + 22a^2z + 20a^3$

Question 5

Given the complex number $u = -\sqrt{3} - i$ then $\text{Arg}\left(\frac{1}{\overline{u}^7}\right)$ is equal to

A. $-\frac{35\pi}{6}$

B. $\frac{5\pi}{6}$

C. $-\frac{2\pi}{3}$

D. $-\frac{\pi}{6}$

E. $\frac{\pi}{6}$

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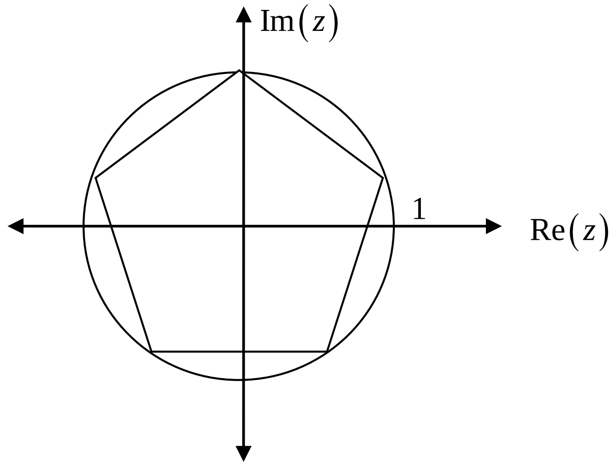
Question 6



The diagram shows an Argand diagram with a regular polygon inscribed in a circle of radius 1. The vertices of the polygon are complex numbers and are roots of the polynomial equation

A. $z^6 - 1 = 0$

B. $z^6 - i = 0$

C. $z^5 + i = 0$

D. $z^5 - i = 0$

E. $z^5 + 1 = 0$

Question 7

Given the vectors $\underset{\sim}{a} = m\underset{\sim}{i} + \sqrt{m}\,\underset{\sim}{j} - \sqrt{m}\,\underset{\sim}{k}$ and $\underset{\sim}{b} = -\sqrt{m}\,\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$,
where m is a non-zero real constant.

Matilda stated that if $m = 1$, then the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are parallel.

Nick stated that if $m = 4$, then the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are parallel.

Yvonne stated that if $m = 1$, then the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are perpendicular.

Zach stated that if $m = -2$, then the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are perpendicular. Then

A. Only Matilda is correct.

B. Only Nick is correct.

C. Matilda and Nick are both correct.

D. Matilda and Zach are both correct.

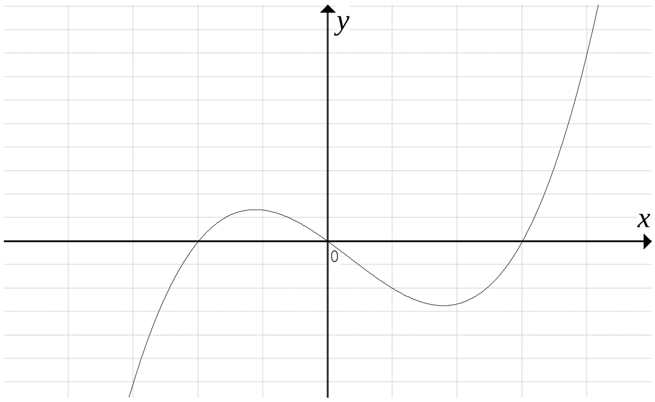
E. Nick and Yvonne are both correct.

Question 8

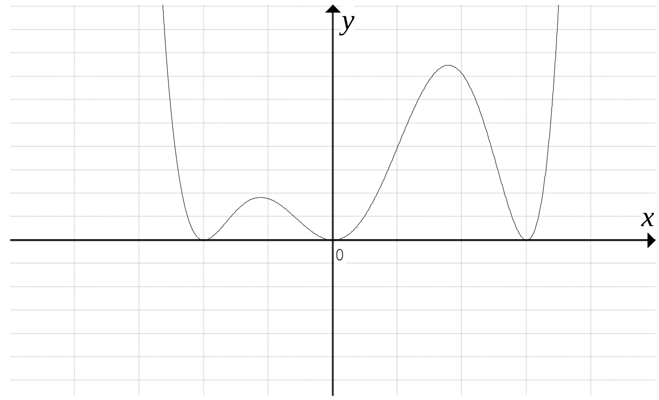
The graphs of $y = f(x)$ and $y = g(x)$ are shown.

The same scale has been used on both graphs.

$y = f(x)$



$y = g(x)$



Then

A. $g(x) = \sqrt{f(x)}$

B. $g(x) = \left[f(x)\right]^2$

C. $g(x) = \left|f(x)\right|$

D. $g(x) = \dfrac{1}{f(x)}$

E. $g(x) = f'(x)$

Question 9

Given the vectors $\underset{\sim}{a} = -2\underset{\sim}{i} + 3\underset{\sim}{j} + 5\underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} + \underset{\sim}{k}$. Which of the following is **false**?

A. The length of the vector $\underset{\sim}{a}$ is $\sqrt{38}$

B. The length of the vector $\underset{\sim}{a} + \underset{\sim}{b}$ is $\sqrt{38} + \sqrt{2}$

C. The scalar resolute of $\underset{\sim}{a}$ in the direction $\underset{\sim}{b}$ is $\dfrac{3\sqrt{2}}{2}$

D. The vector resolute of $\underset{\sim}{a}$ in the direction $\underset{\sim}{b}$ is $\dfrac{3}{2}\left(\underset{\sim}{i} + \underset{\sim}{k}\right)$

E. The vector resolute of $\underset{\sim}{a}$ perpendicular $\underset{\sim}{b}$ is $\dfrac{1}{2}\left(-7\underset{\sim}{i} + 6\underset{\sim}{j} + 7\underset{\sim}{k}\right)$

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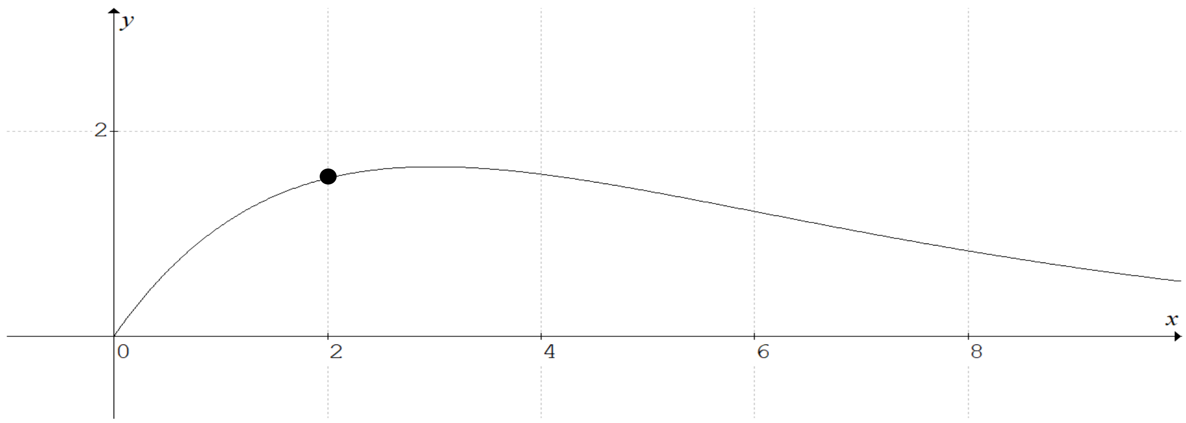
Question 10

The graph of $y = f(x)$ is shown below.



Which of the following is true?

A. $f''(2) < f'(2) < f(2) < 2$

B. $f''(2) < f(2) < f'(2) < 2$

C. $f(2) < f'(2) < f''(2) < 2$

D. $f(2) < 2 < f'(2) < f''(2)$

E. $f'(2) < f''(2) < f(2) < 2$

Question 11

Given the vector $\underset{\sim}{a} = \frac{1}{2}\left(\sqrt{2}\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}\right)$, then the vector $\underset{\sim}{a}$

A. makes an angle of 135^o with the *x*-axis and 150^0 with the *y*-axis.

B. makes an angle of 45^o with the *x*-axis and 120^0 with the *y*-axis.

C. makes an angle of 45^o with the *x*-axis and 150^0 with the *y*-axis.

D. makes an angle of 120^o with the *y*-axis and 30^0 with the *z*-axis.

E. makes an angle of 150^o with the *y*-axis and 60^0 with the *z*-axis.

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Question 12

The differential equation which best represents the direction field shown is

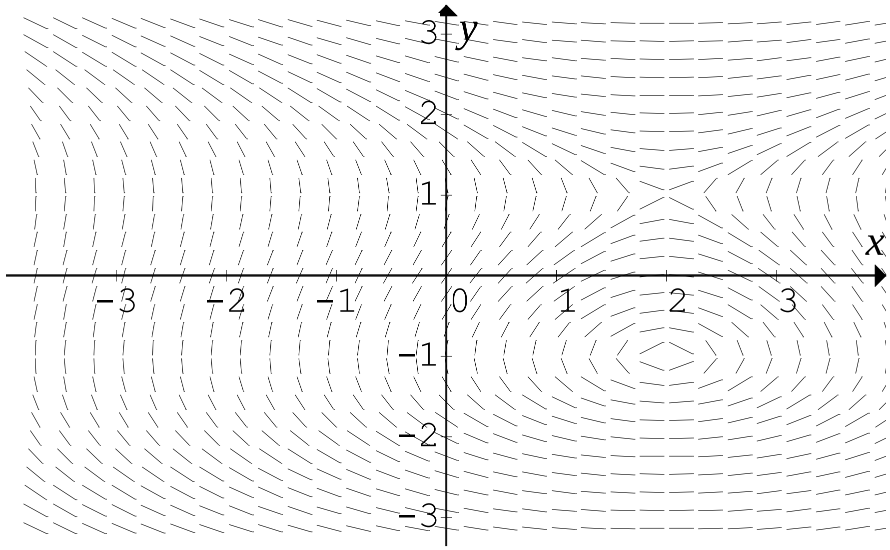
A. $\dfrac{dy}{dx} = \dfrac{y-2}{x^2-1}$

B. $\dfrac{dy}{dx} = \dfrac{x-2}{y-1}$

C. $\dfrac{dy}{dx} = \dfrac{x-2}{y+1}$

D. $\dfrac{dy}{dx} = \dfrac{x-2}{y^2-1}$

E. $\dfrac{dy}{dx} = (x-2)(y+1)$



Question 13

When Euler's method, with a step size of $\dfrac{\pi}{12}$, is used to solve the differential equation $\dfrac{dy}{dx} = \tan^2(2x)$ with $x_0 = 0$ and $y_0 = a$, where a is a real number, then the value of y_3 is

A. $a + \dfrac{\pi}{36}$

B. $a + \dfrac{\pi}{12}$

C. $a + \dfrac{5\pi}{18}$

D. $a + \dfrac{\pi}{9}$

E. undefined

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Question 14

The differential equation $\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0$ is satisfied by $y=axe^{-3x}$, where a, b and c are real constants then

A. $b=-3$ and $c=-18$.

B. $b=-3$ and $c=0$.

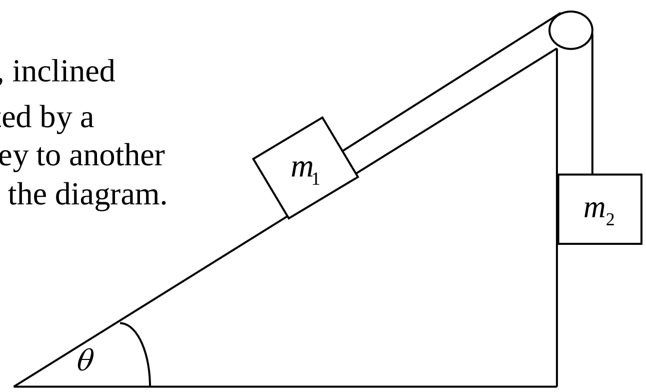
C. $b=0$ and $c=-9$.

D. $b=-6$ and $c=9$.

E. $b=6$ and $c=9$.

Question 15

A particle of mass m_1 kg is on a smooth plane, inclined at an angle of θ to the horizontal. It is connected by a light string which passes around a smooth pulley to another mass of m_2 kg hanging vertically, as shown in the diagram.



Then which of the following is **false?**

A. If $\theta=30^0$ and $\frac{m_2}{m_1}=\frac{1}{2}$ then the system is in equilibrium.

B. If $\theta=30^0$ and $\frac{m_2}{m_1}<\frac{1}{2}$ then the mass m_2 moves upwards.

C. If $\theta=45^0$ and $\frac{m_2}{m_1}=\frac{\sqrt{2}}{2}$ then the system is in equilibrium.

D. If $\theta=60^0$ and $\frac{m_2}{m_1}=\frac{\sqrt{3}}{2}$ then the system is in equilibrium.

E. If $\theta=60^0$ and $\frac{m_2}{m_1}<\frac{\sqrt{3}}{2}$ then the mass m_2 moves downwards.

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Question 16

A body is moving in a straight line. When its displacement is x metres from the origin at time t seconds, then $t = e^{kx}$, where k is a non-zero constant. The acceleration in ms^{-2} is given by

A. $-e^{-kx}$

B. $-\dfrac{e^{-kx}}{k^2}$

C. $-\dfrac{e^{-2kx}}{k}$

D. $-k$

E. e^{-2kx}

Question 17

A body is moving in a straight line. Its velocity v ms^{-1} is given by $t\sqrt{x}$ when it is x metres from the origin at a time t seconds. Initially the particle is 4 metres from the origin.
The rule relating x to t is given by

A. $x = \dfrac{1}{16}\left(t^2 + 8\right)^2$

B. $x = \dfrac{1}{4}\left(t^2 + 4\right)^2$

C. $x = \dfrac{t^4}{16} + 4$

D. $x = \dfrac{t^4}{16}$

E. $x = \sqrt[3]{\dfrac{9t^4}{16}}$

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Question 18

A rectangle has a length of X and a width of Y, where X and Y are independent variables.

X has a mean of 30 and a variance of 9, while Y has a mean of 20 and a variance of 4.

If the perimeter of the rectangle is P, then the mean and variance of P are respectively

A. 100 , 52

B. 100 , 26

C. 100 , 20

D. 50 , 5

E. 50 , 13

Question 19

X is normally distributed with a mean of 10.5 and a standard deviation of 0.5.

A random sample of 25 observations is obtained from X and the mean of these observations is denoted by $\bar{X}$. Then $\Pr\left(\bar{X} > 10.55\right)$ is closest to

A. 0.159

B. 0.309

C. 0.406

D. 0.480

E. 0.5

Question 20

The heights of a random sample of 25 junior footballers of a certain age are measured. If the sample mean is 150 cm, with a standard deviation of 5 cm then a 95% confidence interval for the junior footballers of this age is closest to

A. 140 to 160

B. 145.6 to 154.4

C. 147.4 to 152.6

D. 148.04 to 151.96

E. 149.6 to 150.4

END OF SECTION A

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SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (9 marks)

Consider the function $f : D \to R$, $f(x) = \dfrac{2x^3 + 10x^2 + 4x - 16}{x^3 - 2x^2 - x + 2}$

a.i. Find D, the maximal domain of the function f.

1 mark

ii. State the equations of all straight line asymptotes of the graph of f.

1 mark

iii. Find $f'(x)$ and state the coordinates of any stationary points on the graph of f, correct to two decimal places.

2 marks

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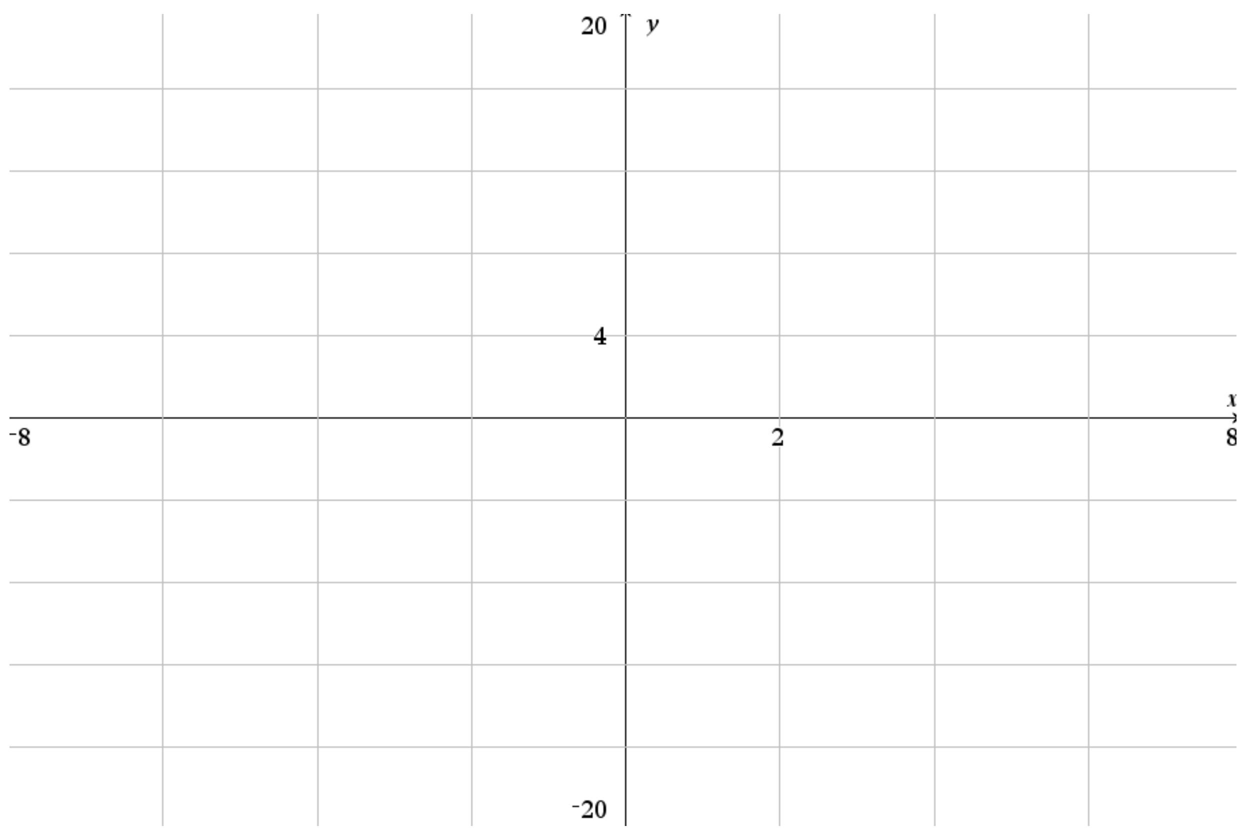
iv. Find $f''(x)$ and state the coordinates of any points of inflexion on the graph of *f*.

2 marks

b. Sketch the graph of $f(x)=\dfrac{2x^3+10x^2+4x-16}{x^3-2x^2-x+2}$ on the axes below, marking all stationary points, points of inflexion and intercepts with axes, labelling with their coordinates. Show any asymptotes and label them with their equations and state any other special features of the graph.

3 marks



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Question 2 (12 marks)

Sets of points in the complex plane are defined by $S=\{z:|z+3+i|=5\}$

$T=\{z:\text{Arg}(z+3)=-\frac{3\pi}{4}\}$ and $R=\{z:|z|=|z+3+3i|\}$, $z\in C$.

a. Find and describe the Cartesian equation of S .

1 mark

b. Find and describe the Cartesian equation of T .

2 marks

c. Find and describe the Cartesian equation of R .

2 marks

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d. If $u \in C$, find u where $u \in S \cap T$.

2 marks

e. If $v \in C$, find v where $v \in S \cap R$.

2 marks

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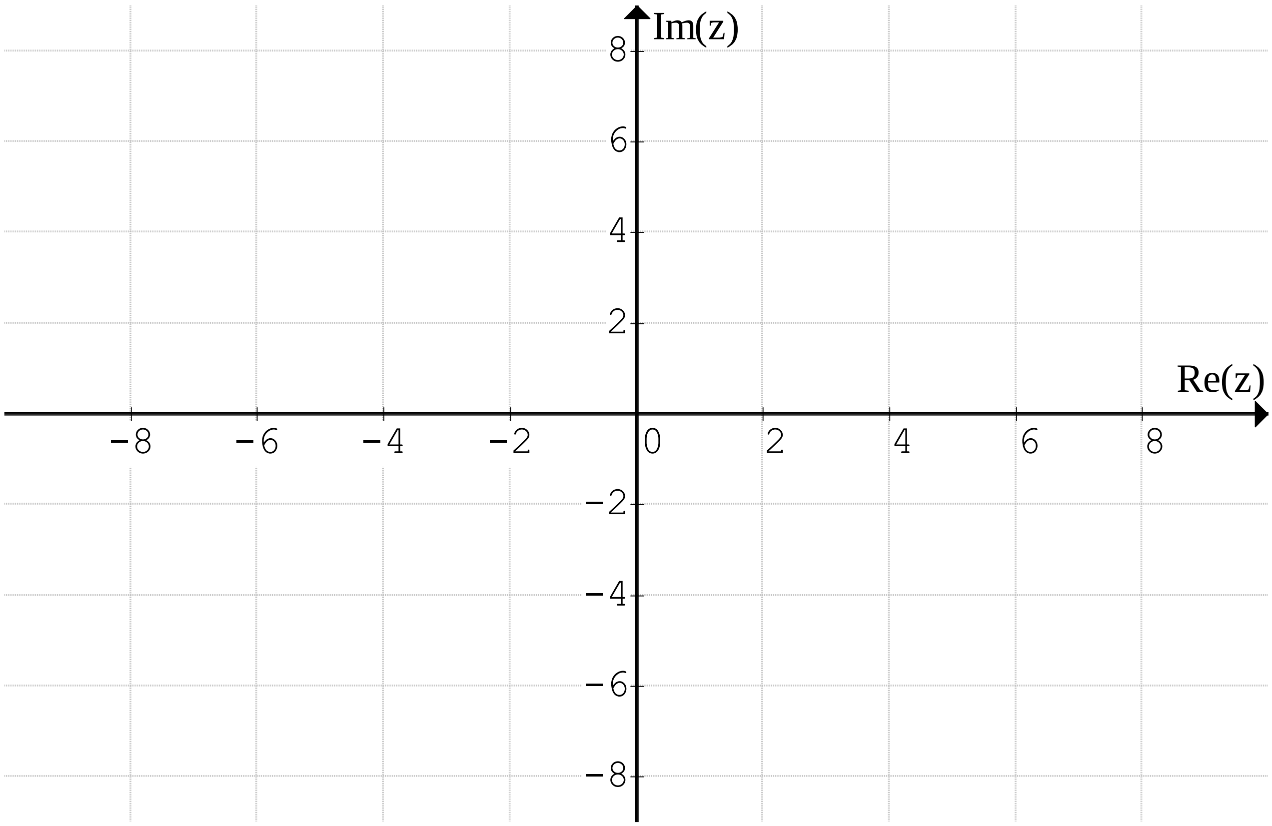
f. Sketch the graphs of S, T and R, and the points u and v on the Argand diagram.

2 marks



g. Find the area in the third quadrant bounded by the graphs of S, T and the real axis.

1 mark

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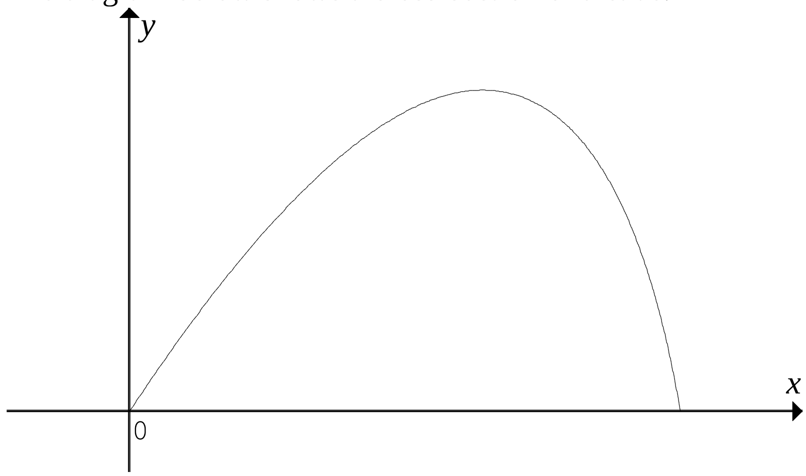
Question 3 (11 marks)

The diagram below shows a cross-section of a cave.



The curve of the cave wall is defined by the parametric equations $x = t^3 - 8t^2 + 16t$ and $y = 3t^3 - 14t^2 + 11t$, for $t \in [0,1]$, where x is the horizontal distance and y is the distance measured vertically upwards above ground level, all distances are measured in metres.

a. State the width of the cave in metres.

1 mark

b.i. Find an expression for the gradient of the curve in terms of t.

2 marks

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ii. Find the co-ordinates of the highest point on the cave, giving your answer in metres correct to two decimal places.

1 mark

c.i. The cross-sectional area of the cave in square metres is given by the definite integral

$$\int_0^1 y\frac{dx}{dt}\,dt = \int_0^1 \left(b_5t^5 + b_4t^4 + b_3t^3 + b_2t^2 + b_1t + b_0\right)dt\,.$$

State the values of b_5, b_4, b_3, b_2, b_1 and b_0.

2 marks

ii. Determine the cross-sectional area of the cave in square metres.

1 mark

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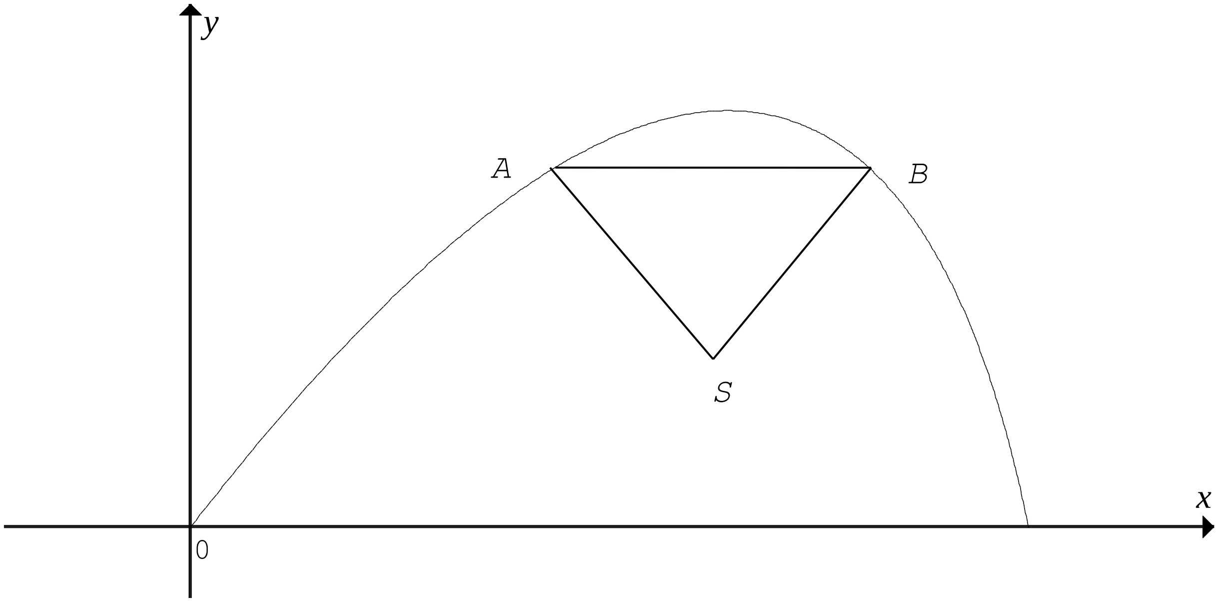
d.



A swing is to be constructed which consists of two ropes each having lengths of two metres. The ends of the rope are attached to two points A and B on the cave wall, on the same horizontal level and also a distance of two metres apart. The two ropes join to the swing at the point S.

i. Find the vertical distance of the swing S above ground level, giving your answer correct to four decimal places.

2 marks

ii. The swing consists of a plastic seat of mass one kilogram and the ropes can withstand a maximum tension of 400 newtons. Find the largest mass that the swing can support, giving your answer in kilograms, correct to one decimal place.

2 marks

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Question 4 (12 marks)

Consider the curve C: $\sqrt{x}+\sqrt{y}=\sqrt{a}$ where a is a positive real number.

a.i. **Show** that the tangent to the curve at the point $P(c,d)$ where $c>0$ and $d>0$ is given by $y\sqrt{c}+x\sqrt{d}=d\sqrt{c}+c\sqrt{d}$

2 marks

ii. If this tangent crosses the x-axis at the point A, and the y-axis at the point B, write down in simplest form the co-ordinates of the points A and B.

1 mark

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Consider the parametric equations $x = a\cos^4(t)$ and $y = a\sin^4(t)$ where $0 \le t \le \frac{\pi}{2}$.

b.i. Find an implicit relationship involving x, y and a.

1 mark

ii. Find the Cartesian co-ordinates on the parametric curve where the tangent is parallel to the line $y = -3x$, and hence determine the equation of the tangent line $t(x)$ at this point.

3 marks

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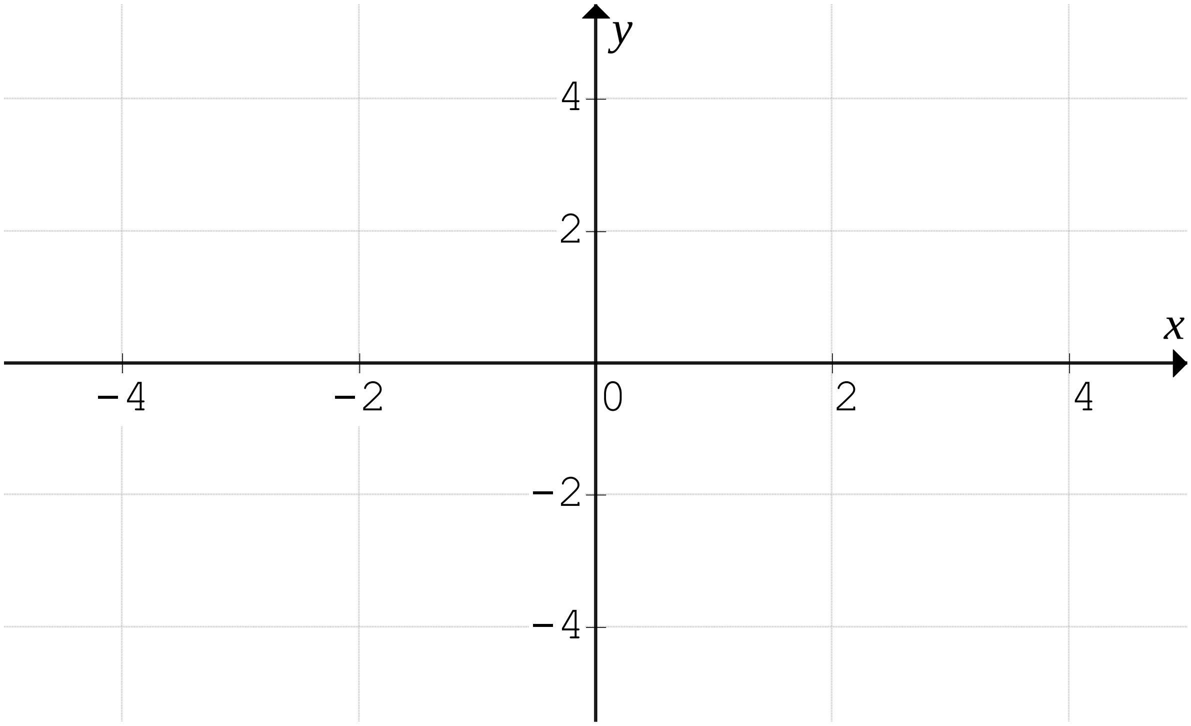
c. Sketch the curve C: $\sqrt{x}+\sqrt{y}=\sqrt{a}$, on the axis below, for the case when $a=4$.

1 mark



Let A be the area bounded by the curve C: $\sqrt{x}+\sqrt{y}=\sqrt{a}$, the tangent line $t(x)$ and the x-axis

d.i. Write down using definite integrals the area A.

1 mark

ii. Hence or otherwise find this area A in terms of a.

1 mark

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e. The length of the curve C is given by La. Find the value of L, giving your answer correct to three decimal places.

2 marks

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Question 5 (9 marks)

A particle of mass one kg is projected vertically upwards in a medium such that the resistance force is equal to kv^2 newtons, where k is a positive constant and v ms^{-1} is its speed as it rises.

a. If the initial velocity is $\frac{1}{3}\sqrt{\frac{g}{k}}$, **show** that the particle rises to a height of $\frac{1}{2k}\log_e\left(\frac{10}{9}\right)$ metres.

3 marks

bi. Write down a definite integral which gives the time T in seconds to reach its maximum height.

1 mark

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Now consider the case when $k = 0.02$

ii. Find value of T.

3 marks

As the particle now falls downwards, it has no resistance forces

c.i. Find the speed in ms^{-1} at which it returns to the initial point from which it was projected. Give your answer correct to two decimal places.

1 mark

ii. Find the how long in seconds the particle takes on its downward journey to reach the initial point from which it was projected. Give your answer correct to two decimal places.

1 mark

Question 6 (7 marks)

Ashley loves his breakfast and tends to over-eat. One morning he had three slices of toast, two eggs and four pieces of bacon. The mean weight of a slice of toast is 50 grams, with a standard deviation of 3 grams, the mean weight of the eggs are 60 grams, with a standard deviation of 5 grams, and the mean weight of a piece of bacon is 65 grams, with a standard deviation of 8 grams. Assume that the weights of slices of toast, eggs and pieces of bacon are independent.

a. Find the probability that Ashley had more than half a kilogram of food for breakfast. Give your answer correct to four decimal places.

2 marks

b. A random sample of n slices of toast was found to have a mean of 51 grams, with a standard deviation of 3 grams. The probability that the mean of these n slices of toast exceeds 50 grams is 0.83, determine the value of n.

2 marks

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c. Ashley also has apple juice for breakfast. A plastic container of apple juice is meant to contain one litre. Ashley has n containers checked by the suppliers. They find the mean of the sample is 999 mls and they know standard deviation of these containers is 5 mls. A 90% confidence interval for the volume of apple juice in these sample containers was found to be 997.16 -1000.84 mls.
Determine the sample size n in this case.

3 marks

END OF EXAMINATION

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EXTRA WORKING SPACE

END OF QUESTION AND ANSWER BOOKLET

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SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

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Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a pyramid	$\frac{1}{3}Ah$
area of triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule	$c^2=a^2+b^2-2ab\cos(C)$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$	
$1+\tan^2(x)=\sec^2(x)$	$\cot^2(x)+1=\operatorname{cosec}^2(x)$
$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$	$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$
$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$	$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$
$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$	
$\sin(2x)=2\sin(x)\cos(x)$	$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

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Circular (trigonometric) functions - continued

Function	$\sin^{-1}$ (arcsin)	$\cos^{-1}$ (arcos)	$\tan^{-1}$ (arctan)
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$	
$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \text{Arg}(z) \le \pi$
$z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$
$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$E(aX + b) = aE(X) + b$ $E(aX + bY) = aE(X) + bE(Y)$ $\text{Var}(aX + b) = a^2\,\text{Var}(X)$
for independent random variables X and Y	$\text{Var}(aX + bY) = a^2\,\text{Var}(X) + b^2\,\text{Var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\bar{X}$	mean $E(\bar{X}) = \mu$ variance $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$
$\lvert\underset{\sim}{r}\rvert = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

Mechanics

momentum	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R} = m\underset{\sim}{a}$

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Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$	$\int \frac{1}{x}dx=\log_e\lvert x\rvert+c$
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$	$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$	$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=a\sec^2(ax)$	$\int \sec^2(ax)\,dx=\frac{1}{a}\tan(ax)+c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right)=\frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right)=\frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right)=\frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$
	$\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$
	$\int (ax+b)^{-1}\,dx=\frac{1}{a}\log_e\lvert ax+b\rvert+c$
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx}=f(x),\ x_0=a$ and $y_0=b,$ then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf\left(x_n\right)$
acceleration	$a=\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2}\sqrt{1+\left(f'(x)\right)^2}\,dx$ or $\int_{t_1}^{t_2}\sqrt{\left(x'(t)\right)^2+\left(y'(t)\right)^2}\,dt$

END OF FORMULA SHEET

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ANSWER SHEET

STUDENT NUMBER

Figures										Letter
Words										

SIGNATURE ______________________________

SECTION A

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
3	**A**	**B**	**C**	**D**	**E**
4	**A**	**B**	**C**	**D**	**E**
5	**A**	**B**	**C**	**D**	**E**
6	**A**	**B**	**C**	**D**	**E**
7	**A**	**B**	**C**	**D**	**E**
8	**A**	**B**	**C**	**D**	**E**
9	**A**	**B**	**C**	**D**	**E**
10	**A**	**B**	**C**	**D**	**E**
11	**A**	**B**	**C**	**D**	**E**
12	**A**	**B**	**C**	**D**	**E**
13	**A**	**B**	**C**	**D**	**E**
14	**A**	**B**	**C**	**D**	**E**
15	**A**	**B**	**C**	**D**	**E**
16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
20	**A**	**B**	**C**	**D**	**E**

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