Neap:

Trial Examination 2018

VCE Specialist Mathematics Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 20 pages
Formula sheet
Answer sheet for multiple-choice questions

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on your answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2018 VCE Specialist Mathematics Units 3&4 Written Examination 2.

SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1

The maximum value of the function $g(x) = \left|\sin(x) - \frac{1}{2}\right|$ is

A. $\frac{3\pi}{2}$

B. $\frac{\pi}{2}$

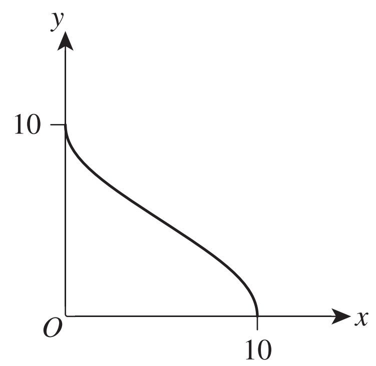
C. $\frac{3}{2}$

D. 1

E. $\frac{1}{2}$

Question 2

The graph of $y = a\cos^{-1}\left(\frac{x}{5} - 1\right)$ for $0 \le x \le 10$ is shown below.



The value of a is

A. 10

B. -10

C. 5

D. $-\frac{10}{\pi}$

E. $\frac{10}{\pi}$

Question 3

Given that $f(x) = \sec\left(\frac{x}{2}\right) + 1,\ 0 \le x < \pi,\ f^{-1}(x)$ is equal to

A. $f^{-1}(x) = 2\cos^{-1}\left(\frac{1}{x-1}\right),\ 2 \le x < \infty$

B. $f^{-1}(x) = 2\cos^{-1}\left(\frac{1}{x-1}\right),\ 0 \le x < \pi$

C. $f^{-1}(x) = 2\cos^{-1}(x-1),\ 2 \le x < \infty$

D. $f^{-1}(x) = 2\cos^{-1}(x-1),\ 0 \le x < \pi$

E. $f^{-1}(x) = 2\cos(x-1),\ 2 \le x < \infty$

Question 4

If $P(z) = z^3 - 3z^2 + z - 3,\ z \in C$, then a linear factor of $P(z)$ is

A. 3

B. i

C. $z + 3$

D. $z + i$

E. $z^2 + 1$

Question 5

The set of points, S, in the complex plane defined by $|z| = |z + a|,\ a \in R^+$ is the

A. point $z = -\frac{a}{2}$.

B. line $\text{Re}(z) = -\frac{a}{2}$.

C. line $\text{Re}(z) = \frac{a}{2}$.

D. circle with centre $(a, 0)$ and radius a.

E. circle with centre $(-a, 0)$ and radius a.

Question 6

For $k \in Z$, the equation $(r\text{cis}(\theta))^n = r_1\text{cis}(\alpha)$ has solutions

A. $(r_1)^n \text{cis}\left(\frac{1}{n}(\alpha + 2k\pi)\right)$

B. $(r_1)^{\frac{1}{n}} \text{cis}\left(\frac{1}{n}(\alpha + 2k\pi)\right)$

C. $(r_1)^{\frac{1}{n}} \text{cis}(n(\alpha + 2k\pi))$

D. $(r_1)^n \text{cis}(n(\alpha + 2k\pi))$

E. $\frac{(r_1)^{\frac{1}{n}}}{n} \text{cis}(\alpha + 2k\pi)$

Question 7

The algebraic fraction $\frac{x-3}{(x-1)^2(x^2+1)}$ could be expressed in partial fraction form as

A. $\frac{A}{(x-1)^2} + \frac{B}{x^2+1}$

B. $\frac{A+B}{x-1} + \frac{C}{x+1}$

C. $\frac{A}{(x-1)^2} + \frac{Bx+C}{x^2+1}$

D. $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1}$

E. $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x^2+1}$

Question 8

With a suitable substitution, $\int_{0}^{\frac{\pi}{6}} \cot\left(\frac{\pi}{2} - 2x\right) dx$ can be expressed as

A. $\int_{\frac{1}{2}}^{1} \frac{1}{u} du$

B. $\int_{\frac{1}{2}}^{1} \frac{1}{2u} du$

C. $\int_{1}^{\frac{1}{2}} \frac{1}{2u} du$

D. $\int_{1}^{\frac{1}{2}} \frac{1}{u} du$

E. $\int_{\frac{1}{2}}^{1} \frac{1}{2} \log_e(u) du$

Question 9

A spherical balloon of volume V and surface area A is inflated.

A correct expression for $\frac{dV}{dt}$ in terms of A and $\frac{dA}{dt}$ is

A. $\frac{1}{6\sqrt{\pi}} A^{\frac{3}{2}} \frac{dA}{dt}$

B. $\sqrt{\frac{A}{\pi}} \frac{dA}{dt}$

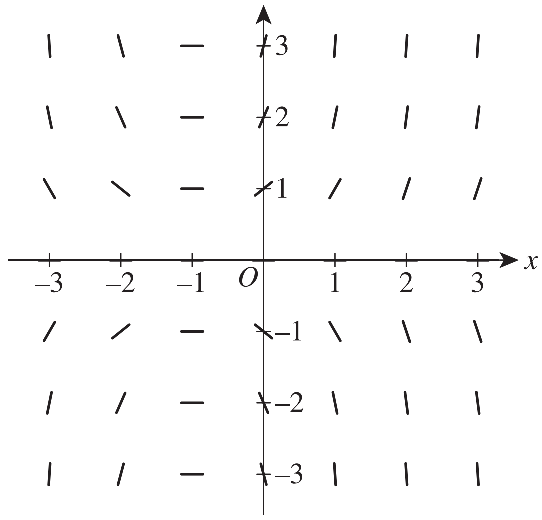
C. $\frac{1}{4}\sqrt{\frac{A}{\pi}} \frac{dA}{dt}$

D. $\frac{1}{2}\sqrt{\frac{A}{\pi}} \frac{dA}{dt}$

E. $\frac{1}{4}\sqrt{A} \frac{dA}{dt}$

Question 10

Consider the direction field below.



The differential equation that is best represented by the above direction field is

A. $\frac{dy}{dx} = (x + 1)^3$

B. $\frac{dy}{dx} = x(y + 1)$

C. $\frac{dy}{dx} = (x + 1)y$

D. $\frac{dy}{dx} = (x - 1)y$

E. $\frac{dy}{dx} = xy$

Question 11

The length of the graph of $y = \sin(\sqrt{x})$ between $x = a$ and $x = b$, where $0 < a < b$ is represented by

A. $\int_a^b \sqrt{\frac{1 + \cos^2\sqrt{x}}{4x}}\, dx$

B. $\int_a^b \sqrt{x + \cos^2\sqrt{x}}\, dx$

C. $\int_a^b \sqrt{1 + \cos^2\sqrt{x}}\, dx$

D. $\int_a^b \sqrt{1 + \frac{\cos^2\sqrt{x}}{2x}}\, dx$

E. $\int_a^b \sqrt{1 + \frac{\cos^2\sqrt{x}}{4x}}\, dx$

Question 12

An infection spreads among a population of N people at a rate proportional to the product of the number of people who have been infected, P, and the number of people who have not been infected.

The differential equation that could be used to model this situation with respect to time t, where k is a positive constant is

A. $\frac{dP}{dt} = kt(t - N)$

B. $\frac{dP}{dt} = kt(N - t)$

C. $\frac{dP}{dt} = kP(P - N)$

D. $\frac{dP}{dt} = kP(N - P)$

E. $\frac{dP}{dt} = kP$

Question 13

If the gradient of a curve is given by $\frac{dy}{dx} = \sqrt{4 - y^2}$, then the equation of the curve that passes through the point $\left(\frac{\pi}{6}, 1\right)$ is

A. $y = 2\sin(x)$

B. $y = 2\sin\left(x + \frac{\pi}{6}\right)$

C. $y = 2\cos\left(x + \frac{\pi}{6}\right)$

D. $y = 2\cos(x)$

E. $y = 2\sin\left(x + \frac{\pi}{3}\right)$

Question 14

The position vector of a particle at time t is given by $\underset{\sim}{r}(t) = \tan(t)\underset{\sim}{i} + \sec^2(t)\underset{\sim}{j}$, $0 \le t < \frac{\pi}{2}$.

At $t = \frac{\pi}{4}$, the particle's direction of motion is given by

A. $\frac{1}{\sqrt{5}}\underset{\sim}{i} + \frac{2}{\sqrt{5}}\underset{\sim}{j}$

B. $\frac{1}{\sqrt{33}}\underset{\sim}{i} + \frac{4\sqrt{2}}{\sqrt{33}}\underset{\sim}{j}$

C. $\frac{1}{\sqrt{5}}\underset{\sim}{i} - \frac{1}{\sqrt{5}}\underset{\sim}{j}$

D. $\frac{1}{\sqrt{5}}\underset{\sim}{i} - \frac{2}{\sqrt{5}}\underset{\sim}{j}$

E. $\frac{1}{\sqrt{5}}\underset{\sim}{i} + \frac{1}{\sqrt{5}}\underset{\sim}{j}$

Question 15

The points R, S and T are collinear.

Given that $\overrightarrow{OR} = \underset{\sim}{i} + \underset{\sim}{j}$, $\overrightarrow{OS} = 2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ and $\overrightarrow{OT} = 3\underset{\sim}{i} + m\underset{\sim}{j} + n\underset{\sim}{k}$, the values of m and n are

A. $m = 6, n = -1$

B. $m = -1, n = 0$

C. $m = 0, n = 1$

D. $m = 3, n = -2$

E. $m = -3, n = 2$

Question 16

A particle of mass 4 kg is acted on by two forces $\underset{\sim}{F}_1 = 6\underset{\sim}{i} - 8\underset{\sim}{j}$ and $\underset{\sim}{F}_2 = -10\underset{\sim}{i} + 24\underset{\sim}{j}$. Both forces are measured in newtons.

The acceleration of the particle has magnitude

A. $\sqrt{5}$ ms^{-2}

B. $\sqrt{15}$ ms^{-2}

C. $\sqrt{17}$ ms^{-2}

D. $4\sqrt{2}$ ms^{-2}

E. $4\sqrt{17}$ ms^{-2}

Question 17

An adult of mass 80 kg is standing on a set of scales in a lift with an upward acceleration of magnitude 1 ms^{-2}.

The reading on the scales in newtons is

A. $80g$

B. 80

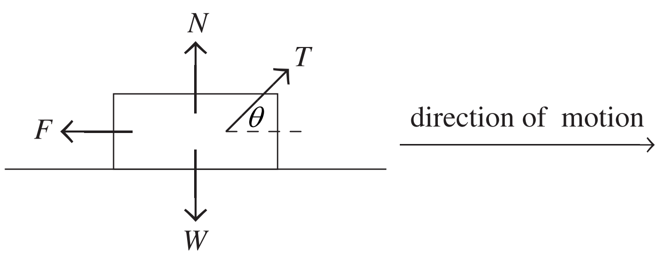
C. $80(g - 1)$

D. 81

E. $80(g + 1)$

Question 18

The diagram below shows all the forces, measured in newtons, acting on a body moving at a constant velocity along a rough horizontal surface.



Which one of the following is true?

A. $T > F$ and $N = W$

B. $T > F$ and $N < W$

C. $T = F$ and $N = W$

D. $T = F$ and $N < W$

E. $T > F$ and $N > W$

Question 19

A random variable X is normally distributed with an unknown mean, μ. A random sample of 16 observations is selected from this population.

If the sample mean is 15.8 and sample variance is 6.1, an approximate 95% confidence interval for μ is

A. (9.7, 21.9)
B. (12.8, 18.8)
C. (15.1, 16.5)
D. (14.6, 17.0)
E. (14.8, 16.8)

Question 20

Random variable X follows a normal distribution with mean 80 and standard deviation 7. Random variable Y follows a normal distribution with mean 54 and standard deviation 5.

If a random variable D is such that $D = X - 2Y$, and D also follows a normal distribution, then the mean and variance of D are

A. –28 and 149.
B. –28 and 27.
C. –28 and $\sqrt{149}$.
D. 188 and 149.
E. 188 and $\sqrt{149}$.

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1 (9 marks)

Consider the function, $f(x) = \dfrac{1}{\sqrt{1+x^2}}$, $x \geq 0$.

a. State the range of f. 1 mark

b. Find $f''(x)$. 1 mark

c. Hence find the coordinates of the point of inflection on the graph of $y = f(x)$. 2 marks

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d. The region bounded by the graph of $y = f(x)$, the x-axis, the y-axis and the line with equation $x = \sqrt{3}$ is rotated 360° about the x-axis to form a volume of the solid of revolution.

By setting up an appropriate definite integral, find the volume of the solid of revolution. 2 marks

e. The region described in **part d.** is now rotated 360° about the y-axis to form another volume of solid of revolution.

By setting up an appropriate definite integral, find the volume of the solid of revolution. 3 marks

Question 2 (11 marks)

The function f is defined by $f(x) = \dfrac{(x-a)(x-b)}{x-c}$ where a, b and c are constants, and $0 < a < b < c$.

a. Express the function f in the form $x + q + \dfrac{r}{x-c}$ giving the constants q and r in terms of a, b and c. 3 marks

The graph of $y = f(x)$ has a vertical asymptote and a non-vertical asymptote.

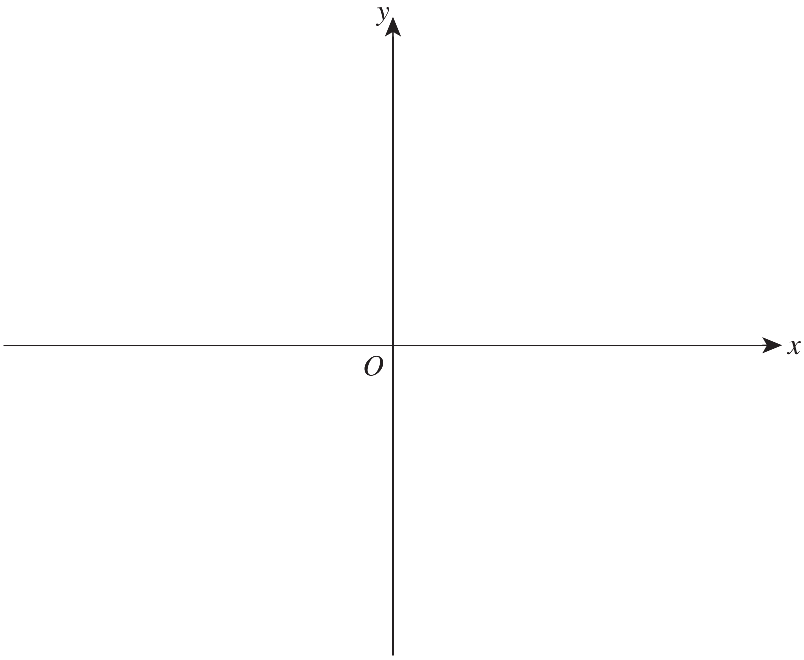
b. **i.** State the equation of the vertical asymptote. 1 mark

ii. State the equation of the non-vertical asymptote, giving your answer in terms of a, b and c. 1 mark

c. Show that the graph of $y = f(x)$ has two stationary points. 3 marks

It is given that $a + b > c$.

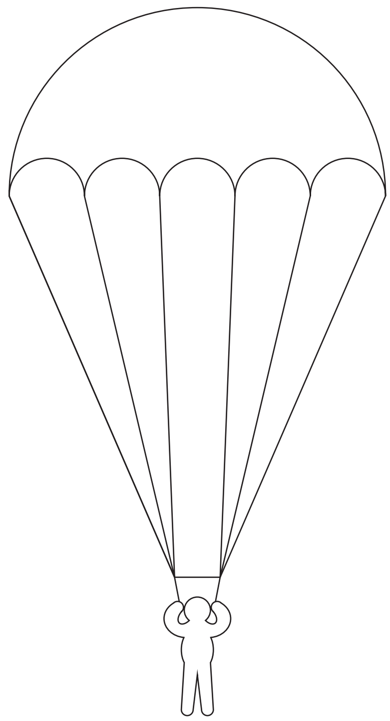
d. Sketch the graph of $y = f(x)$ on the axes below, showing the asymptotes with their equations and the coordinates of any axes intercepts. 3 marks



Question 3 (12 marks)

A parachutist of mass 90 kg falls vertically from rest. The forces acting on him are his weight and the resistance to motion, R newtons. At time t seconds, the velocity of the parachutist is v ms^{-1} and the distance he has fallen is x metres.

a. On the diagram below, mark in the forces acting on the parachutist. 1 mark



Before the parachute opens, the parachutist is in free-fall. The resistance to motion is modelled by $R = kv^2$, where k is a constant. The terminal velocity of the parachutist in free-fall is 60 ms^{-1}.

b. Show that $k = \frac{g}{40}$. 1 mark

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c. By using integration to solve an appropriate differential equation, show that $v^2 = 3600\left(1 - e^{-\frac{gx}{1800}}\right)$. 5 marks

When the parachutist has fallen 1800 metres, he opens his parachute. At the moment the parachute opens, the resistance to motion instantly changes and is now modelled by $R = 90v$.

d. Find the parachutist's velocity in the instant just before he opens his parachute. Give your answer correct to four decimal places. 1 mark

e. Find the time it takes after opening the parachute for the parachutist's velocity to decrease to 20 ms^{-1}. Give your answer correct to two decimal places. 4 marks

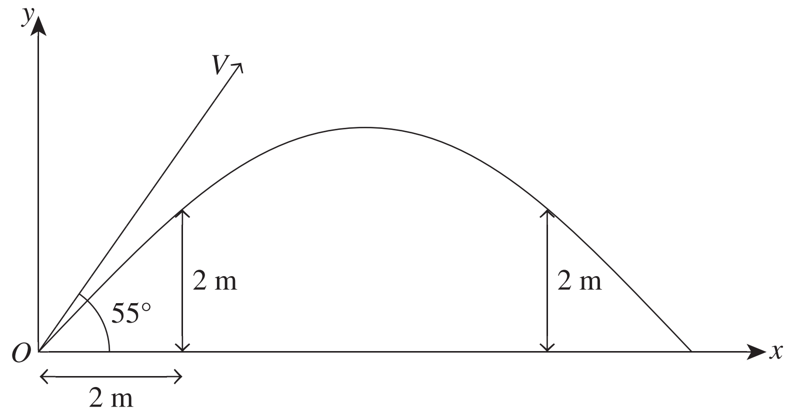
Question 4 (10 marks)

A projectile is fired from O with velocity $V\cos(\theta)\underset{\sim}{i} + V\sin(\theta)\underset{\sim}{j}$ ms^{-1} at an angle θ degrees to the horizontal, where $0° < \theta < 90°$. The acceleration, $\underset{\sim}{a}_y$, of the projectile moving in the x–y plane is $-g\underset{\sim}{j}$ ms^{-2}. Let $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j}$ be the projectile's position vector after t seconds of flight.

a. Show that $\underset{\sim}{r} = (V\cos(\theta)t)\underset{\sim}{i} + \left(V\sin(\theta)t - \frac{1}{2}gt^2\right)\underset{\sim}{j}$. 4 marks

b. Hence show that the cartesian equation describing the projectile's path is given by $y = \tan(\theta)x - \dfrac{gx^2}{2V^2\cos^2(\theta)}$. 2 marks

A particle is projected from a point O at ground level at an angle of elevation of 55°. It just clears the top of two walls that are both 2 metres high. The first wall is situated 2 metres from O as shown in the diagram below.



c. Find the particle's speed of projection. Give your answer correct to three decimal places. 2 marks

d. Find the distance of the second wall from O. Give your answer correct to one decimal place. 2 marks

Question 5 (8 marks)

A sample of size 100 is taken from a normal population with unknown mean μ and variance 64.

A researcher wishes to test the hypotheses $H_0 : \mu = 35$, $H_1 : \mu > 35$. He decides on the following acceptance criteria:

- Accept H_0 if the sample mean $\bar{x} \leq 36.5$.
- Accept H_1 if $\bar{x} > 36.5$.

a. Find the probability of a type I error. Give your answer correct to four decimal places. 2 marks

A second researcher decides to use the same data to test the hypotheses $H_0 : \mu = 35$, $H_1 : \mu = 37.9$. He decides to use the same acceptance criteria as the first researcher.

b. **i.** Find the probability of a type II error. Give your answer correct to four decimal places. 3 marks

ii. Find the critical value for $\bar{x}$ if the researcher wants to satisfy the condition Pr(type I error) = Pr(type II error). 3 marks

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Question 6 (10 marks)

Consider the quadratic equation $z^2 + bz + 1 = 0$ where $b \in R$ and $z \in C$.

Let $x + yi$ be a root of this quadratic equation.

a. Show that $x^2 - y^2 + bx + 1 = 0$ and $(2x + b)y = 0$. 3 marks

b. Hence, by first considering $(2x + b)y = 0$, show that either $b = -2x$ or $b = -\frac{x^2 + 1}{x}$, $x \neq 0$. 3 marks

Consider the case $b = -2x$.

c. **i.** Find the cartesian equation of the relation represented in the complex plane. 1 mark

ii. Describe geometrically the relation found in **part c. i.** 1 mark

Consider the case $b = -\frac{x^2 + 1}{x}, x \neq 0$.

d. **i.** Find the cartesian equation of the relation represented in the complex plane. 1 mark

ii. Describe geometrically the relation found in **part d. i.** 1 mark

END OF QUESTION AND ANSWER BOOKLET