

STUDENT NUMBER

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SPECIALIST MATHEMATICS

Written examination 2

Monday 12 November 2018

Reading time: 3.00 pm to 3.15 pm (15 minutes)

Writing time: 3.15 pm to 5.15 pm (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	Number of questions	Number of questions to be answered	Number of marks
A	20	20	20
B	6	6	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 25 pages
- Formula sheet
- Answer sheet for multiple-choice questions

Instructions

- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Multiple-choice questions**Instructions for Section A**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

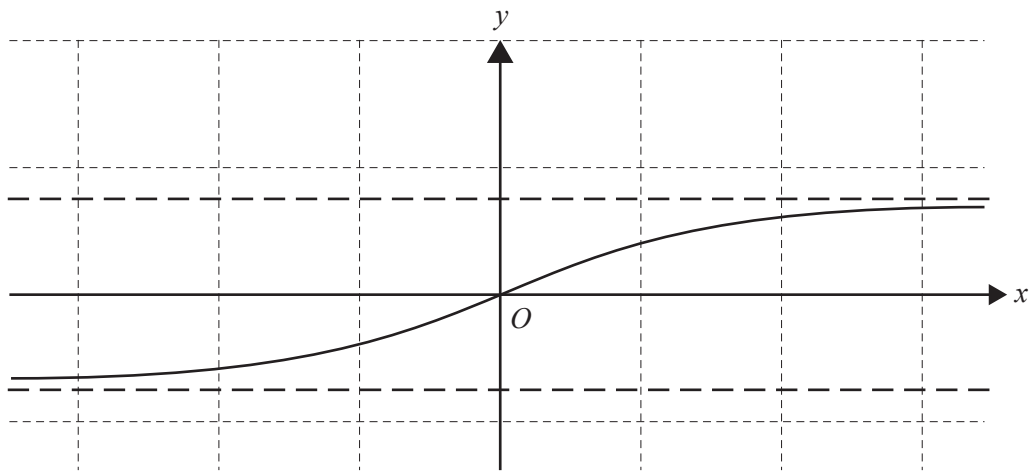
No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$

Question 1

Part of the graph of $y = \frac{1}{2} \tan^{-1}(x)$ is shown below.



The equations of its asymptotes are

- A. $y = \pm \frac{1}{2}$
- B. $y = \pm \frac{3}{4}$
- C. $y = \pm 1$
- D. $y = \pm \frac{\pi}{2}$
- E. $y = \pm \frac{\pi}{4}$

Question 2

Consider the function f with rule $f(x) = \frac{1}{\sqrt{\sin^{-1}(cx+d)}}$, where $c, d \in R$ and $c > 0$.
The domain of f is

- A. $x > -\frac{d}{c}$
- B. $-\frac{d}{c} < x \leq \frac{1-d}{c}$
- C. $\frac{-1-d}{c} \leq x \leq \frac{1-d}{c}$
- D. $x \in R \setminus \left\{ -\frac{d}{c} \right\}$
- E. $x \in R$

Question 3

Which one of the following, where A, B, C and D are non-zero real numbers, is the partial fraction form for the expression $\frac{2x^2 + 3x + 1}{(2x+1)^3(x^2-1)}$?

- A. $\frac{A}{2x+1} + \frac{B}{x-1} + \frac{C}{x+1}$
- B. $\frac{A}{2x+1} + \frac{B}{(2x+1)^2} + \frac{C}{(2x+1)^3} + \frac{Dx}{x^2-1}$
- C. $\frac{A}{2x+1} + \frac{Bx+C}{x^2-1}$
- D. $\frac{A}{2x+1} + \frac{B}{(2x+1)^2} + \frac{C}{x-1}$
- E. $\frac{A}{2x+1} + \frac{Bx+C}{(2x+1)^2} + \frac{D}{x-1}$

Question 4

If $\cos(x) = -a$ and $\cot(x) = b$, where $a, b > 0$, then $\operatorname{cosec}(-x)$ is equal to

- A. $\frac{b}{a}$
- B. $-\frac{b}{a}$
- C. $-\frac{a}{b}$
- D. $\frac{a}{b}$
- E. $-ab$

Question 5

Let $z = a + bi$, where $a, b \in \mathbb{R} \setminus \{0\}$.

If $z + \frac{1}{z} \in \mathbb{R}$, which one of the following must be **true**?

- A. $\text{Arg}(z) = \frac{\pi}{4}$
- B. $a = -b$
- C. $a = b$
- D. $|z| = 1$
- E. $z^2 = 1$

Question 6

The complex numbers z , iz and $z + iz$, where $z \in \mathbb{C} \setminus \{0\}$, are plotted in the Argand plane, forming the vertices of a triangle.

The area of this triangle is given by

- A. $|z|$
- B. $|z| + |z|^2$
- C. $\frac{|z|^2}{2}$
- D. $|z|^2$
- E. $\frac{\sqrt{3}|z|^2}{2}$

Question 7

A curve is described parametrically by $x = \sin(2t)$, $y = 2\cos(t)$ for $0 \leq t \leq 2\pi$.

The length of the curve is closest to

- A. 9.2
- B. 9.5
- C. 12.2
- D. 12.5
- E. 38.3

Question 8

Using a suitable substitution, $\int_0^{\frac{\pi}{6}} \tan^2(x) \sec^2(x) dx$ can be expressed as

A. $\int_0^{\frac{1}{\sqrt{3}}} (u^4 + u^2) du$

B. $\int_1^{\frac{2}{\sqrt{3}}} (u^4 + u^2) du$

C. $\int_0^{\frac{1}{\sqrt{3}}} u du$

D. $\int_0^{\frac{\pi}{6}} u^2 du$

E. $\int_0^{\frac{1}{\sqrt{3}}} u^2 du$

Question 9

A solution to the differential equation $\frac{dy}{dx} = \frac{2}{\sin(x+y) - \sin(x-y)}$ can be obtained from

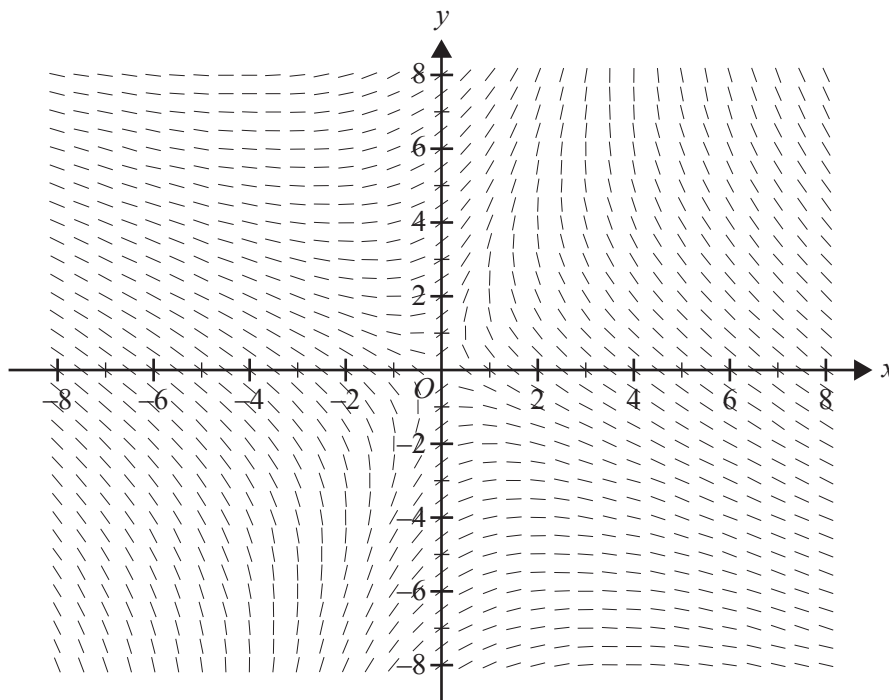
A. $\int 1 dx = \int 2 \sin(y) dy$

B. $\int \cos(y) dy = \int \operatorname{cosec}(x) dx$

C. $\int \cos(x) dx = \int \operatorname{cosec}(y) dy$

D. $\int \sec(x) dx = \int \sin(y) dy$

E. $\int \sec(x) dx = \int \operatorname{cosec}(y) dy$

Question 10

The differential equation that best represents the direction field above is

- A. $\frac{dy}{dx} = \frac{2x + y}{y - 2x}$
- B. $\frac{dy}{dx} = \frac{x + 2y}{2x - y}$
- C. $\frac{dy}{dx} = \frac{2x - y}{x + 2y}$
- D. $\frac{dy}{dx} = \frac{x - 2y}{y - 2x}$
- E. $\frac{dy}{dx} = \frac{2x + y}{2y - x}$

Question 11

Consider the vectors given by $\mathbf{a} = m\mathbf{i} + \mathbf{j}$ and $\mathbf{b} = \mathbf{i} + m\mathbf{j}$, where $m \in R$.

If the acute angle between \mathbf{a} and \mathbf{b} is 30° , then m equals

- A. $\sqrt{2} \pm 1$
- B. $2 \pm \sqrt{3}$
- C. $\sqrt{3}, \frac{1}{\sqrt{3}}$
- D. $\frac{\sqrt{3}}{4 - \sqrt{3}}$
- E. $\frac{\sqrt{39}}{13}$

Question 12

If $|\underline{a} + \underline{b}| = |\underline{a}| + |\underline{b}|$ and $\underline{a}, \underline{b} \neq \underline{0}$, which one of the following is **necessarily true**?

- A. \underline{a} is parallel to \underline{b}
- B. $|\underline{a}| = |\underline{b}|$
- C. $\underline{a} = \underline{b}$
- D. $\underline{a} = -\underline{b}$
- E. \underline{a} is perpendicular to \underline{b}

Question 13

The position vector of a particle that is moving along a curve at time t is given by $\underline{r}(t) = 3\cos(t)\underline{i} + 4\sin(t)\underline{j}$, $t \geq 0$.

The **first** time when the speed of the particle is a minimum is

- A. 3
- B. $\frac{\pi}{2}$
- C. $\tan^{-1}\left(\frac{4}{3}\right)$
- D. $\frac{3\pi}{2}$
- E. 9

Question 14

The scalar resolute of $\underline{a} = 3\underline{i} - 2\underline{k}$ in the direction of $\underline{b} = -\underline{i} + 2\underline{j} + 3\underline{k}$ is

- A. $-\frac{9\sqrt{13}}{13}$
- B. $-\frac{9}{14}(-\underline{i} + 2\underline{j} + 3\underline{k})$
- C. $-\frac{9\sqrt{14}}{14}$
- D. $-\frac{9}{13}(3\underline{i} - 2\underline{k})$
- E. $-\frac{\sqrt{14}}{2}$

Question 15

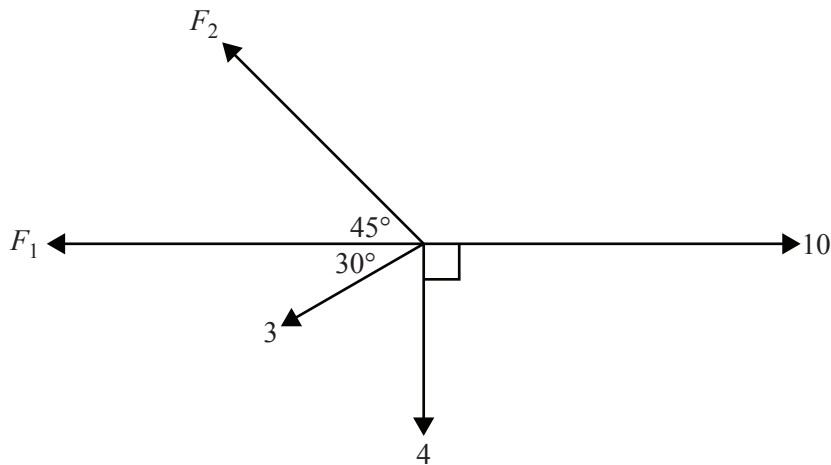
A constant force of magnitude P newtons accelerates a particle of mass 8 kg in a straight line from a speed of 4 ms^{-1} to a speed of 20 ms^{-1} over a distance of 15 m.

The magnitude of P is

- A. 9.8
- B. 12.5
- C. 12.8
- D. 100
- E. 102.4

Question 16

The diagram below shows a mass being acted on by a number of forces whose magnitudes are labelled. All forces are measured in newtons and the system is in equilibrium.



The value of F_2 is

- A. $\frac{\sqrt{2}}{2}(8+3\sqrt{3})$
- B. $\frac{11\sqrt{2}}{2}$
- C. $\frac{3\sqrt{2}}{2}$
- D. 7.78
- E. 7.0

Question 17

A tourist standing in the basket of a hot air balloon is ascending at 2 ms^{-1} . The tourist drops a camera over the side when the balloon is 50 m above the ground.

Neglecting air resistance, the time in seconds, correct to the nearest tenth of a second, taken for the camera to hit the ground is

- A. 2.3
- B. 2.4
- C. 3.0
- D. 3.2
- E. 3.4

Question 18

A 95% confidence interval for the mean height μ , in centimetres, of a random sample of 36 Irish setter dogs is $58.42 < \mu < 67.31$

The standard deviation of the height of the population of Irish setter dogs, in centimetres, correct to two decimal places, is

- A. 2.26
- B. 2.27
- C. 13.60
- D. 13.61
- E. 62.87

Question 19

The gestation period of cats is normally distributed with mean $\mu = 66$ days and variance $\sigma^2 = \frac{16}{9}$.

The probability that a sample of five cats chosen at random has an average gestation period greater than 65 days is closest to

- A. 0.5000
- B. 0.7131
- C. 0.7734
- D. 0.8958
- E. 0.9532

Question 20

The scores on the Mathematics and Statistics tests, expressed as percentages, in a particular year were both normally distributed. The mean and the standard deviation of the Mathematics test scores were 71 and 10 respectively, while the mean and the standard deviation of the Statistics test scores were 75 and 7 respectively.

Assuming the sets of test scores were independent of each other, the probability, correct to four decimal places, that a randomly chosen Mathematics score is higher than a randomly chosen Statistics score is

- A. 0.2877
- B. 0.3716
- C. 0.4070
- D. 0.7123
- E. 0.9088

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

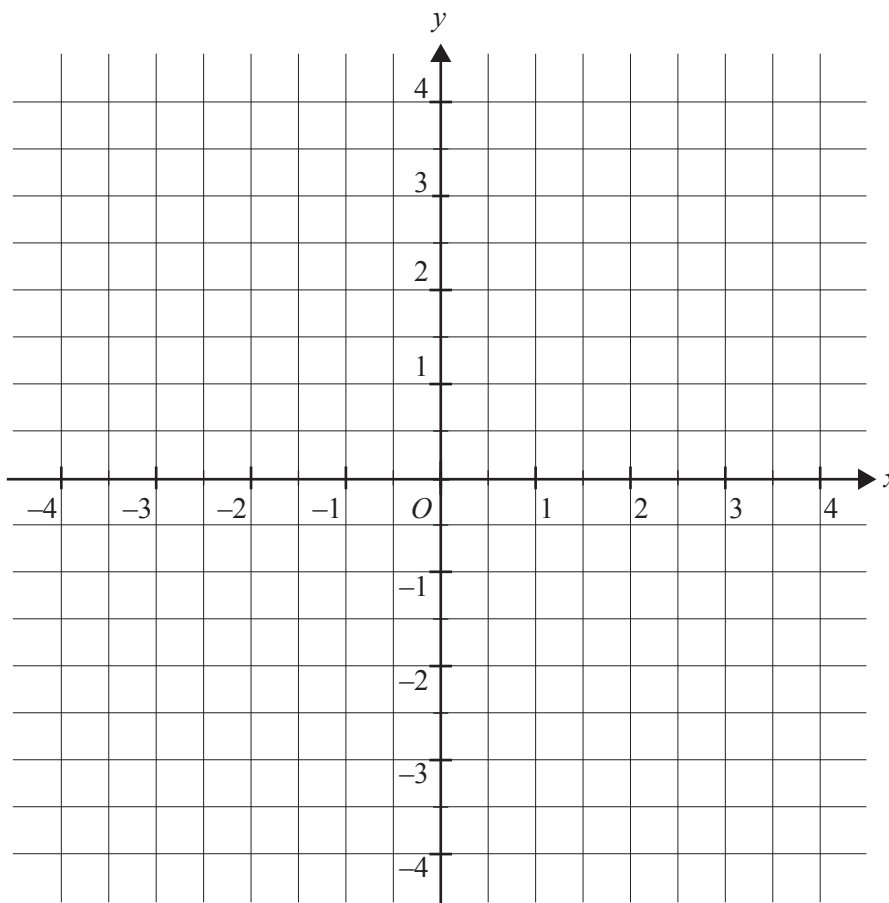
Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$

Question 1 (11 marks)

Consider the function $f: D \rightarrow R$, where $f(x) = 2\arcsin(x^2 - 1)$.

- a. Determine the maximal domain D and the range of f . 2 marks

- b. Sketch the graph of $y = f(x)$ on the axes below, labelling any endpoints and the y -intercept with their coordinates. 3 marks



- c. Find $f'(x)$ for $x > 0$, expressing your answer in the form $f'(x) = \frac{A}{\sqrt{2-x^2}}$, $A \in R$. 1 mark

- d. Write down $f'(x)$ for $x < 0$, expressing your answer in the form $f'(x) = \frac{B}{\sqrt{2-x^2}}$, $B \in R$. 1 mark

e. The derivative $f'(x)$ can be expressed in the form $f'(x) = \frac{g(x)}{\sqrt{2-x^2}}$ over its maximal domain.

i. Find the maximal domain of f' .

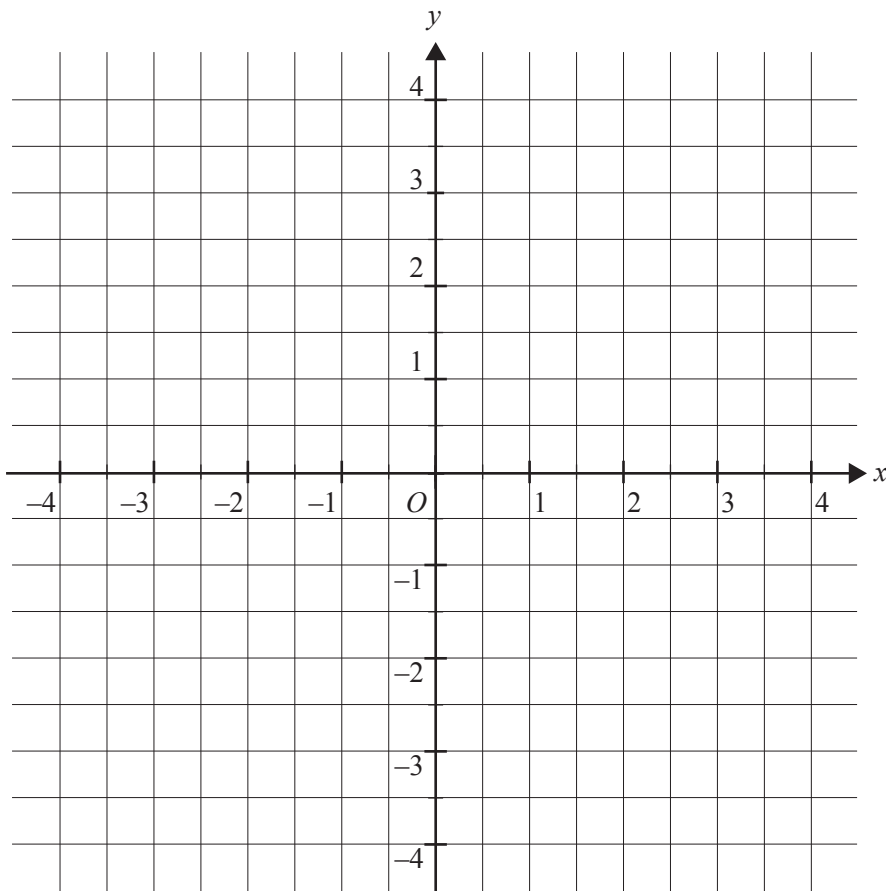
1 mark

ii. Find $g(x)$, expressing your answer as a piecewise (hybrid) function.

1 mark

iii. Sketch the graph of g on the axes below.

2 marks



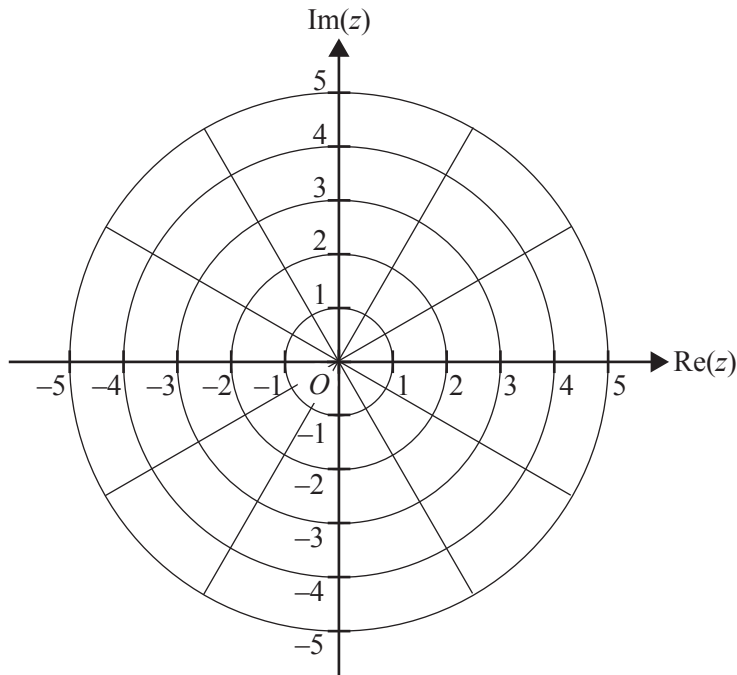
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Question 2 (10 marks)

- a. State the centre in the form (x, y) , where $x, y \in R$, and state the radius of the circle given by $|z - (1 + 2i)| = 2$, where $z \in C$. 1 mark

- b. By expressing the circle given by $|z + 1| = \sqrt{2}|z - i|$ in cartesian form, show that this circle has the same centre and radius as the circle given by $|z - (1 + 2i)| = 2$. 2 marks

- c. Graph the circle given by $|z + 1| = \sqrt{2}|z - i|$ on the Argand diagram below, labelling the intercepts with the vertical axis. 2 marks

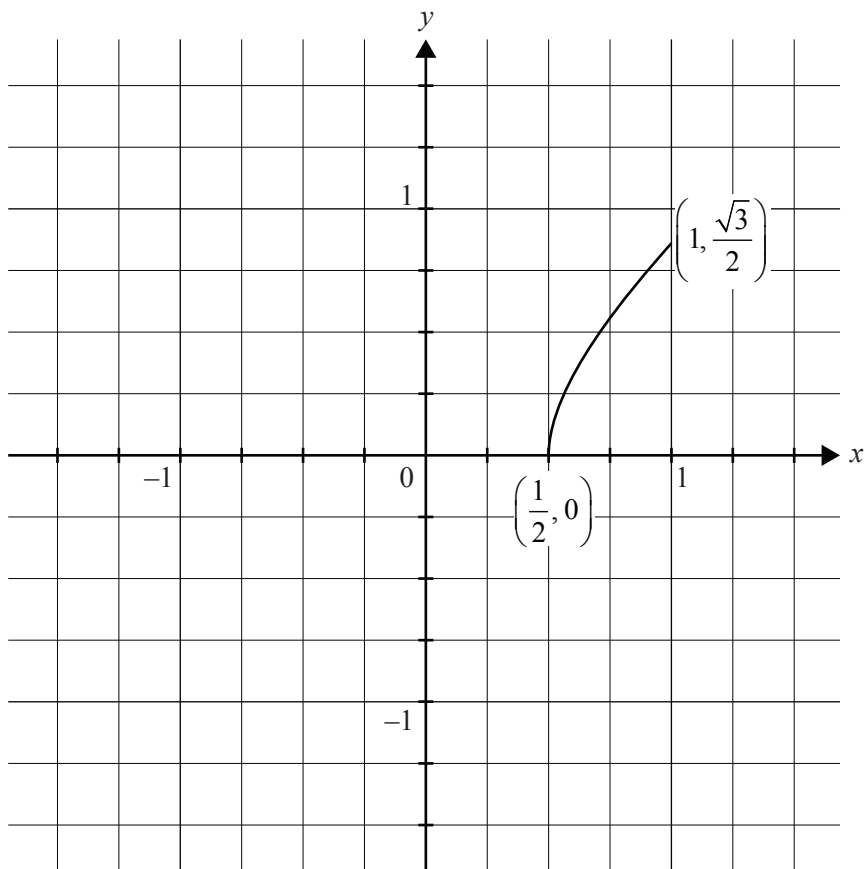


The line given by $|z-1|=|z-3|$ intersects the circle given by $|z+1|=\sqrt{2}|z-i|$ in two places.

- d.** Draw the line given by $|z-1|=|z-3|$ on the Argand diagram in **part c**. Label the points of intersection with their coordinates. 2 marks
- e.** Find the area of the minor segment enclosed by an arc of the circle given by $|z+1|=\sqrt{2}|z-i|$ and part of the line given by $|z-1|=|z-3|$. 3 marks

Question 3 (13 marks)

Part of the graph of $y = \frac{1}{2}\sqrt{4x^2 - 1}$ is shown below.



The curve shown is rotated about the y -axis to form a volume of revolution that is to model a fountain, where length units are in metres.

- a. Show that the volume, V cubic metres, of water in the fountain when it is filled to a depth of h metres is given by $V = \frac{\pi}{4} \left(\frac{4}{3}h^3 + h \right)$.

2 marks

- b.** Find the depth h when the fountain is filled to half its volume. Give your answer in metres, correct to two decimal places. 2 marks

The fountain is initially empty. A vertical jet of water in the centre fills the fountain at a rate of 0.04 cubic metres per second and, at the same time, water flows out from the bottom of the fountain at a rate of $0.05\sqrt{h}$ cubic metres per second when the depth is h metres.

- c. i.** Show that $\frac{dh}{dt} = \frac{4 - 5\sqrt{h}}{25\pi(4h^2 + 1)}$. 2 marks

- ii.** Find the rate, in metres per second, correct to four decimal places, at which the depth is increasing when the depth is 0.25 m. 1 mark

- d.** Express the time taken for the depth to reach 0.25 m as a definite integral and evaluate this integral correct to the nearest tenth of a second. 2 marks

- e.** After 25 seconds the depth has risen to 0.4 m.
Using Euler's method with a step size of five seconds, find an estimate of the depth 30 seconds after the fountain began to fill. Give your answer in metres, correct to two decimal places. 2 marks

- f.** How far from the top of the fountain does the water level ultimately stabilise? Give your answer in metres, correct to two decimal places. 2 marks

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Question 4 (10 marks)

Two yachts, A and B, are competing in a race and their position vectors on a certain section of the race after time t hours are given by

$$\underline{r}_A(t) = (t+1)\underline{i} + (t^2 + 2t)\underline{j} \quad \text{and} \quad \underline{r}_B(t) = t^2\underline{i} + (t^2 + 3)\underline{j}, \quad t \geq 0$$

where displacement components are measured in kilometres from a given reference buoy at origin O .

- a.** Find the cartesian equation of the path for each yacht. 2 marks

- b.** Show that the two yachts will not collide if they follow these paths. 2 marks

- c.** Find the coordinates of the point where the paths of the two yachts cross. Give your coordinates correct to three decimal places. 2 marks

One of the rules for the race is that the yachts are not allowed to be within 0.2 km of each other. If this occurs there is a time penalty for the yacht that is travelling faster.

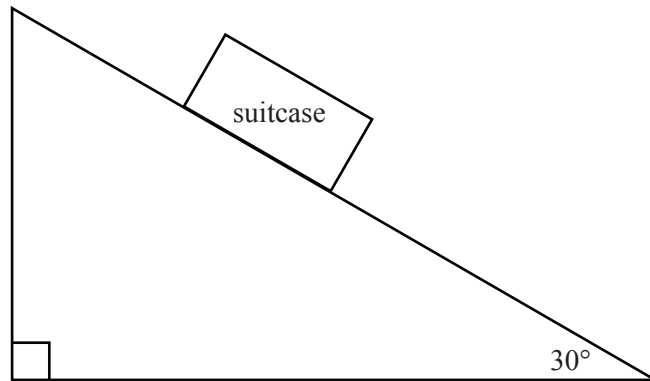
- d. For what values of t is yacht A travelling faster than yacht B? 2 marks

- e. If yacht A does not alter its course, for what period of time will yacht A be within 0.2 km of yacht B? Give your answer in **minutes**, correct to one decimal place. 2 marks

Question 5 (8 marks)

Luggage at an airport is delivered to its owners via a 15 m ramp that is inclined at 30° to the horizontal. A 20 kg suitcase, initially at rest at the top of the ramp, slides down the ramp against a resistance of v newtons per kilogram, where $v \text{ ms}^{-1}$ is the speed of the suitcase.

- a. On the diagram below, show all forces acting on the suitcase during its motion down the ramp. 1 mark



- b. i. By resolving forces parallel to the ramp, write down an equation of motion for the 20 kg suitcase. 1 mark

- ii. Hence, show that the magnitude of the acceleration, $a \text{ ms}^{-2}$, of the suitcase down the ramp is given by $a = \frac{g - 2v}{2}$. 1 mark

- c. By expressing a in an appropriate form, find the distance x metres that the suitcase has slid as a function of v . Give your answer in the form $x = bv + c \log_e \left(\frac{c}{c-v} \right)$, where $b, c \in R$. 2 marks

- d. Find the velocity of the suitcase just before it reaches the end of the ramp. Give your answer in ms^{-1} , correct to two decimal places. 1 mark

- e. i. Write down a definite integral that gives the time taken for the suitcase to reach a speed of 4.5 ms^{-1} . 1 mark

- ii. Find the time taken for the suitcase to reach a speed of 4.5 ms^{-1} . Give your answer in seconds, correct to two decimal places. 1 mark

Question 6 (8 marks)

The heights of mature water buffaloes in northern Australia are known to be normally distributed with a standard deviation of 15 cm. It is claimed that the mean height of the water buffaloes is 150 cm.

To decide whether the claim about the mean height is true, rangers selected a random sample of 50 mature water buffaloes. The mean height of this sample was found to be 145 cm.

A one-tailed statistical test is to be carried out to see if the sample mean height of 145 cm differs significantly from the claimed population mean of 150 cm.

Let \bar{X} denote the mean height of a random sample of 50 mature water buffaloes.

- a. State suitable hypotheses H_0 and H_1 for the statistical test. 1 mark

- b. Find the standard deviation of \bar{X} . 1 mark

- c. Write down an expression for the p value of the statistical test and evaluate your answer correct to four decimal places. 2 marks

- d. State with a reason whether H_0 should be rejected at the 5% level of significance. 1 mark

- e. What is the smallest value of the sample mean height that could be observed for H_0 to be **not** rejected? Give your answer in centimetres, correct to two decimal places. 1 mark

- f. If the true mean height of all mature water buffaloes in northern Australia is in fact 145 cm, what is the probability that H_0 will be accepted at the 5% level of significance? Give your answer correct to two decimal places.

1 mark

- g. Using the observed sample mean of 145 cm, find a **99% confidence interval** for the mean height of all mature water buffaloes in northern Australia. Express the values in your confidence interval in centimetres, correct to one decimal place.

1 mark

**Victorian Certificate of Education
2018**

SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc \sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab \cos(C)$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

Circular functions – continued

Function	\sin^{-1} or arcsin	\cos^{-1} or arccos	\tan^{-1} or arctan
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r \operatorname{cis}(\theta)$	
$ z = \sqrt{x^2 + y^2} = r$	$-\pi < \operatorname{Arg}(z) \leq \pi$
$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$
$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$E(aX + b) = aE(X) + b$ $E(aX + bY) = aE(X) + bE(Y)$ $\operatorname{var}(aX + b) = a^2 \operatorname{var}(X)$
for independent random variables X and Y	$\operatorname{var}(aX + bY) = a^2 \operatorname{var}(X) + b^2 \operatorname{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z \frac{s}{\sqrt{n}}, \bar{x} + z \frac{s}{\sqrt{n}}\right)$
distribution of sample mean \bar{X}	mean $E(\bar{X}) = \mu$ variance $\operatorname{var}(\bar{X}) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$	$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$
$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)} (ax+b)^{n+1} + c, n \neq -1$
	$\int (ax+b)^{-1} dx = \frac{1}{a} \log_e ax+b + c$
product rule	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2} \sqrt{1+(f'(x))^2} dx$ or $\int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2} dt$

Vectors in two and three dimensions

$\underline{r} = x\hat{i} + y\hat{j} + z\hat{k}$
$ \underline{r} = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$
$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

Mechanics

momentum	$\underline{p} = m\underline{v}$
equation of motion	$\underline{R} = m\underline{a}$