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Specialist Mathematics

2019

Trial Examination 1
(1 hour)

Instructions

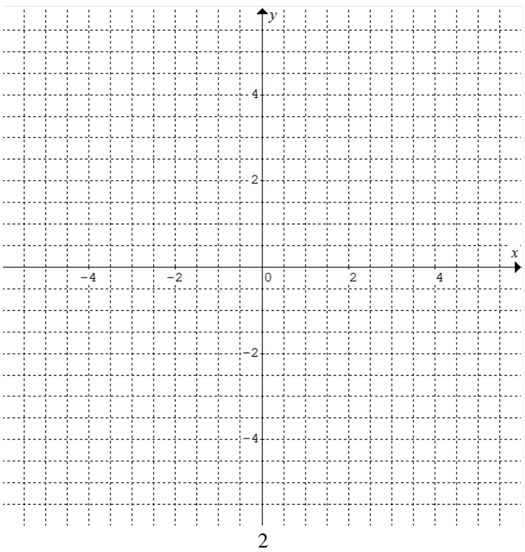
Answer **all** questions. Do **not** use calculators.
Unless otherwise specified, an **exact** answer is required to a question.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
In questions where more than one mark is available, show appropriate working or explanation.
Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

Question 1 Consider $f(x) = \frac{x^2}{4} - \frac{4}{3x^2}$.

a. Determine the asymptotic behaviour, axis intercepts, nature and coordinates of stationary points, and coordinates of points of inflection of $f(x)$.

4 marks

b. Sketch the graph of $f(x) = \frac{x^2}{4} - \frac{4}{3x^2}$. Include features found in part a. 2 marks



Question 2 Evaluate $\int_0^1 \left(\frac{xe^{x^2}}{1+e^{x^2}} \right) dx$. Hint: Let $u = 1 + e^{x^2}$. 3 marks

Question 3 $-\frac{\sqrt{6}}{2} + i\frac{\sqrt{2}}{2}$ is a root of $z^6 + n = 0$ where n is a positive integer.

a. Show that $n = 8$. 2 marks

b. Find the other roots of $z^6 + n = 0$. 2 marks

Question 4 $\underset{\sim}{p} = \sqrt{2}\,\underset{\sim}{i} - 2\,\underset{\sim}{j} + \sqrt{3}\,\underset{\sim}{k}$ is perpendicular to $\underset{\sim}{q} = \alpha\,\underset{\sim}{j} - \beta\,\underset{\sim}{k}$ where $\alpha, \beta \in R$.

a. Show that $\dfrac{\alpha}{\beta} = -\dfrac{\sqrt{3}}{2}$. 1 mark

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b. The angle between $\underset{\sim}{p}$ and $\underset{\sim}{j}$ is θ. Evaluate $\cos\theta$. 1 mark

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c. In terms of β only, find a possible $\underset{\sim}{r}$ such that $\underset{\sim}{p}$, $\underset{\sim}{q}$ and $\underset{\sim}{r}$ are linearly dependent.
2 marks

Question 5 Solve $\dfrac{dy}{dx} = -\dfrac{x\left(y^2 - 1\right)}{y\left(x^2 - 1\right)}$ for y in terms of x, given that $(-2, 2)$ satisfies the relation. 4 marks

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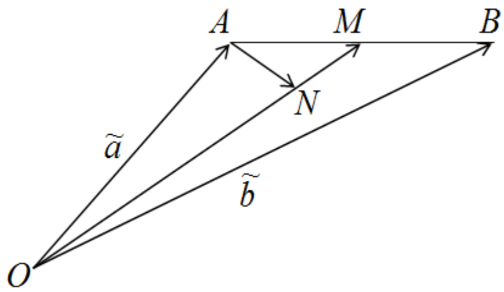
Question 6 In the following diagram, M is the midpoint of line segment AB, and point N divides line segment CM into a ratio of $4:1$. $\overrightarrow{OA} = \tilde{a}$ and $\overrightarrow{OB} = \tilde{b}$.

Express $\overrightarrow{AN}$ in terms of $\tilde{a}$ and $\tilde{b}$. 3 marks



Question 7 Solve for x.

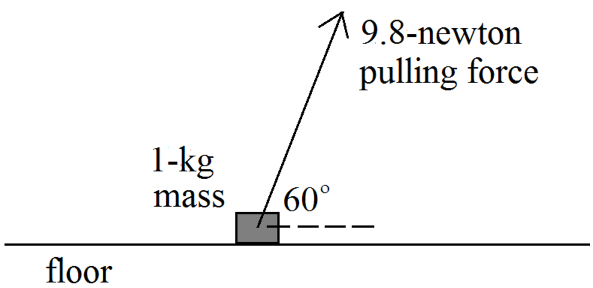
a. $\sin^2\left(-\frac{\pi}{x}\right) - \cos^2\left(\frac{\pi}{x}\right) = \frac{1}{2}$ where $x \in [-1, 1]$ 2 marks

b. $\sin^{-1}\left(\frac{x}{\pi}\right) - \cos^{-1}\left(-\frac{x}{\pi}\right) = \frac{\pi}{2}$ where $x \in [-\pi, \pi]$ 3 marks

Question 8 Show that $y(x^2-1)=2x(y^2-1)$ has the same gradient at $(-1,-1)$, $(-1,1)$, $(1,1)$ and $(1,-1)$.

3 marks

Question 9 A 1-kg mass is pulled by a 9.8-newton force at $60°$ angle to the horizontal floor. The mass remains at rest on the floor.



The reaction force of the floor on the mass makes an acute angle of $\theta°$ to the horizontal. Determine the exact value of $\tan\theta°$.

3 marks

Question 10 A factory produces nuts and bolts.
Random variables X and Y are nut weight and bolt weight respectively.
Bolts are produced to fit nuts such that $Y = 3(X + 10)$, $\mu_X = 55$ and $\sigma_X = 2$.
Weights are measured in grams.

a. Calculate σ_Y. 1 mark

b. The factory packages its products in bags containing 2 nuts and 2 bolts in each bag.
Let random variable W be the total weight of a bag of 2 nuts and 2 bolts.
Ignore the weight of packaging bag.
Show that $\mu_W = 500$ and $\sigma_W = 16$. 2 marks

100 bags (of 2 nuts and 2 bolts) are sampled randomly. 50 random samples in total are taken.
The distribution of $\overline{W}$ across the 50 samples is approximately normal.

c. Show that 1.6 is the standard deviation of $\overline{W}$ across the samples. 1 mark

d. Estimate $\Pr(484 < \overline{W} < 516)$. 1 mark

End of Exam 1