Neap:

Trial Examination 2019

VCE Specialist Mathematics Units 3&4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of booklet

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 9 pages

Formula sheet

Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2019 VCE Specialist Mathematics Units 3&4 Written Examination 1.

Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

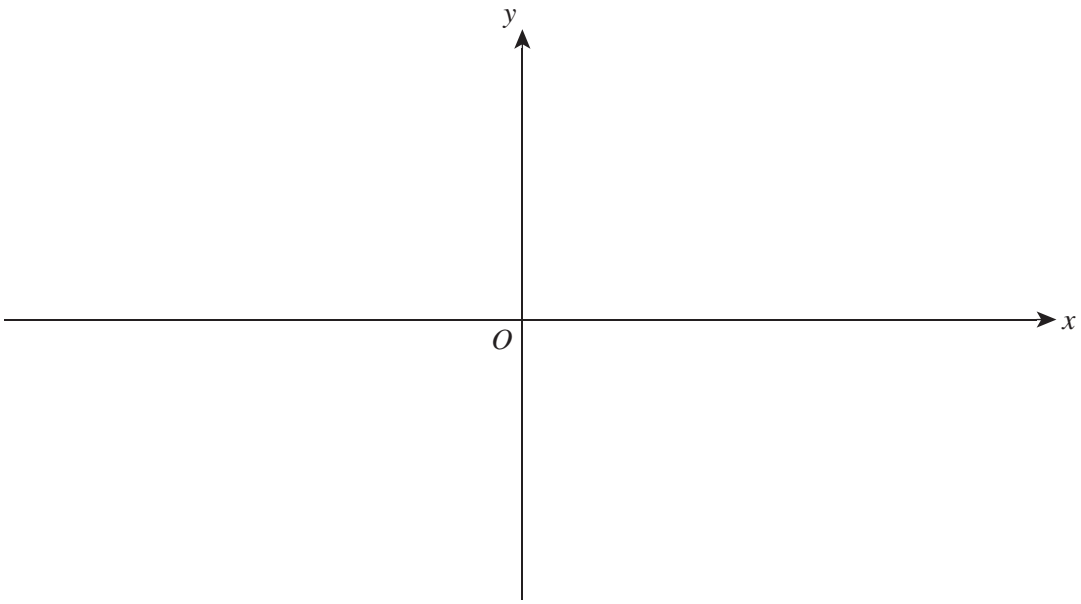
Question 1 (3 marks)

A particle of mass m kg moves in a straight line on a smooth horizontal surface. Initially at rest, the particle moves under a variable force of magnitude $(2pt + q)$ newtons at time t seconds, where p and q are real constants.

Find an expression for the velocity of the particle at time t.

Question 2 (4 marks)

Sketch the graph of $y = \dfrac{x^2 - 2}{x - 1}$ on the axes below. Label any asymptotes with their equations and label any intercepts with the axes and stationary points, expressing them as coordinates.



Question 3 (3 marks)

The random variable Z is defined as $Z = 4X - 3Y$, where $X \sim \text{N}(30, 3^2)$, $Y \sim \text{N}(20, 2^2)$ and X and Y are independent.

a. Find the mean of Z. 1 mark

b. Find the variance of Z. 2 marks

Question 4 (3 marks)

Relative to an origin O, the points A, B and C are defined respectively by the position vectors $\overrightarrow{OA} = 2\underset{\sim}{\text{i}} + \underset{\sim}{\text{j}} - 3\underset{\sim}{\text{k}}$, $(\overrightarrow{OB} = 5\underset{\sim}{\text{i}} - \underset{\sim}{\text{j}} + m\underset{\sim}{\text{k}})$ and $\overrightarrow{OC} = 2\underset{\sim}{\text{i}} + 6\underset{\sim}{\text{j}} - 3\underset{\sim}{\text{k}}$, where m is a real constant.

Find the possible values of m for which $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{OC}\right|$.

Question 5 (3 marks)

Consider the equation $5\tan(2x) = 4\cot(x)$ for $0 < x < \pi$.

Find the possible values of $\tan(x)$.

Question 6 (4 marks)

Find $\int_0^{\frac{\pi}{6}} \frac{1 + \cos^4(2x)}{\cos^2(2x)} dx$. Give your answer in the form $\frac{a\sqrt{b} + c\pi}{d}$, where a, b, c and d are integers.

Question 7 (5 marks)

a. Show that $\frac{d}{dx}\left(\frac{1}{5}e^{2x}(2\sin(x)-\cos(x))\right)=e^{2x}\sin(x)$. 2 marks

b. Solve the differential equation $\frac{dy}{dx}=\sqrt{1-y^2}e^{2x}\sin(x)$ with the condition $y(0)=0$.
Express your answer in the form $y=f(x)$. 3 marks

Question 8 (5 marks)

Solve the equation $z^2 = 7 - 6\sqrt{2}i$, where $z \in C$. Express your answers in the form $a + bi$, where $a, b \in R$.

Question 9 (5 marks)

The normal to the curve $xe^{-y} + e^{y} - x = 0$ at the point $(k, \log_e(k))$ crosses the y-axis at the point $\left(0, \frac{2k^2 + 1}{2}\right)$.

Find the value of k.

Question 10 (5 marks)

Consider the function $f(x) = \arctan\left(\frac{\sqrt{1-x^2}}{x}\right)$.

a. State the maximal domain of f. 1 mark

b. Show that $f'(x) = -\frac{1}{\sqrt{1-x^2}}$. 4 marks

END OF QUESTION AND ANSWER BOOKLET