Neap:

Trial Examination 2019

VCE Specialist Mathematics Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 21 pages
Formula sheet
Answer sheet for multiple-choice questions

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on your answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1

The graph of $y = a + b\tan^{-1}(cx + d)$, where $a, b, c, d \in R$, has asymptotes with equations

A. $y = b \pm \frac{a\pi}{2}$

B. $y = a \pm \frac{b\pi}{2}$

C. $y = c \pm \frac{d\pi}{2}$

D. $y = d \pm \frac{c\pi}{2}$

E. $y = \pm\frac{\pi}{2}$

Question 2

The graph of $y = \sec(m(x - n))$, where m and n are positive real constants, has stationary points at

A. $x = \frac{\pi}{2m} + n, \frac{3\pi}{2m} + n, \frac{5\pi}{2m} + n, \ldots$

B. $x = 0, \frac{\pi}{m}, \frac{2\pi}{m}, \ldots$

C. $x = m, \frac{\pi}{n} + m, \frac{2\pi}{n} + m, \ldots$

D. $x = \frac{\pi}{2n} + m, \frac{3\pi}{2n} + m, \frac{5\pi}{2n} + m, \ldots$

E. $x = n, \frac{\pi}{m} + n, \frac{2\pi}{m} + n, \ldots$

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Question 3

If $\cos(x) = -\frac{3}{4}$ and $\pi < x < \frac{3\pi}{2}$, then cosec(x) is equal to

A. $-\frac{\sqrt{23}}{4}$

B. $-\frac{4}{\sqrt{7}}$

C. $-\frac{4}{3}$

D. $-\frac{4}{\sqrt{23}}$

E. $-\frac{4}{5}$

Question 4

The algebraic fraction $\frac{4x^2 - 5x - 6}{(4x+3)^3(x^2-4)}$ could be expressed in partial fraction form as

A. $\frac{A}{4x+3} + \frac{Bx+C}{(4x+3)^2} + \frac{D}{x+2}$

B. $\frac{A}{4x+3} + \frac{B}{x-2} + \frac{C}{x+2}$

C. $\frac{A}{4x+3} + \frac{B}{(4x+3)^2} + \frac{C}{x+2}$

D. $\frac{A}{4x+3} + \frac{B}{(4x+3)^2} + \frac{C}{(4x+3)^3} + \frac{Dx}{x^2-4}$

E. $\frac{A}{4x+3} + \frac{Bx+C}{x^2-4}$

Question 5

The points A and B represent the complex numbers $1 - 7i$ and $3 - i$ respectively. The circle C has AB as a diameter.

The equation of C is

A. $|z - 2 + 4i| = 10$

B. $|z - 2 + 4i| = \sqrt{10}$

C. $|z + 2 - 4i| = \sqrt{10}$

D. $|z - 2 - 4i| = \sqrt{10}$

E. $|z - 2 + 4i| = 2\sqrt{10}$

Question 6

The complex number $1 + z$, where $z = \text{cis}(\theta)$, is equal to

A. $2\cos\left(\frac{\theta}{2}\right)\left(\text{cis}\left(\frac{\theta}{2}\right)\right)$

B. $2\cos(\theta)(\text{cis}(\theta))$

C. $2\cos\left(\frac{\theta}{2}\right)(\text{cis}(\theta))$

D. $2\sin\left(\frac{\theta}{2}\right)\left(\text{cis}\left(\frac{\theta}{2}\right)\right)$

E. $(1 + \cos(\theta))(\text{cis}(\theta))$

Question 7

With a suitable substitution, $\int x^2\sqrt{2x^3 + 1}\,dx$ can be expressed as

A. $\frac{1}{6}\int \sqrt{u}\,du$

B. $6\int \sqrt{u}\,du$

C. $\frac{1}{6}\int u\,du$

D. $\int \sqrt{u}\,du$

E. $6\int u^{\frac{3}{2}}\,du$

Question 8

To solve the differential equation $\frac{dy}{dx} = e^{x^2}$ with the initial condition $y = 1$ when $x = 0$, Euler's method is used with a step size of 0.1.

When $x = 0.2$, the approximation for y is given by

A. $1 + 0.1e^{0.01}$

B. $1 + 0.2e^{0.02}$

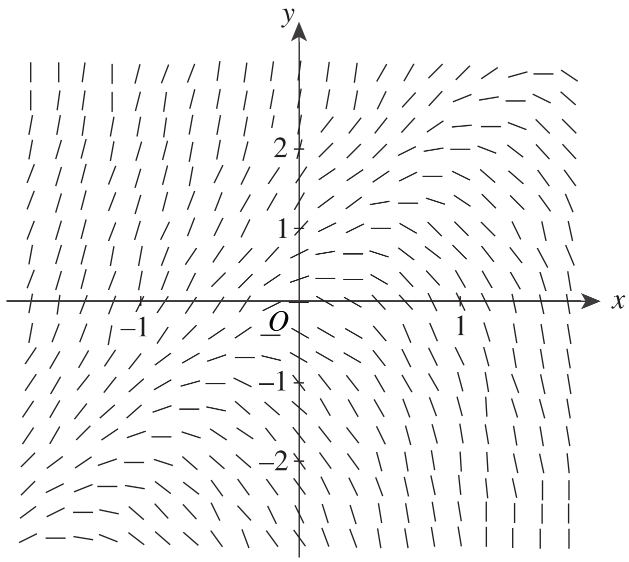
C. $1.1 + 0.1e^{0.01}$

D. $1 + 0.2e^{0.01}$

E. $1.1 + 0.2e^{0.01}$

Question 9

Consider the direction field below.



The differential equation that is best represented by the direction field above is

A. $\frac{dy}{dx} = x - y$

B. $\frac{dy}{dx} = y - x$

C. $\frac{dy}{dx} = 2x + 1$

D. $\frac{dy}{dx} = 2(x - 2y)$

E. $\frac{dy}{dx} = 2(y - 2x)$

Question 10

A drone is moving vertically upwards with a speed of 10 m s^{-1}. The drone is h metres directly above the point P, which is situated on level ground. The drone is observed from the point O, also at ground level, such that $\overline{OP} = 40$ metres.

When $h = 30$, the rate of change of angle HOP in radians per second is

A. $\frac{2}{125}$

B. $\frac{2}{45}$

C. $\frac{1}{5}$

D. $\frac{4}{25}$

E. $\frac{25}{64}$

Question 11

Consider the points $P(3, -2, 4)$, $Q(5, 4, 0)$ and $R(11, 6, -4)$.

The magnitude of angle PQR is

A. $\arccos\left(\frac{5}{7}\right)$

B. $\arccos\left(-\frac{2}{7}\right)$

C. $\arccos\left(-\frac{3}{7}\right)$

D. $\arccos\left(\frac{1}{7}\right)$

E. $\arccos\left(-\frac{5}{7}\right)$

Question 12

Consider non-zero vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$.

If $\underset{\sim}{a} = 4\underset{\sim}{b} - \underset{\sim}{c}$, which one of the following statements must be true?

A. $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are linearly independent.

B. $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are linearly independent.

C. $\underset{\sim}{b}$ is perpendicular to $\underset{\sim}{c}$.

D. $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are linearly dependent.

E. $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are linearly dependent.

Question 13

The position vector of a particle at time t is given by $\underset{\sim}{r}(t) = k(1 + \cos(t))\underset{\sim}{i} + k(t + \sin(t))\underset{\sim}{j}$, $0 \le t \le \frac{\pi}{2}$ and $k > 0$.

The speed of the particle at time t is

A. $2k\cos\left(\frac{t}{2}\right)$

B. $2k\sin\left(\frac{t}{2}\right)$

C. $2k^2\cos\left(\frac{t}{2}\right)$

D. $2k^2\sin\left(\frac{t}{2}\right)$

E. $\sqrt{2}k\cos\left(\frac{t}{2}\right)$

Question 14

A particle moves so that its position vector at time t is given by $\underset{\sim}{r}(t) = 2\text{cosec}(t)\underset{\sim}{i} + 2\cot(t)\underset{\sim}{j}$, $0 < t \le \frac{\pi}{2}$.

The cartesian equation of the path of the particle is

A. $\frac{x^2}{4} - \frac{y^2}{4} = 1$ for $x \ge 2$ and $y \ge 0$

B. $\frac{x^2}{4} + \frac{y^2}{4} = 1$ for $x \ge 2$ and $y \ge 0$

C. $\frac{x^2}{4} - \frac{y^2}{4} = 1$ for $x \ge 0$ and $y \ge 0$

D. $\frac{x^2}{4} + \frac{y^2}{4} = 1$ for $x \ge 0$ and $y \ge 0$

E. $4x^2 - 4y^2 = 1$ for $x \ge 2$ and $y \ge 0$

Question 15

A boy of mass 40 kg is standing in an elevator that is moving with a downwards acceleration of magnitude $\frac{g}{5}$ m s^{-2}.

The magnitude of the force, in newtons, exerted by the floor of the elevator on the boy is

A. g

B. $8g$

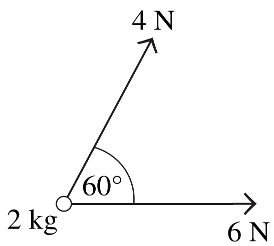
C. $32g$

D. $40g$

E. $48g$

Question 16

A particle of mass 2 kg moves under the action of two forces of magnitude 6 N and 4 N respectively.



Given that these two forces act at 60° to each other, the magnitude of the particle's acceleration, a m s^{-2}, is

A. $\sqrt{7}$

B. $2\sqrt{3}$

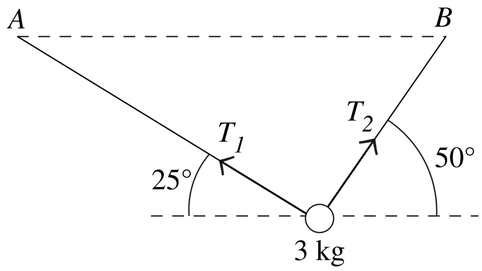
C. $\sqrt{3} + 4$

D. $\sqrt{19}$

E. 10

Question 17

A particle of mass 3 kg hangs in equilibrium supported by two light, inelastic strings attached at points A and B, which are in a horizontal line. The strings are inclined at 25° and 50° respectively to the horizontal.



Using the sine rule, the magnitude of the tensions, T_1 and T_2, in each string are given by

A. $T_1 = \dfrac{3g\sin(75°)}{\sin(40°)}$ and $T_2 = \dfrac{3g\sin(75°)}{\sin(65°)}$

B. $T_1 = \dfrac{3\sin(75°)}{\sin(40°)}$ and $T_2 = \dfrac{3\sin(75°)}{\sin(65°)}$

C. $T_1 = \dfrac{3g}{\sin(40°)\sin(75°)}$ and $T_2 = \dfrac{3g}{\sin(65°)\sin(75°)}$

D. $T_1 = \dfrac{3\sin(40°)}{\sin(75°)}$ and $T_2 = \dfrac{3\sin(65°)}{\sin(75°)}$

E. $T_1 = \dfrac{3g\sin(40°)}{\sin(75°)}$ and $T_2 = \dfrac{3g\sin(65°)}{\sin(75°)}$

Question 18

A p-value of 0.03 is calculated from a hypothesis test.

Which one of the following conclusions is true?

A. The probability that the null hypothesis is false is equal to 0.03.

B. The probability that the null hypothesis is true is equal to 0.03.

C. If the null hypothesis is true, the probability of observing a sample statistic as extreme or more extreme as the sample statistic that is observed is equal to 0.03.

D. If the null hypothesis is false, the probability of observing a sample statistic as extreme or more extreme as the sample statistic that is observed is equal to 0.03.

E. The probability that the null hypothesis is true is equal to 0.97.

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Question 19

A random variable, X, is normally distributed with an unknown mean, μ. A random sample of 100 observations is selected from this population.

If the sample mean is 497 and sample variance is 13, an approximate 95% confidence interval for μ is

A. $(494.1, 499.9)$

B. $(494.5, 499.5)$

C. $(496.1, 497.9)$

D. $(496.3, 497.7)$

E. $(496.7, 497.3)$

Question 20

The independent random variables L and W denote the length and width in cm, respectively, of a rectangular-shaped manufactured part.

Given that $L \sim \text{N}(2, 0.1^2)$ and $W \sim \text{N}(5, 0.2^2)$, the probability that the perimeter of the manufactured part is greater than 14.5 cm, correct to four decimal places, is

A. 0.0062

B. 0.1318

C. 0.3240

D. 0.8682

E. 0.9938

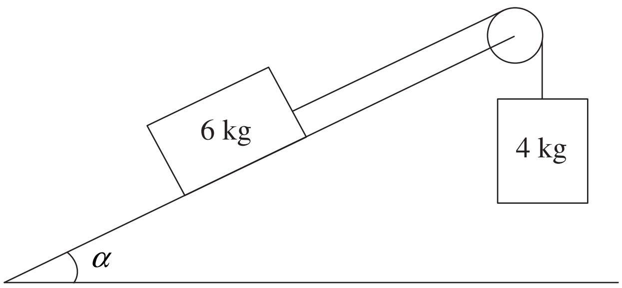
SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1 (10 marks)

Two objects of mass 6 kg and 4 kg respectively are attached by a light, inextensible string of length 2 m that passes over a smooth pulley. The 6 kg object is on a smooth plane inclined at an angle α to the horizontal, where $\sin(\alpha) = \frac{3}{5}$. The 4 kg object is hanging vertically as shown in the diagram below.



a. On the diagram above, show all of the forces acting on both objects. 1 mark

At $t = 0$, the two objects are held at rest with the string taut. The 6 kg object is on the inclined plane and the 4 kg object hangs vertically just over the pulley. The two objects are then released.

b. Show that the magnitude of the acceleration, a m s^{-2}, of the two objects is given by $a = \frac{g}{25}$. 2 marks

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c. Find the magnitude of the tension in the string. 1 mark

At $t = 0$, the 4 kg object is 1.5 m above the ground.

d. Find the speed with which the 4 kg object hits the ground. Give your answer in the form of $\frac{\sqrt{pg}}{q}$ m s^{-1}, where p and q are positive integers. 2 marks

e. The 6 kg object does not reach the top of the inclined plane.

Find the time that elapses between the 4 kg object reaching the ground and the string becoming taut again. Give your answer correct to two decimal places. 4 marks

Question 2 (9 marks)

Consider the function $f(x) = \dfrac{ne^x + m}{me^x + n}$, where $0 < n < m$.

a. Show that the graph of f has no vertical asymptotes. 1 mark

b. **i.** Show that the graph of f has a point of inflection, and state its coordinates. 3 marks

ii. State, in terms of m and n, the set of values of x for which the graph of f is concave down. 1 mark

iii. Show that the point of inflection on the graph of f is always above the x-axis. 1 mark

c. Consider the case where $m = 3$ and $n = 1$. The region R is enclosed by the graph of f, the y-axis and the line $y = \frac{2}{3}$.

Find the volume of the solid formed when R is rotated 360° about the x-axis. Give your answer correct to three decimal places. 3 marks

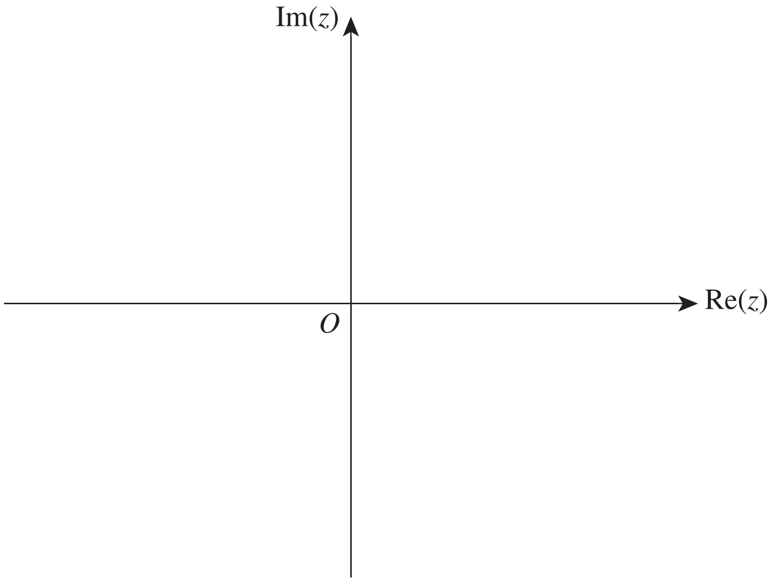
Question 3 (10 marks)

The equation $z^4 - 4z^3 + az^2 - 24z + 30 = 0$, where $z \in C$ and $a \in R$, has a root ki, where $k \in R$, $k \neq 0$.

a. Explain why there is more than one possible value for k. 1 mark

b. Find the possible exact values of k and hence show that $a = 11$. 3 marks

c. Plot the roots of the equation $z^4 - 4z^3 + 11z^2 - 24z + 30 = 0$ on the Argand diagram below. 2 marks



The points in the Argand diagram representing the roots of the equation $z^4 - 4z^3 + 11z^2 - 24z + 30 = 0$ form the vertices of a quadrilateral.

d. Find the area of this quadrilateral. Give your answer in the form $a(\sqrt{b} + c)$ where a, b and c are positive integers. 2 marks

e. By using a suitable transformation from z to w, find the roots of the equation $1 - 4w + 11w^2 - 24w^3 + 30w^4 = 0,\ w \in C$. 2 marks

Question 4 (12 marks)

At time $t \geq 0$, a particle moving along a curve in the *xy*-plane has velocity vector $\underset{\sim}{v}(t) = \cos(t^4)\underset{\sim}{i} + e^{0.75t}\underset{\sim}{j}$, where $\underset{\sim}{i}$ is a unit vector to the east and $\underset{\sim}{j}$ is a unit vector to the north. Time is measured in seconds and distance is measured in metres.

At $t = 1$, the particle is at the point (4, 6).

a. Show that the vertical movement of the particle is in a northwards direction at time $t = 2$. 2 marks

b. Find the x-coordinate of the particle at time $t = 2$. Give your answer correct to two decimal places. 2 marks

c. There is a point P on the curve at which the tangent to the curve has a gradient of 3.

Find the time when the particle is at the point P. Give your answer correct to two decimal places. 3 marks

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d. Find the time when the particle is moving at a speed of 5 m s^{-1}. Give your answer correct to two decimal places. 3 marks

e. Find the total distance travelled by the particle in the first second of motion. Give your answer correct to two decimal places. 2 marks

Question 5 (11 marks)

A small stone of mass m kg is dropped from a height of 25 m above a lake. The forces acting on the stone are its weight, W N, and the resistance to motion, R N, given by $R = \frac{mv^2}{1000}$, where v m s^{-1} is the velocity of the stone. At time t seconds, the stone has fallen a distance of x metres.

a. On the diagram below, mark in the forces acting on the stone. 1 mark

b. Show that $v\frac{dv}{dx} = g - \frac{v^2}{1000}$. 1 mark

c. Use integration to solve the differential equation in **part b.** and hence show that the stone hits the surface of the lake at a speed of 21.86 m s^{-1}, correct to two decimal places. 4 marks

Assume that when the stone hits the lake's surface, there is no instantaneous change in the stone's velocity. As the stone descends through the water, the only forces acting are its weight and the resistance to motion, which is now modelled by $R = 5mv$.

d. Find the distance below the surface of the lake at which the speed of the stone is reduced to 10 m s^{-1}. Give your answer correct to one decimal place. 4 marks

e. According to this model, determine the terminal velocity of the stone under water. 1 mark

Question 6 (8 marks)

The weights, W g, of packets of coffee granules are known to be normally distributed with a standard deviation of 4 g. The manufacturer claims that the mean weight of coffee granule packets is 125 g.

To decide whether the manufacturer's claim about the mean weight is true, researchers selected a random sample of 100 coffee granule packets. The mean weight of this sample was found to be 124 grams. Let $\overline{W}$ g denote the mean weight of a random sample of 100 coffee granule packets.

A one-tailed test is to be carried out to determine if the sample mean weight of 124 g differs significantly from the manufacturer's claim of 125 g.

a. State suitable hypotheses to test the manufacturer's claim. 1 mark

b. Find the p value for this test. Give your answer correct to four decimal places. 3 marks

c. State whether the null hypothesis should be rejected at the 5% level of significance. Give a reason for your answer. 1 mark

d. Find the minimum sample mean weight that could be observed for the null hypothesis to **not** be rejected. Give your answer correct to two decimal places. 1 mark

e. After further research, the manufacturer determined that the true mean weight of coffee granule packets is in fact 124 g.

Find the probability that the null hypothesis will be accepted at the 5% level of significance. Give your answer correct to two decimal places. 2 marks

END OF QUESTION AND ANSWER BOOKLET