TSSM
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2019 Trial Examination

STUDENT NUMBER | | | | | | | | | Letter |

SPECIALIST MATHEMATICS

Units 3&4 - Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper and/or white out liquid/tape.
- No calculator is permitted in this examination.

Materials supplied

- Question and answer book of 16 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Take the acceleration due to gravity to have magnitude $g\ ms^{-2}$, where $g = 9.8$.

Question 1 (3 marks)

Find the gradient of $3x^2y + 2x - y^3 = 13$ at the point $(2, 3)$.

3 marks

Question 2 (2 marks)

Find the vector resolute of $\underset{\sim}{a} = -3\underset{\sim}{\mathrm{i}} + 2\underset{\sim}{\mathrm{j}} - \underset{\sim}{\mathrm{k}}$ in the direction of $\underset{\sim}{b} = \underset{\sim}{\mathrm{i}} - 2\underset{\sim}{\mathrm{j}} + 2\underset{\sim}{\mathrm{k}}$

2 marks

Question 3 (4 marks)

Forces $\underset{\sim}{M} = 4\underset{\sim}{\mathrm{i}} - 2\underset{\sim}{\mathrm{j}} - \underset{\sim}{\mathrm{k}}$ and $\underset{\sim}{N} = \underset{\sim}{\mathrm{i}} + 2\underset{\sim}{\mathrm{j}} + 13\underset{\sim}{\mathrm{k}}$ act upon a stationary 3.25 kg mass.

a. Given that no other forces act on the mass, find the magnitude of its acceleration.

1 mark

Question 3 - continued

TURN OVER

b. When will the speed of the mass first reach $12\ ms^{-1}$?

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1 mark

c. If the initial position of the mass is $\underline{O} = \frac{4}{13}(3\underline{i} + 2\underline{k})$, find its position after 2 seconds.

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2 marks

1 + 1 + 2 = 4 marks

The following may be useful for questions 4 and 5.

Let Z be an observation from a standard normal distribution.

$\Pr(Z \leq 1.00) \approx 0.84$
$\Pr(Z \leq 1.28) \approx 0.90$
$\Pr(Z \leq 1.64) \approx 0.95$
$\Pr(Z \leq 1.96) \approx 0.975$
$\Pr(Z \leq 2.33) \approx 0.99$
$\Pr(Z \leq 2.58) \approx 0.995$

Question 4 (3 marks)

Two independent, normally distributed random variables X and Y are such that:

$E(X) = 10, Var(X) = 4$
$E(Y) = 25, Var(Y) = 9$

a. Given $A = 2X - Y$, evaluate $E(A)$ and $Var(A)$.

2 marks

Question 4 - continued

TURN OVER

b. Estimate $\Pr(A \geq 1.4)$

1 mark

Question 5 (4 marks)

The weights of 20 kg cement bags is normally distributed with a mean of 20.0 kg and a standard deviation of 0.4 kg.

a. What percentage of cement bags are expected to weigh less than 19.6 kg?

1 mark

Question 5 - continued

b. Several complaints have been registered by builders who claim that many of the bags they purchase are underweight (less than 20.0 kg).

In order to test this claim, 25 bags are checked by an appropriate authority and are found to have a mean weight of $\bar{x} = 19.8\ kg$.

i. State the null hypothesis H_0 and the alternative hypothesis H_1

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1 mark

ii. Justify whether or not the null hypothesis should be accepted or rejected at the 0.05 level of significance.

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2 marks

1 + 1 + 2 = 4 marks

TURN OVER

Question 6 (3 marks)

Given $\operatorname{cosec} 2x = 1.25$, where $2x$ is in the first quadrant, find $tan\left(x + \frac{\pi}{4}\right)$.

3 marks

Question 7 (5 marks)

Consider the function $f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow R$ where $f(x) = 2\,tan\,x$

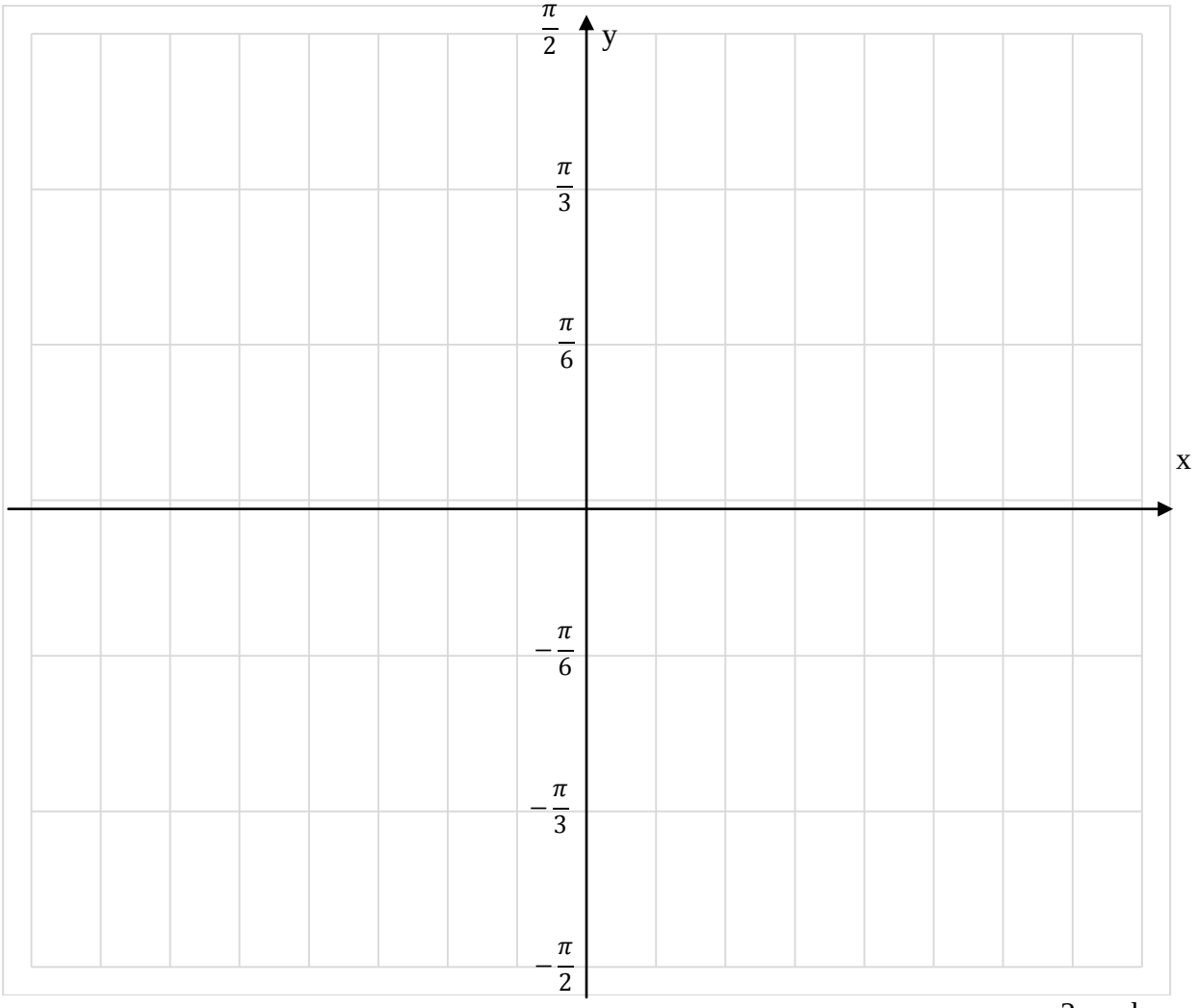
a. Write down, in function notation the inverse of f.

_____

_____

1 mark

b. Sketch the inverse of f on the axes provided and shade the region bound by f^{-1}, the x-axis and $x = \sqrt{12}$.



2 marks

Question 7 - continued

TURN OVER

c. Find the exact value of the region shaded in part **b.**

2 marks

1 + 2 + 2 = 5 marks

Question 8 (5 marks)

Consider a fourth degree polynomial P(z) with only real coefficients where $P(2i) = 0$.

a. Write down two linear factors of $P(z)$.

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_____

1 mark

b. Hence find real numbers a and b given $P(z) = z^4 + 10z^3 + 31z^2 + az + b$

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2 mark

Question 8 - continued

TURN OVER

c. Hence find all solutions to $P(z) = z^4 + 10z^3 + 31z^2 + az + b = 0$

2 marks

1 + 2 + 2 = 5 marks

Question 9 (5 marks)

A cylindrical vessel is leaking water consistent with the differential equation $\frac{dV}{dt} = -4\sqrt{h}$

$V =$ volume in cm^3, $h =$ height of water in cm, $t =$ time in minutes.

The base of the vessel has an area of 480 cm² and the water in the vessel initially has a height of 100 cm.

a. Write down the rule for the volume of water in the vessel in terms of *h*.

1 mark

b. Find $\frac{dh}{dt}$ in terms of *h*, hence find the rule for h in terms of *t*.

3 marks

Question 9 - continued

TURN OVER

c. In how many **hours** after $t = 0$ will the vessel be empty?

_____

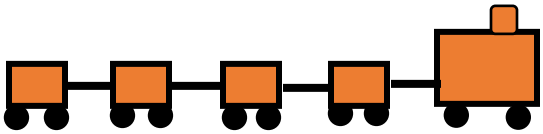
_____

1 mark

1 + 3 + 1 = 5 marks

Question 10 (6 marks)

A toy train with an engine of mass 10 kg has 4 identical carriages each of mass 5 kg. Each carriage is connected to the one directly in front by a light cable. (See diagram.)



Assume the contact between the train and the track is smooth.

a. Find the force needed to be generated by the engine so that the train moves with constant acceleration from rest to a speed of $4\ ms^{-1}$ over a distance of $16\ m$.

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2 marks

Question 10 - continued

b. Find the tension in each cable if the train accelerates at the rate specified in **part a**.

2 marks

Question 10 - continued

TURN OVER

c. When the train reaches a speed of $4\ ms^{-1}$ the force generated by the engine is removed and a braking force of $12v$ newtons is applied until the train comes to rest. (v is the speed of the train in ms^{-1} and x is the number of metres that the train has been braking).
Over what distance does the train brake?

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2 marks

2 + 2 + 2 = 6 marks

END OF QUESTION AND ANSWER BOOK