Victorian Certificate of Education
2019

Letter

STUDENT NUMBER

SPECIALIST MATHEMATICS
Written examination 1

Tuesday 4 June 2019

Reading time: 2.00 pm to 2.15 pm (15 minutes)
Writing time: 2.15 pm to 3.15 pm (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	Number of questions to be answered	Number of marks
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 8 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1 (4 marks)

A 10 kg mass is placed on a rough plane that is inclined at 30° to the horizontal, as shown in the diagram below. A force of 40 N is applied to the mass up the slope and parallel to the slope. There is also a frictional resistance force of magnitude F that opposes the motion of the mass.
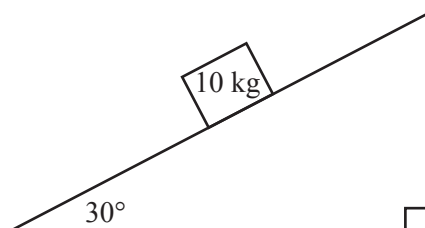
a. Find the magnitude of the frictional resistance force, in newtons, acting up the slope if the force is just sufficient to stop the mass from sliding down the slope.   2 marks

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b. An **additional** force of magnitude P newtons is applied to the mass up the slope and parallel to the slope. The sum of the additional force and the frictional resistance force of magnitude F that now acts down the slope is such that it is just sufficient to stop the mass from sliding up the slope.

Find P.   2 marks

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Question 2 (4 marks)

A cubic polynomial has the form $p(z) = z^3 + bz^2 + cz + d$, $z \in C$, where $b, c, d \in R$.

Given that a solution of $p(z) = 0$ is $z_1 = 3 - 2i$ and that $p(-2) = 0$, find the values of b, c and d.

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Question 3 (4 marks)

The number of cars per day making a U-turn at a particular location is known to be normally distributed with a standard deviation of 17.5. In a sample of 25 randomly selected days, a total of 1450 cars were observed making the U-turn.

a. Based on this sample, calculate an approximate 95% confidence interval for the number of cars making the U-turn each day. Use an integer multiple of the standard deviation in your calculations. 3 marks

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b. The average number of U-turns made at the location is actually 60 per day.

Find an approximation, correct to two decimal places, for the probability that on 25 randomly selected days the average number of U-turns is less than 53. 1 mark

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Question 4 (3 marks)

Evaluate $\displaystyle\int_{e^3}^{e^4} \frac{1}{x\log_e(x)}\,dx.$

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Question 5 (5 marks)

A triangle has vertices $A(\sqrt{3}+1, -2, 4)$, $B(1, -2, 3)$ and $C(2, -2, \sqrt{3}+3)$.

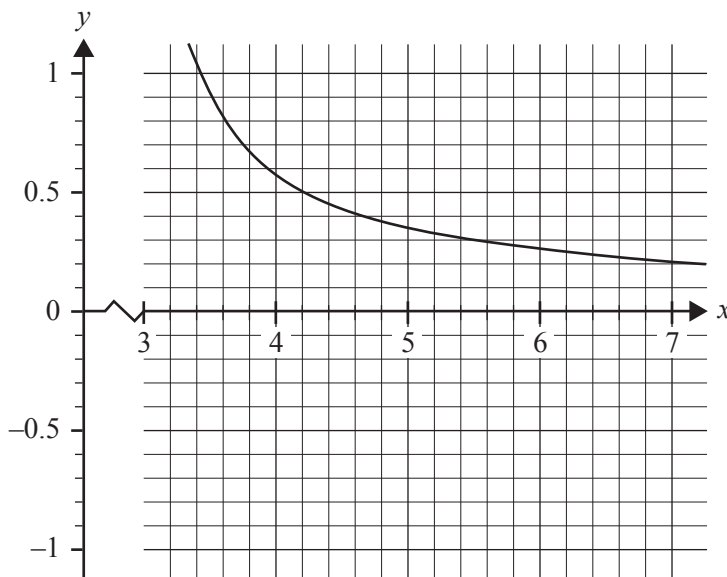**a.** Find angle ABC.   3 marks

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b. Find the area of the triangle.   2 marks

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Question 6 (5 marks)

Part of the graph of $y = \dfrac{2}{\sqrt{x^2 - 4x + 3}}$, where $x > 3$, is shown below.

Find the volume of the solid of revolution formed when the graph of $y = \dfrac{2}{\sqrt{x^2 - 4x + 3}}$ from $x = 4$ to $x = 6$ is rotated about the x-axis. Give your answer in the form $a\log_e(b)$, where a and b are real numbers.

Question 7 (5 marks)

Given that $3x^2 + 2xy + y^2 = 6$, find $\dfrac{d^2y}{dx^2}$ at the point (1, 1).
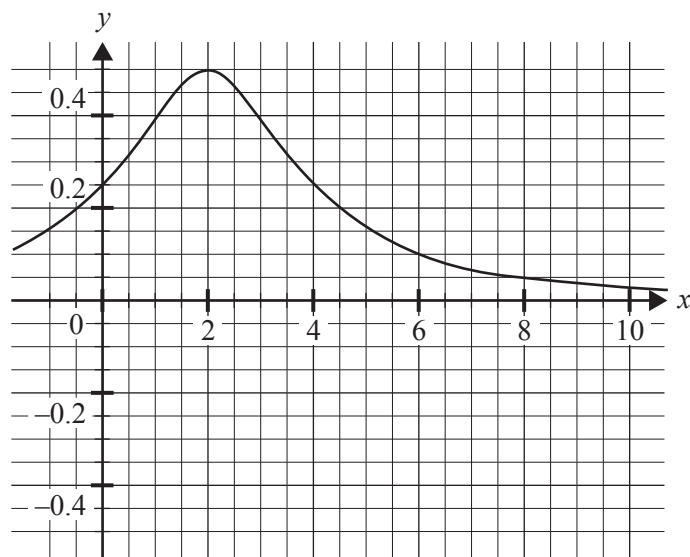
Question 8 (4 marks)

Find the length of the arc of the curve defined by $y = \dfrac{x^4}{4} + \dfrac{1}{8x^2} + 3$ from $x = 1$ to $x = 2$. Give your answer in the form $\dfrac{a}{b}$, where a and b are positive integers.

Question 9 (6 marks)

a.   Show that $\tan\left(\dfrac{5\pi}{12}\right) = \sqrt{3} + 2.$ 2 marks

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b.

Hence, find the area bounded by the graph of $f(x) = \dfrac{2}{x^2 - 4x + 8}$ shown above, the x-axis and

the lines $x = 0$ and $x = 2\sqrt{3} + 6.$ 4 marks

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END OF QUESTION AND ANSWER BOOK

Victorian Certificate of Education
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SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.

A question and answer book is provided with this formula sheet.

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Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\dfrac{a}{\sin(A)} = \dfrac{b}{\sin(B)} = \dfrac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \dfrac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y) = \dfrac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \dfrac{2\tan(x)}{1-\tan^2(x)}$

Circular functions – continued

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r\left(\cos(\theta) + i\sin(\theta)\right) = r\operatorname{cis}(\theta)$	
$|z| = \sqrt{x^2 + y^2} = r$	$-\pi < \operatorname{Arg}(z) \le \pi$
$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$	$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$
$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$ $\mathrm{E}(aX + bY) = a\mathrm{E}(X) + b\mathrm{E}(Y)$ $\operatorname{var}(aX + b) = a^2\operatorname{var}(X)$
for independent random variables X and Y	$\operatorname{var}(aX + bY) = a^2\operatorname{var}(X) + b^2\operatorname{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\dfrac{s}{\sqrt{n}},\ \bar{x} + z\dfrac{s}{\sqrt{n}}\right)$
distribution of sample mean $\overline{X}$	mean $\quad \mathrm{E}\left(\overline{X}\right) = \mu$ variance $\quad \operatorname{var}\left(\overline{X}\right) = \dfrac{\sigma^2}{n}$

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Calculus

$\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\displaystyle\int x^n\,dx = \dfrac{1}{n+1}x^{n+1} + c,\, n \neq -1$
$\dfrac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\displaystyle\int e^{ax}\,dx = \dfrac{1}{a}e^{ax} + c$
$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$	$\displaystyle\int \dfrac{1}{x}\,dx = \log_e|x| + c$
$\dfrac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\displaystyle\int \sin(ax)\,dx = -\dfrac{1}{a}\cos(ax) + c$
$\dfrac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\displaystyle\int \cos(ax)\,dx = \dfrac{1}{a}\sin(ax) + c$
$\dfrac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$	$\displaystyle\int \sec^2(ax)\,dx = \dfrac{1}{a}\tan(ax) + c$
$\dfrac{d}{dx}\left(\sin^{-1}(x)\right) = \dfrac{1}{\sqrt{1-x^2}}$	$\displaystyle\int \dfrac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c,\, a > 0$
$\dfrac{d}{dx}\left(\cos^{-1}(x)\right) = \dfrac{-1}{\sqrt{1-x^2}}$	$\displaystyle\int \dfrac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\dfrac{x}{a}\right) + c,\, a > 0$
$\dfrac{d}{dx}\left(\tan^{-1}(x)\right) = \dfrac{1}{1+x^2}$	$\displaystyle\int \dfrac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\dfrac{x}{a}\right) + c$
	$\displaystyle\int (ax+b)^n\,dx = \dfrac{1}{a(n+1)}(ax+b)^{n+1} + c,\, n \neq -1$
	$\displaystyle\int (ax+b)^{-1}\,dx = \dfrac{1}{a}\log_e|ax+b| + c$
product rule	$\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
quotient rule	$\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
chain rule	$\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$
Euler's method	If $\dfrac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$
arc length	$\displaystyle\int_{x_1}^{x_2}\sqrt{1 + \left(f'(x)\right)^2}\,dx$ or $\displaystyle\int_{t_1}^{t_2}\sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\,dt$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$
$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underset{\sim}{r}} = \dfrac{d\underset{\sim}{r}}{dt} = \dfrac{dx}{dt}\underset{\sim}{i} + \dfrac{dy}{dt}\underset{\sim}{j} + \dfrac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

Mechanics

momentum	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R} = m\underset{\sim}{a}$

END OF FORMULA SHEET