

 Student Name:………………………………………

### SPECIALIST MATHEMATICS UNITS 3 & 4

### TRIAL EXAMINATION 2

**2020**

**(Adjusted Study Design)**

#### Reading Time: 15 minutes

Writing time: 2 hours

######  Instructions to students

This exam consists of Section A and Section B.

Section A consists of 20 multiple-choice questions and should be answered on the detachable answer sheet which can be found on page 26 of this exam.

Section B consists of 5 extended-answer questions.

Section A begins on page 2 of this exam and is worth 20 marks.

Section B begins on page 10 of this exam and is worth 60 marks.

There is a total of 80 marks available.

All questions in Section A and B should be answered.

In Section B, where more than one mark is allocated to a question, appropriate working must be shown. An exact value is required to a question unless otherwise directed.

Unless otherwise stated, diagrams in this exam are not drawn to scale.

The acceleration due to gravity should be taken to have magnitude  where 

Students may bring one bound reference into the exam.

Students may bring into the exam one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory does not need to be cleared. For approved computer-based CAS, full functionality may be used.

A formula sheet can be found on pages 23-25 of this exam.

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This Trial Exam is licensed on a non transferable basis to the purchasing school. It may be copied by the school which has purchased it. This license does not permit distribution or copying of this Trial Exam by any other party.**SECTION A – Multiple-choice questions**

**Question 1**

The implied domain of  is

1. 
2. 
3. 
4. 
5. 

**Question 2**

Let 

The range of *f* is

1. 
2. 
3. 
4. 
5. 

**Question 3**

The graph of the function  has the same asymptotes as the graph of

1. ****
2. ****
3. ****
4. ****
5. ****

**Question 4**

The algebraic fraction  can be expressed in partial fraction form, where *A, B, C, D* and *E*  are non-zero real numbers, as

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 5**

The number of distinct roots of the equation  is

**A.** 3

**B.** 4

**C.** 5

**D.** 6

**E.** 7

**Question 6**

The solutions to the equation  lie on the graph of which one of the following relations?

1. 
2. 
3. 
4. 
5. 

**Question 7**

Let  are acute angles.

The complex numbers , where *k* is a scalar, are shown on the Argand diagram below.



Which one of the following statements **canno**t be true?

1. 
2. 
3. 
4. 
5. 

**Question 8**

Let .

The values of *n* for which  is real are given by

1. 
2. 
3. 
4. 
5. 

**Question 9**

Using a suitable substitution,  can be written as

1. 
2. 
3. 
4. 
5. 

**Question 10**

If  equals

1. 
2. 
3. 
4. 
5. 

**Question 11**



The differential equation that best represents the direction field shown in the diagram above is

1. 
2. 
3. 
4. 
5. 

**Question 12**

The length, in units, of the curve with rule  is closest to

1. 3.26
2. 3.38
3. 3.86
4. 4.79
5. 5.02

**Question 13**

The differential equation  has a solution that can be obtained from

1. 
2. 
3. 
4. 
5. 

**Question 14**

The scalar resolute of , where *m* is a positive constant.

The value of *m* is

1. 0
2. 2
3. 3
4. 5
5. 8

**Question 15**

The position vectors of particles *A* and *B* are given by



where  and *k* is a real constant.

The particles will collide if the value of *k* is

1. 
2. 
3. 
4. 
5. 

**Question 16**

The vectors , where *n* is a real constant, are linearly dependent.

The value of *n* is

1. –1
2. 0
3. 1
4. 2
5. 3

**Question 17**

A particle starts from rest at the origin *O* and moves in a straight line so that its acceleration,

*a* ms-2, is given by , where *v* is the velocity, in ms-1, of the particle at time *t* seconds.

The displacement of the particle from *O*, when its velocity is 1 ms-1, is

1. 1
2. 
3. 
4. 
5. 

**Question 18**

The diagram below shows a particle being acted on by three coplanar forces with magnitudes *P*, *Q* and *R* The forces are all measured in newtons and the system is in equilibrium.



Which one of the following statements is **not** true?

1. 
2. 
3. 
4. 
5. 

**Question 19**

A body of mass 15 kg, which is initially at rest, is dragged along a rough horizontal surface by a pulling force of 30 newtons, acting at an angle of 25° upwards from the horizontal. This motion is opposed by a friction force of 8 newtons.

The magnitude of the momentum, in kg ms-1, of the body 10 seconds after the pulling force began to act on it, is closest to

1. 13
2. 30
3. 127
4. 192
5. 288

**Question 20**

Two particles of mass  kg and  kg rest on smooth planes inclined at an angle of  respectively to the horizontal The particles are connected by a light, inextensible string that passes over a smooth pulley as shown in the diagram below.



The system is released and the particle with mass  kg accelerates at  down the plane.

The equation of motion of this particle down the plane is given by

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**SECTION B**

**Question 1**  (12 marks)

In the complex plane, a circle has the equation .

1. State the centre, in the form , and state the radius of this circle. 1 mark

A second circle *L*, has its centre at and passes through the point .

1. Sketch *L* on the Argand diagram below. Intercepts with the coordinate axes do **not** need to be calculated or labelled.1 mark



1. The circle *L* can be expressed in the form .
2. Find  in polar form. 1 mark
3. Find *a*. 1 mark

**d.** **i.** Find the cartesian equation of the line . 1 mark

 **ii.** Find the cartesian coordinates of the points of intersection of *L* and the line

 . 2 marks

**e.** Find the area enclosed by *L* and the lines . 2 marks

**f.** A ray with equation , intersects with *L.*

Find the possible values of *n*. 3 marks

**Question 2** (12 marks)

1. State the equations of the asymptotes of the graph of**. 2 marks
2. For the graph of the function given in part **a.**, find the coordinates of
3. the stationary point, correct to two decimal places. 1 mark
4. the point of inflection. 1 mark
5. Sketch the graph of  on the axes below, labelling the stationary point and any axis intercepts with their coordinates correct to two decimal places. Label the point of inflection with its coordinates. Show any asymptotes and label them with their equations. 3 marks



Let .

1. Find the coordinates of the point of intersection of the two asymptotes of the graph

of *f.* 1 mark

1. For ,

**i.** find the value of *k* for which exactly two solutions exist. Give your answer

 correct to two decimal places. 2 marks

**ii.** find the values of *k* for which three solutions exist. Give the endpoints of any

 intervals correct to two decimal places. 1 mark

1. Find the value of *k* for which the point of inflection lies on the *x*-axis. 1 mark

**Question 3** (11 marks)

The graph of , is rotated about the *y*-axis to form a solid of revolution which is used to model a bird bath.

The graph, together with a dotted outline of the bird bath is shown below.



The unit of measurement is the centimetre.

1. **i.** Show that the volume of this solid of revolution is given by . 1 mark
	1. Find the volume of the solid of revolution in cubic centimetres. 1 mark

The bird bath is initially filled with water but at some point, a leak develops in its base.

As a result, cubic centimetres per second, where *V*, in cm3, is the volume of water and *h*, in cm, is the height of the water remaining in the bird bath *t* seconds after the leak develops and *k* is a positive constant.

When the bird bath is leaking water at the rate of 6 cm3 per second, the height of the water remaining in the bird bath is 9 cm.

1. Find the value of *k*. 1 mark
2. **i.** Show that . 3 marks

**ii**. Find therate, in cm per second, correct to four decimal places, at which the

 height of the water is decreasing when the height is 5 cm. 1 mark

1. Find the time taken, in seconds, correct to one decimal place, for the bird bath to empty. 2 marks
2. The height of the water in the bird bath drops to 9 cm, 196 seconds after the leak develops.

Using Euler’s method with a step size of 10 seconds, find an estimate for the height of the water remaining in the bird bath 206 seconds after the leak develops.

Give your answer in centimetres, correct to two decimal places. 2 marks

**Question 4** (13 marks)

Surf lifesavers, on duty in a tower, spot a boat sailing just out from the shoreline.

The position vector of the boat, relative to the tower at origin *O*, is given by

 

where *t* is the time, in hours, after the boat is first spotted.

The boat is visible to the lifesavers for .

The displacement components are measured in kilometres.

1. Show that the Cartesian equation of the path of the boat has the rule . 2 marks
2. Sketch the path of the boat, when it is visible to the lifesavers, on the axes below.

Label endpoints and show the direction of motion of the boat with an arrow. 3 marks



1. Find the speed of the boat, in km per hour, when . 2 marks
2. Find the distance travelled by the boat for .

Give your answer, in kilometres, correct to two decimal places. 2 marks

The Cartesian equation of the shoreline, relative to the tower at origin *O*, is given by .

A woman walks in a straight line from the tower along the shoreline for , where *t* is the time, in hours, after the boat is first spotted.

1. Show that the woman is **never** travelling in the same direction as the boat during this period. 2 marks

**f.** Find the acute angle, in degrees correct to one decimal place, between the path of the

woman and that of the boat at . 2 marks

**Question 5** (12 marks)

A 100 kg container sits on a smooth plane inclined at an angle of  to the horizontal. It is connected by a light inextensible cable that passes over a smooth pulley to a hanging 50 kg block. The tension in the cable is *T* newtons.

The system is in equilibrium.



1. Label all the forces acting on the container and on the block on the diagram above. 1 mark
	1. Show that . 1 mark

Unexpectedly a 10 kg component of the block breaks off and falls to the ground. As a consequence, the container starts to move down the plane.

1. Find the time taken, in seconds, for the container to move 20 metres down the plane.

Give your answer correct to two decimal places. 3 marks

When the velocity of the container moving down the plane reaches 6 ms-1, a split occurs in the container. This causes its contents to escape so that the mass of the container decreases at the rate of 2 kg per second. As a consequence, the motion of the container down the plane eventually slows. Assume that the block still has a mass of 40 kg.

1. Write down an expression, in terms of *t*, for the weight force, in newtons, of the container *t* seconds after it splits. 1 mark
2. Show that the acceleration of the container down the plane is given by . 2 marks
3. Find the maximum velocity that the container reaches as it moves down the plane.

Give your answer in metres per second, correct to two decimal places. 2 marks

1. An attempt to halt the container moving down the plane can be made if its velocity is .

After the first attempt to halt the container fails, how long is it until the second attempt can be made? Give your answer in seconds, correct to two decimal places. 2 marks

**Specialist Mathematics Formulas**

**Mensuration**

|  |  |
| --- | --- |
| area of a trapezium |  |
| curved surface area of a cylinder |  |
| volume of a cylinder |  |
| volume of a cone |  |
| volume of a pyramid |  |
| volume of a sphere |  |
| area of a triangle |  |
| sine rule |  |
| cosine rule |  |

**Circular functions**

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**Circular functions – continued**

|  |  |  |  |
| --- | --- | --- | --- |
| **Function** |  |  |  |
| **Domain** |  |  |  |
| **Range** |  |  |  |

**Algebra (complex numbers)**

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|  |  |
|  |  |
|  (de Moivre’s theorem) |  |

**Calculus**

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|  |  |
|  |  |
|  |  |
| product rule |  |
| quotient rule |  |
| chain rule |  |
| Euler’s method |  |
| acceleration |  |
| arc length |  |

**Vectors in two and three dimensions Mechanics**

|  |  |
| --- | --- |
| momentum |  |
| equation of motion |  |

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