Neap

Trial Examination 2020

VCE Specialist Mathematics Units 1&2

Written Examination 2

Suggested Solutions

SECTION A – MULTIPLE-CHOICE QUESTIONS

1	A	**B**	C	D	E	11	A	B	C	**D**	E
2	A	B	**C**	D	E	12	A	B	**C**	D	E
3	A	B	C	**D**	E	13	A	B	C	D	**E**
4	A	B	**C**	D	E	14	A	B	**C**	D	E
5	A	B	C	D	**E**	15	A	B	C	**D**	E
6	A	B	C	D	**E**	16	**A**	B	C	D	E
7	A	B	C	**D**	E	17	A	B	**C**	D	E
8	A	B	**C**	D	E	18	A	B	C	**D**	E
9	A	**B**	C	D	E	19	**A**	B	C	D	E
10	A	B	C	**D**	E	20	A	B	**C**	D	E

Question 1 B

$\underset{\sim}{i} + \underset{\sim}{j} = 2(\underset{\sim}{i} - \underset{\sim}{j})$

$= -\underset{\sim}{i} + 3\underset{\sim}{j}$

Question 2 C

$r^4 = \frac{9}{1}$

$r = \sqrt{3}$

$t_6 = r^3 \times t_3$

$= 3\sqrt{3} \times 1$

$= 3\sqrt{3}$

Question 3 D

$c^2 = a^2 + b^2 - 2ab\cos(\theta)$

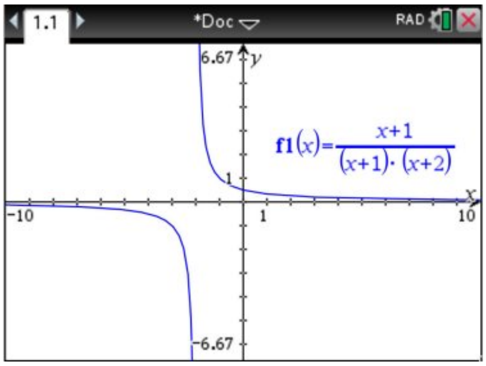
$9^2 = 7^2 + 8^2 - 2 \times 7 \times 8\cos(\theta)$

$\cos(\theta) = \frac{49 + 64 - 81}{112}$

$\theta = \cos^{-1}\left(\frac{32}{112}\right)$

$\approx 73°$

Question 4 C



Question 5 E

$\underset{\sim}{a} + \underset{\sim}{b} = (m + 4)\underset{\sim}{i} + (2m + 4)\underset{\sim}{j}$

Solving $x = 4 + m$ and $y = 2m + 4$ simultaneously gives infinite solutions for x, y and therefore the solution is vectors $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 6 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 9 \\ 12 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ -1 \end{bmatrix}$.

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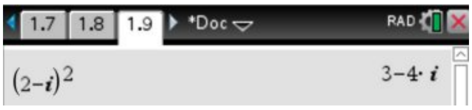
Question 6 E



Question 7 D

$$|m\underset{\sim}{i} - 2m\underset{\sim}{j}| = 1$$

$$\sqrt{m^2 + (-2m)^2} = 1$$

$$m^2 + 4m^2 = 1$$

$$m^2 = \frac{1}{5}$$

$$m = \pm\frac{\sqrt{5}}{5}$$

Question 8 C

$OB = r,\ OC = 2,\ \angle BOC = 120°$

$$c^2 = a^2 + b^2 - 2ab\cos(\theta)$$

$$BC^2 = r^2 + r^2 - 2r \times r \times \cos(120°)$$

$$= 3r^2$$

$$BC = \sqrt{3}r$$

Question 9 B

$$9 \times 4 = x(x + 9)$$

$$36 = x^2 + 9x$$

$$x^2 + 9x - 36 = 0$$

$$(x + 12)(x - 3) = 0$$

$$x = 3 \text{ as } x > 0$$

Question 10 D

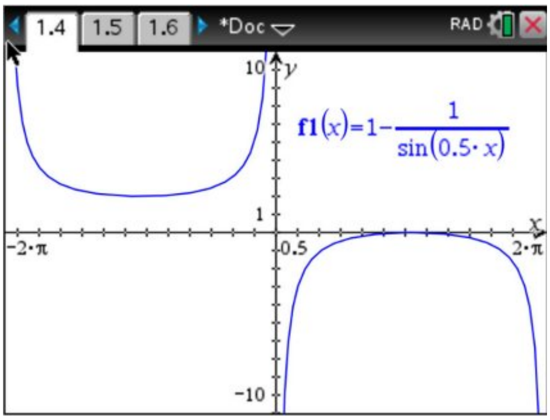
$$S_\infty = \frac{a}{1 - r}$$

$$= 10a$$

$$10 = \frac{1}{1 - r}$$

$$r = \frac{9}{10}$$

Question 11 D

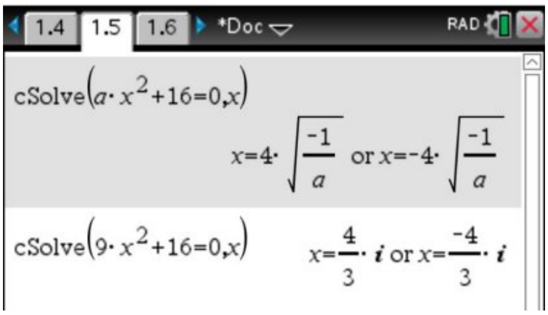


Question 12 C

$$z = \frac{3}{2} - 2i$$

$$\text{Im}(z) = -2$$

Question 13 E



Question 14 C

$$\sqrt{(x-0)^2 + (y-6)^2} = \sqrt{(x-2)^2 + (y-0)^2}$$

$$x^2 + y^2 - 12y + 36 = x^2 - 4x + 4 + y^2$$

$$12y = 4x + 32$$

$$y = \frac{1}{3}x + \frac{8}{3}$$

Question 15 D

$$x^2 + (y-b)^2 - b^2 + c = 0$$

$$x^2 + (y-b)^2 = b^2 - c$$

$$b^2 - c > 0$$

$$c < b^2$$

Question 16 **A**

$$\angle BAC = \frac{1}{2} \times \angle BOC \quad \text{(subtended angle)}$$

$$\angle BAC = 50°$$

$$\text{obtuse } \angle BAC = 260°$$

$$\angle OBA = 360 - 260 - 50 - 20$$

$$= 30°$$

Question 17 **C**

$$\underset{\sim}{a}.\underset{\sim}{b} = 3m \times m^2 + -6 \times -4$$

$$= 0$$

$$3m^3 = 24$$

$$m^3 = 8$$

$$m = 2$$

Question 18 **D**

$$\cos(\theta) = \frac{\underset{\sim}{a}.\underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|}$$

$$= \frac{-2-3}{\sqrt{5} \times \sqrt{10}}$$

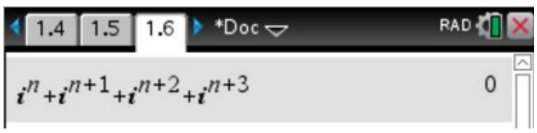
$$= \frac{-5}{\sqrt{50}}$$

$$= -\frac{\sqrt{2}}{2}$$

$$\theta = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$$

$$= \frac{3\pi}{4}$$

Question 19 **A**



Question 20 **C**

length ratio:

$$\frac{AB'}{AB} = \frac{3}{4}$$

area ratio:

$$\left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

$$\begin{aligned}\text{area } \Delta AB'C' &= \frac{9}{16} \times 40 \\ &= \frac{45}{2} \text{ cm}^2\end{aligned}$$

SECTION B

Question 1 (8 marks)

a. $r = \frac{4}{1 + \sin(\theta)}$

$r + r\sin(\theta) = 4$

Let $y = r\sin(\theta)$ and $x^2 + y^2 = r^2$.

$r + y = 4$

$r = 4 - y$ M1

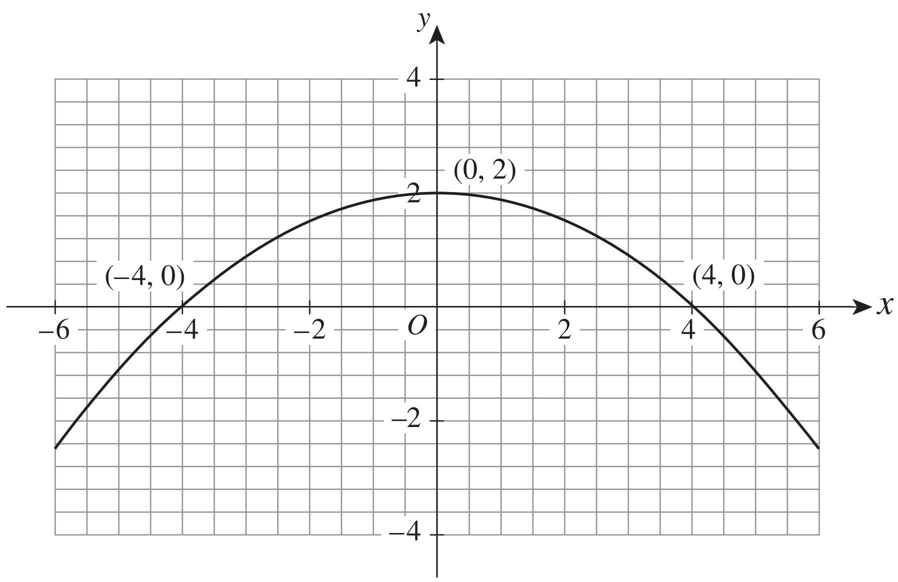
$x^2 + y^2 = (4 - y)^2$

$= 16 - 8y + y^2$

$8y = 16 - x^2$

$y = -\frac{1}{8}x^2 + 2$ A1

b.



2 marks

parabolic shape A1

labelled intercepts A1

c. $r + r\sin(\theta) = \frac{4}{n}$

Let $y = r\sin(\theta)$ and $x^2 + y^2 = r^2$. M1

$r + y = 4$

$r = 4 - y$

$x^2 + y^2 = \left(\frac{4}{n} - y\right)^2$

$= \frac{16}{n^2} - \frac{8}{n}y + y^2$ M1

$\frac{8}{n}y = \frac{16}{n^2} - x^2$

$y = -\frac{n}{8}x^2 + \frac{2}{n}$

y-intercept:

$\left(0, \frac{2}{n}\right)$ A1

x-intercept:

Let $y = 0$.

$-\frac{n}{8}x^2 + \frac{2}{n} = 0$

$x = \pm\frac{4}{n}$

The x-intercepts are therefore $\left(-\frac{4}{n}, 0\right)$ and $\left(\frac{4}{n}, 0\right)$. A1

Note: Alternatively, students may use dilation of factor of $\frac{1}{n}$ *as their method mark.*

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Question 2 (10 marks)

a. Let $z_1 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$.

$$r^2 = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$r = 1$$ A1

$$\theta = \tan^{-1}\left(\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\right)$$

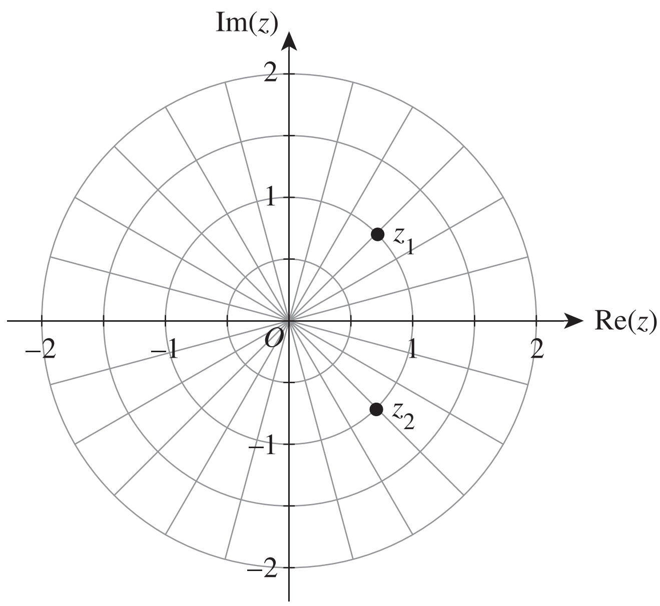
$$= \frac{\pi}{4}$$ A1

$$z_1 = \text{cis}\left(\frac{\pi}{4}\right)$$

b. **i.** $z_2 = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$ A1

ii. $z_2 = \text{cis}\left(-\frac{\pi}{4}\right)$ A1

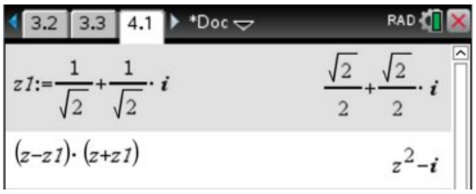
c.



correct z_1 A1
correct z_2 A1

d. **i.** $(z-z_1)(z+z_1)=\left(z-\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)\right)\left(z+\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)\right)$ M1

$(z-z_1)(z+z_1)=z^2-i$ A1



ii.
$$z^2=i$$
$$z^2-i=0$$
$$(z-z_1)(z+z_1)=z^2-i$$ M1
$$=0$$

$z=z_1$ or $z=-z_1$

$z=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i$ or $z=-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i$ A1

Question 3 (9 marks)

a. $\angle PAR$ corresponding to $\angle QRC$

$\angle RPA$ corresponding to $\angle CQR$ M1

$\therefore \Delta RAP$ is similar to ΔCRQ (two angles equal in a triangle) A1

b. $PB=d-x$ A1

c. $|RQ|=|PB|$
$$=d-x$$
$$\frac{|RQ|}{|AP|}=\frac{d-x}{x}$$ M1

$$\text{area ratio}=\frac{(d-x)^2}{x^2}$$ M1
$$=\frac{d^2-2dx+x^2}{x^2}$$
$$=\frac{d^2}{x^2}-\frac{2d}{x}+1$$

d. ΔABC is right-angled (inscribed in semi-circle).

$$\therefore \angle RCQ = \angle ARP$$
$$= 90°$$

$$AR^2 + RP^2 = AP^2$$
$$1^2 + RP^2 = x^2$$
$$RP = \sqrt{x^2 - 1} \qquad \text{M1}$$

$$\text{area of } \Delta ARP = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 1 \times \sqrt{x^2 - 1}$$
$$= \frac{\sqrt{x^2 - 1}}{2} \qquad \text{M1}$$

$$\text{area ratio} = \frac{(d - x)^2}{x^2}$$

$$d = 4 \rightarrow \text{area ratio} = \frac{(4 - x)^2}{x^2} \qquad \text{M1}$$

$$\text{area of } \Delta CRQ = \frac{\sqrt{x^2 - 1}(4 - x)^2}{2x^2} \qquad \text{A1}$$

Question 4 (11 marks)

a. **i.** $\overrightarrow{OA} = 10\underset{\sim}{i} + 18\underset{\sim}{j}$ and $\overrightarrow{OB} = 20\underset{\sim}{i} + 6\underset{\sim}{j}$. A1

ii. $\overrightarrow{AB} = \overrightarrow{OA} - \overrightarrow{OB}$

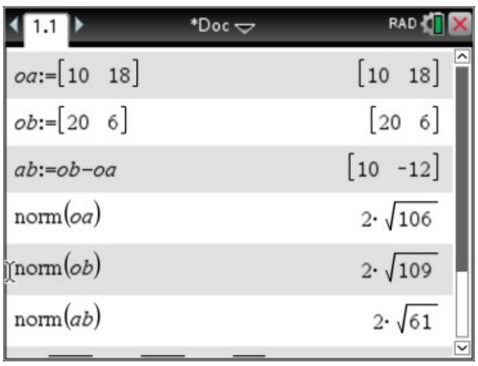
$= (20 - 10)\underset{\sim}{i} + (6 - 18)\underset{\sim}{j}$ M1

$= 10\underset{\sim}{i} - 12\underset{\sim}{j}$ as required

b. $\left|\overrightarrow{OA}\right| = 2\sqrt{106}$ $\quad \left|\overrightarrow{OB}\right| = 2\sqrt{109}$ $\quad \left|\overrightarrow{AB}\right| = 2\sqrt{61}$ A1

distance $= \left|\overrightarrow{OA}\right| + \left|\overrightarrow{OB}\right| + \left|\overrightarrow{AB}\right|$

$= 57$ km A1

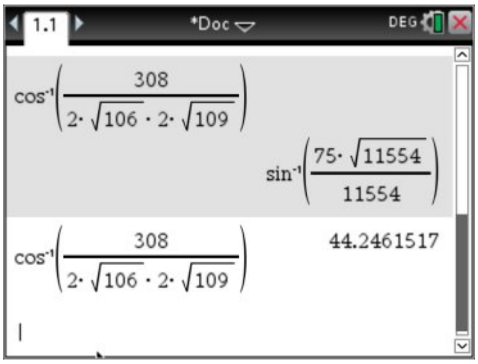


c. **i.** $\overrightarrow{OA} \cdot \overrightarrow{OB} = 308$ A1

ii. $\theta = \cos^{-1}\left(\dfrac{\overrightarrow{OA} \cdot \overrightarrow{OB}}{\left|\overrightarrow{OA}\right|\left|\overrightarrow{OB}\right|}\right)$

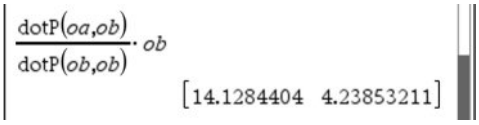
$= \cos^{-1}\left(\dfrac{308}{2\sqrt{106} \times 2\sqrt{109}}\right)$ M1

$\approx 44°$ A1



d. $\underset{\sim}{u} = \dfrac{\underset{\sim}{a}.\underset{\sim}{b}}{\underset{\sim}{b}.\underset{\sim}{b}}\underset{\sim}{b}$ M1

$= 14.1\underset{\sim}{i} + 4.2\underset{\sim}{j}$ A1



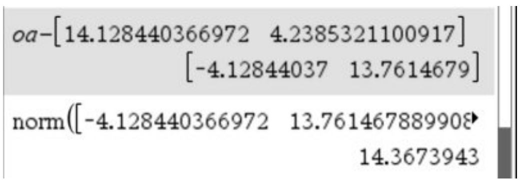
e. $\underset{\sim}{v} = \overrightarrow{OA} - \underset{\sim}{u}$

$= 10\underset{\sim}{i} + 18\underset{\sim}{j} - (14.128\ldots\underset{\sim}{i} + 4.238\ldots\underset{\sim}{j})$

$= -4.128\ldots\underset{\sim}{i} + 13.76\ldots\underset{\sim}{j}$ M1

minimum distance $= \left|\underset{\sim}{v}\right|$

$\left|\underset{\sim}{v}\right| = 14.4$ km A1



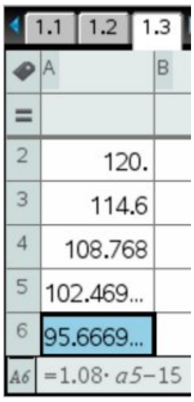
Question 5 (12 marks)

a. **i.** $125 - 4 \times 15 = 65$ A1

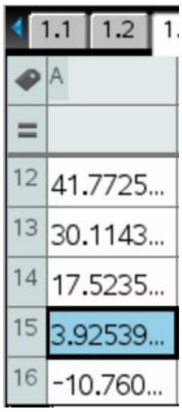
ii. $\dfrac{125}{15} = 8.33\ldots$

$\therefore 8$ years A1

b. **i.** $C_5 = 96$ A1



ii. 14 years A1



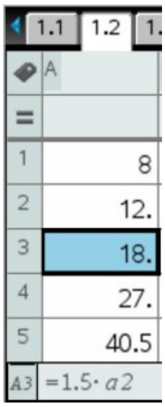
c. $m = 125 \times 0.08$

$= 10$ A1

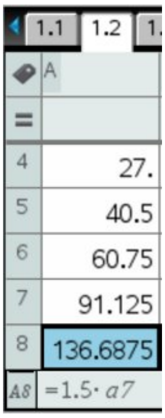
$n = 1 + \dfrac{15}{125}$

$= 1.12$ A1

d. **i.** 18 ducks after 2 years A1



ii. 7 years A1



e. 8×1.5^n A1

f. $D_{n+1} = 1.5D_n,\ D_0 = 8$ A1

g. after 5 years: $C_5 = 96$ and $D_5 = 73$

after 6 years: $C_5 = 88$ and $D_5 = 106$ M1

Therefore there will be more ducks than chickens after 6 years. A1

Question 6 (10 marks)

a. $y = \frac{x^2}{4a}$ and $x = 2at$

$y = \frac{(2at)^2}{4a} = \frac{4a^2t^2}{4a} = at^2$ as required A1

b. **i.** $y - y_1 = m(x - x_1)$

$y - \frac{a}{t^2} = t\left(x - -\frac{2a}{t}\right)$ M1

$= tx + 2a$

$y = tx + 2a + \frac{a}{t^2}$ A1

ii. $m_{RP} = -\frac{1}{m_{QR}}$

$= -\frac{1}{t}$

$y - y_1 = m(x - x_1)$

$y - at^2 = -\frac{1}{t}(x - 2at)$ M1

$y = -\frac{1}{t}x + 2a + at^2$ A1

iii. Let $y_{QR} = y_{RP} \rightarrow tx + 2a + \frac{a}{t^2} = -\frac{1}{t}x + 2a + at^2$

$(t^2 + 1)x = at^3 - \frac{a}{t}$

$x = \frac{\frac{at^4 - a}{t}}{t^2 + 1}$

$= \frac{a(t^4 - 1)}{t(t^2 + 1)}$

$= \frac{a(t^2 - 1)(t^2 + 1)}{t(t^2 + 1)}$ M1

$= a\left(t - \frac{1}{t}\right)$

$y = -\frac{1}{t}x + 2a + at^2$

$= ta\left(t - \frac{1}{t}\right) + 2a + \frac{a}{t^2}$ M1

$= at^2 - a + 2a + \frac{a}{t^2}$

$x = a\left(t - \frac{1}{t}\right),\ y = a\left(t^2 + 1 + \frac{1}{t^2}\right)$ A1

c. $x = a\left(t - \frac{1}{t}\right)$

$x^2 = a^2\left(t^2 - 2 + \frac{1}{t^2}\right)$ M1

$= a^2\left(t^2 + 1 + \frac{1}{t^2} - 3\right)$

$= a.a\left(t^2 + 1 + \frac{1}{t^2}\right) - 3a^2$

$= ay - 3a^2$

$ay = x^2 + 3a^2$

$y = \frac{x^2}{a} + 3a$ A1