Neap

Trial Examination 2020

VCE Specialist Mathematics Units 1&2

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 21 pages

Formula sheet

Answer sheet for multiple-choice questions

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on your answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1

Given that $\underset{\sim}{a} = \underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{b} = \underset{\sim}{i} + \underset{\sim}{j}$, then $\underset{\sim}{b} - 2\underset{\sim}{a}$ is equal to

A. $-\underset{\sim}{i} - 3\underset{\sim}{j}$

B. $-\underset{\sim}{i} + 3\underset{\sim}{j}$

C. $-\underset{\sim}{i} - \underset{\sim}{j}$

D. $\underset{\sim}{i} + 3\underset{\sim}{j}$

E. $\underset{\sim}{i} - \underset{\sim}{j}$

Question 2

If a geometric sequence has $t_3 = 1$ and $t_7 = 9$, then t_6 is equal to

A. $\sqrt{3}$

B. 3

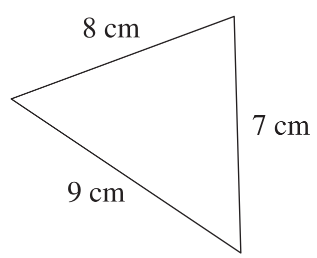
C. $3\sqrt{3}$

D. 5

E. 6

Question 3

Consider the triangle below.



The size of the largest angle in the triangle, to the nearest degree, is

A. 48°

B. 58°

C. 60°

D. 73°

E. 80°

Question 4

The graph of $y = \dfrac{x+1}{(x+1)(x+2)}$ has asymptotes of

A. $x = -2$
B. $x = -1,\ x = -2$
C. $x = -2,\ y = 0$
D. $x = -1,\ x = -2,\ y = 0$
E. $x = 1,\ x = 2$

Question 5

If $\underset{\sim}{a} = \begin{bmatrix} 4 \\ 2m \end{bmatrix}$ and $\underset{\sim}{b} = \begin{bmatrix} m \\ 4 \end{bmatrix}$, then a vector that is parallel to $\underset{\sim}{a} + \underset{\sim}{b}$ could be

A. $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$

B. $\begin{bmatrix} 6 \\ 4 \end{bmatrix}$

C. $\begin{bmatrix} 9 \\ 12 \end{bmatrix}$

D. $\begin{bmatrix} -2 \\ -1 \end{bmatrix}$

E. $\begin{bmatrix} 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \end{bmatrix}, \begin{bmatrix} 9 \\ 12 \end{bmatrix}$ or $\begin{bmatrix} -2 \\ -1 \end{bmatrix}$

Question 6

If $z = 2 - i$, then z^2 is equal to

A. 3
B. 5
C. $3 - 2i$
D. $5 - 4i$
E. $3 - 4i$

Question 7

The vector $m\underset{\sim}{i} - 2m\underset{\sim}{j}$ is a unit vector if m is equal to

A. $\frac{1}{3}$

B. 0

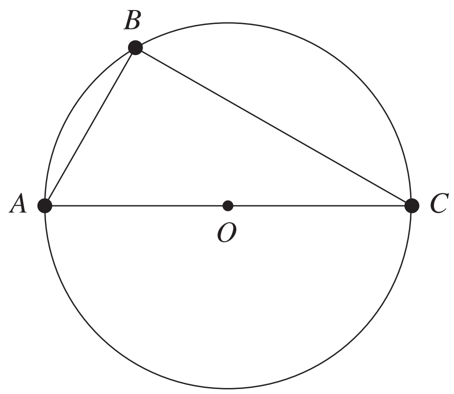
C. $\frac{\sqrt{3}}{3}$

D. $\frac{\sqrt{5}}{5}$

E. 1

Question 8

In the diagram below, AOC is the diameter of a circle with centre O and radius of length r. B is a point on the circumference of the circle.



If $OA = AB$, then the length of BC is equal to

A. $\frac{3r}{2}$

B. $\sqrt{2}r$

C. $\sqrt{3}r$

D. $2r$

E. $\sqrt{5}r$

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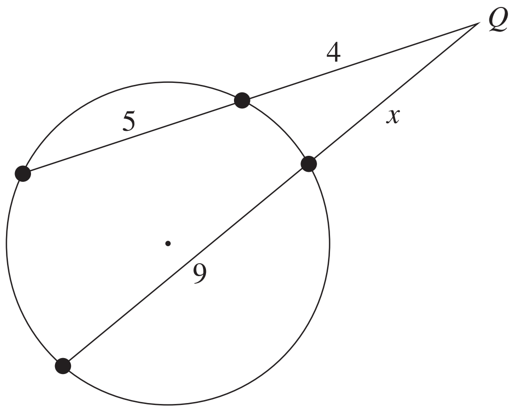
Question 9

In the diagram below, two secants from the point Q intersect a circle.



The value of x is

A. 2

B. 3

C. 4

D. 4.5

E. 8

Question 10

The sum of an infinite geometric series is known to be ten times the first term.

The common ratio, r, is equal to

A. $\frac{1}{100}$

B. $\frac{1}{10}$

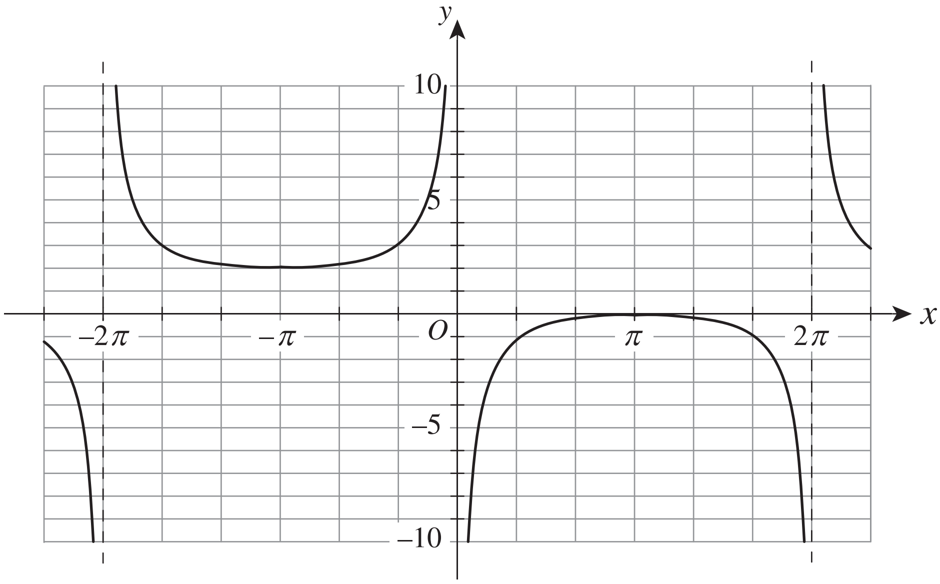
C. $\frac{\sqrt{10}}{10}$

D. $\frac{9}{10}$

E. 10

Question 11

Consider the graph below.



The graph has the equation

A. $y = 1 + \dfrac{1}{\sin(0.5x)}$

B. $y = 1 - \dfrac{1}{\sin(x)}$

C. $y = 1 + \dfrac{1}{\sin(2x)}$

D. $y = 1 - \dfrac{1}{\sin(0.5x)}$

E. $y = 1 - \dfrac{2}{\sin(x)}$

Question 12

If $z = \frac{1}{2}(3 - 4i)$, then Im(z) is equal to

A. –4
B. $-4i$
C. –2
D. $-2i$
E. 3

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Question 13

The equation $ax^2 + 16 = 0$ has solutions of $x = \pm\frac{4}{3}i$ when a is equal to

A. -3

B. 3

C. $\frac{3}{4}$

D. -9

E. 9

Question 14

The equation that describes the locus of points (x, y) that are equidistant from the points with coordinates of (2, 0) and (0, 6) is

A. $y = x$

B. $y = 3x$

C. $y = \frac{1}{3}x + \frac{8}{3}$

D. $y = x + 3$

E. $y = -3x + \frac{10}{3}$

Question 15

If the circle given by $x^2 + y^2 - 2by + c = 0$, then which one of the following must be true?

A. $c > b^2$

B. $c > 0$

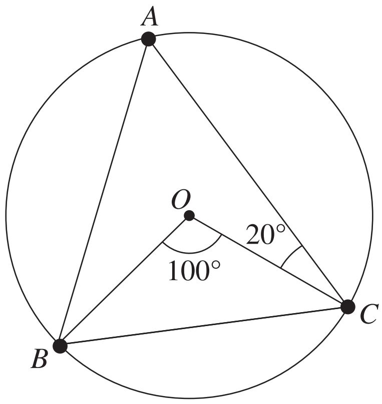
C. $c = b^2$

D. $c < b^2$

E. $c = 0$

Question 16

In the diagram below, the points A, B and C lie on a circle with a centre O.



Given that $\angle BOC = 100°$ and $\angle OCA = 20°$, then the magnitude of $\angle OBA$ is equal to

A. 30°
B. 40°
C. 45°
D. 50°
E. 80°

Question 17

The vectors $\underset{\sim}{a} = 3m\underset{\sim}{i} - 6\underset{\sim}{j}$ and $\underset{\sim}{b} = m^2\underset{\sim}{i} + 4\underset{\sim}{j}$ are perpendicular if m is equal to

A. $2\sqrt{2}$

B. ± 2

C. 2

D. $\frac{8}{3}$

E. 8

Question 18

The angle between the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{b} = -\underset{\sim}{i} + 3\underset{\sim}{j}$ is

A. $\frac{\pi}{3}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{3\pi}{4}$

E. $\frac{4\pi}{3}$

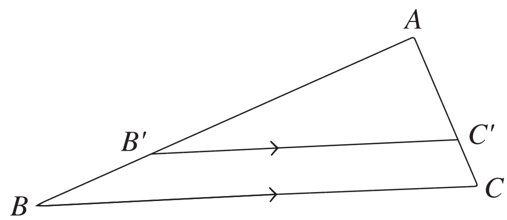
Question 19

In the simplest possible form $i^n + i^{n+1} + i^{n+2} + i^{n+3}$, $n \in Z$ is equal to

A. 0
B. i
C. $1 + i$
D. $1 - i$
E. 2

Question 20

In the diagram below, BC is parallel to $B'C'$ and $BB' = \frac{1}{4}AB$. The area of triangle ABC is 40 cm^2.



The area of triangle $AB'C'$ is equal to

A. 20 cm^2
B. $\frac{40}{9}$ cm^2
C. $\frac{45}{2}$ cm^2
D. 30 cm^2
E. $\frac{40}{3}$ cm^2

SECTION B

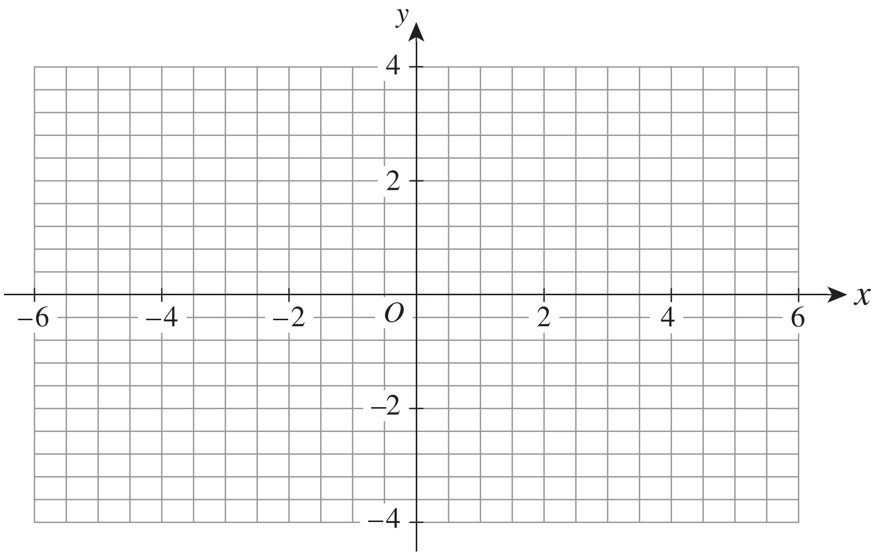
Instructions for Section B

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 (8 marks)

a. Find the cartesian equation for the curve defined by the polar equation $r = \dfrac{4}{1 + \sin(\theta)}$. 2 marks

b. Sketch the graph of the relation found in **part a.** on the set of axes below. Include coordinates of axes intercepts as required. 2 marks



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c. Find the coordinates of the axes intercepts of the graph of the cartesian equation for the curve defined by the polar equation $r = \dfrac{4}{n + n\sin(\theta)}$. 4 marks

Question 2 (10 marks)

Let $z_1 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$.

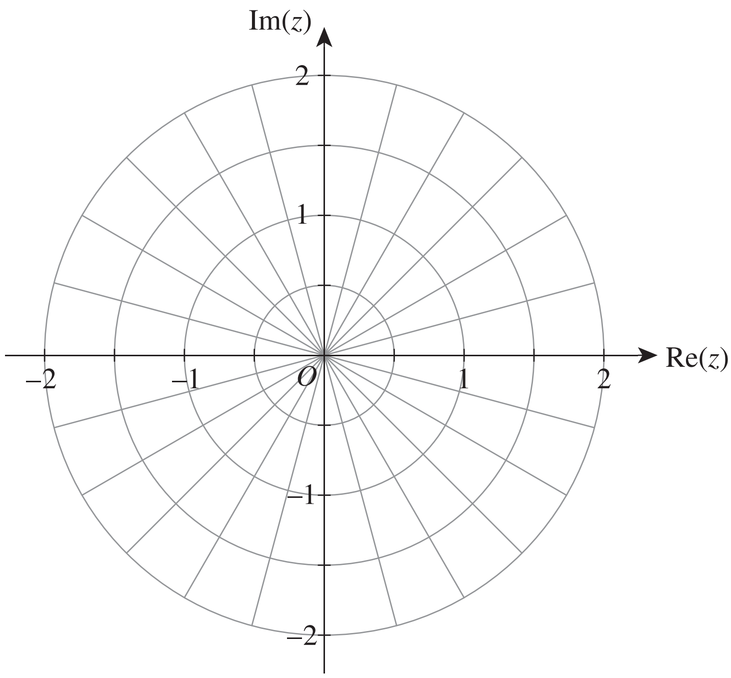
a. Express z_1 in polar form. 2 marks

Let $z_2 = \bar{z}_1$.

b. **i.** Find z_2 in cartesian form. 1 mark

ii. Express z_2 in polar form. 1 mark

c. Plot and label the complex numbers z_1 and z_2 on the Argand diagram below. 2 marks



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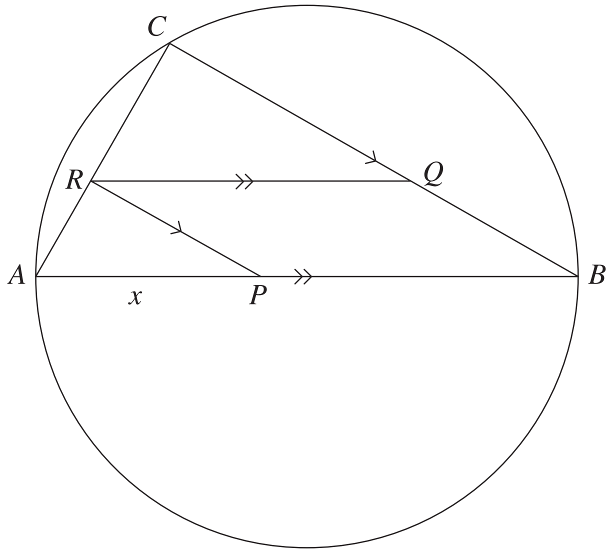
d. **i.** Expand $(z - z_1)(z + z_1)$. 2 marks

ii. Hence, solve the equation $z^2 = i$. 2 marks

Question 3 (9 marks)

The triangle ABC is inscribed in a circle with a diameter of AB. The line RQ is parallel to AB and the line CB is parallel to RP. The length AP is represented by x.



a. Show that ΔRAP is similar to ΔCRQ. 2 marks

The length of the diameter AB is represented by d.

b. Express the length of PB in terms of x and d. 1 mark

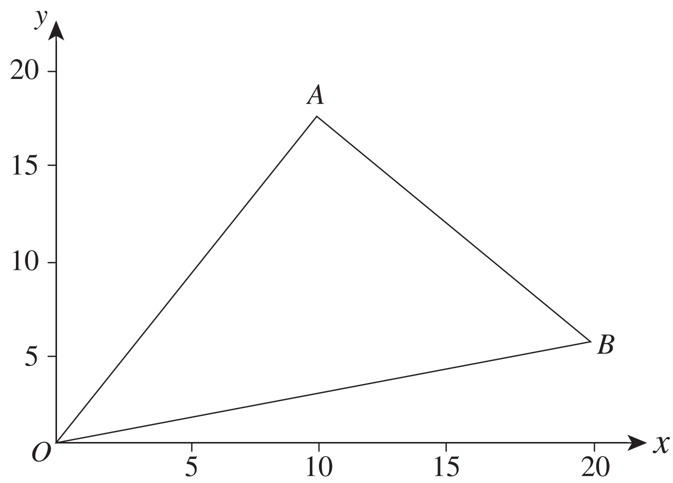
c. Show that the ratio of the area of ΔCRQ to the area of ΔRAP is given by $\frac{d^2}{x^2} - \frac{2d}{x} + 1$. 2 marks

d. Given the length AR is equal to 1 cm and the diameter of the circle AB is equal to 4 cm, find the area of ΔCRQ in terms of x. 4 marks

Question 4 (11 marks)

A triangular course has been planned for a yacht race. Point O is the start and finish of the race. The race goes from O to A to B to O, with the coordinates of A and B being (10, 18) and (20, 6) respectively. This information is presented in the diagram below.

The coordinates represent distances in kilometres East and North of O. Take $\underset{\sim}{i}$ and $\underset{\sim}{j}$ as unit vectors along the x-axis and y-axis.



a. **i.** Write vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ in terms of $\underset{\sim}{i}$ and $\underset{\sim}{j}$. 1 mark

ii. Hence, show that $\overrightarrow{AB} = 10\underset{\sim}{i} - 12\underset{\sim}{j}$. 1 mark

b. Find the magnitudes of $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{AB}$ and, hence, determine the distance of the race to the nearest kilometre. 2 marks

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Let θ be the angle between OA and OB.

c. **i.** Find the dot product $\overrightarrow{OA} \cdot \overrightarrow{OB}$. 1 mark

ii. Hence, find the value of θ to the nearest degree. 2 marks

Once the yacht race is over, two yachts leave O. One moves towards A along the path OA, and the other moves towards B along the path OB.

d. Find $\underset{\sim}{u}$, the vector resolute of $\overrightarrow{OA}$ in the direction of $\overrightarrow{OB}$. Give your answer correct to one decimal place. 2 marks

e. The first yacht reaches a buoy at A and stops.

By using a vector resolute, or otherwise, find the **closest distance** between the buoy and the path OB. Give your answer in kilometres correct to one decimal place. 2 marks

Question 5 (12 marks)

Michael operates a hobby farm producing chicken and duck eggs. Initially he has 125 chickens.

Michael plans to sell 15 chickens each year. Assume that no new chickens arrive on the farm and that none die.

a. **i.** How many chickens will Michael have after 4 years? 1 mark

ii. For how many full years can Michael sell 15 chickens each year? 1 mark

The following difference equation provides a more realistic model for the number of chickens, C_n, on Michael's farm after n years:

$$C_{n+1} = 1.08C_n - 15 \qquad C_0 = 125$$

b. **i.** Use this difference equation to predict the number of chickens on the farm after 5 years. 1 mark

ii. For how many full years can Michael continue to sell 15 chickens each year? 1 mark

Suppose Michael wants to keep a steady population of 125 chickens on the farm. Two alternative forms of the new difference equation used to model this situation are given below:

$$C_{n+1} = 1.08C_n - m \qquad C_0 = 125$$

$$C_{n+1} = nC_n - 15 \qquad C_0 = 125$$

c. Find the values of m and n. 2 marks

Michael also keeps ducks on his farm. He plans to increase the number of ducks each year. The number of ducks on the farm will form a geometric sequence with a common ratio of 1.5. Michael began with 8 ducks on the farm.

d. **i.** How many ducks will be on the farm after 2 years? 1 mark

____

ii. After how many whole years will the number of ducks exceed 100? 1 mark

____

e. Write an expression in terms of n that can be used to predict the number of ducks on the farm after n years. 1 mark

____

f. Let D_n be the number of ducks on the farm after n years.

Write a difference equation for D_{n+1} in terms of D_n that can be used to predict the number of ducks on the farm after n years. 1 mark

____

____

g. The following difference equation provides a more realistic model for the number of ducks, D_n, on Michael's farm after n years:

$$D_{n+1} = 1.6D_n - 10 \qquad D_0 = 22$$

To predict the number of chickens, C_n, on the farm after n years, the following difference equation is used:

$$C_{n+1} = 1.08C_n - 15 \qquad C_0 = 125$$

Determine after how many years it will be before the number of ducks on the farm first exceeds the number of chickens on the farm. 2 marks

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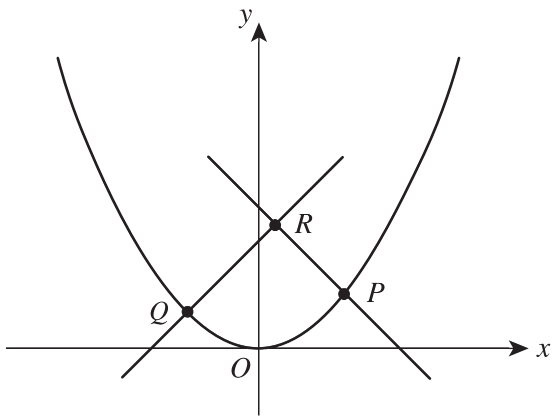
Question 6 (10 marks)

Consider the parabola $y = \frac{x^2}{4a}$ shown in the diagram below. The line through PR is perpendicular to the curve at the point P. The line through QR is perpendicular to the curve at the point Q.



a. Verify that the point $P(2at, at^2)$ lies on the graph of $y = \frac{x^2}{4a}$. 1 mark

The gradient of the line through the points Q and R is equal to t. The point Q has coordinates of $\left(-\frac{2a}{t}, \frac{a}{t^2}\right)$.
The line through the points Q and R is perpendicular to the line through the points R and P.

b. **i.** Find the equation of the line through the points Q and R. 2 marks

ii. Find the equation of the line through the points R and P. 2 marks

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iii. Hence, show that the point R has coordinates of $x = a\left(t - \frac{1}{t}\right)$, $y = a\left(t^2 + 1 + \frac{1}{t^2}\right)$. 3 marks

c. Find the equation in cartesian form of the locus of the point R. 2 marks

END OF QUESTION AND ANSWER BOOKLET

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Trial Examination 2020

VCE Specialist Mathematics Units 1&2

Written Examination 2

Multiple-choice Answer Sheet

Student's Name: ______________________________

Teacher's Name: ______________________________

Instructions

Use a pencil for all entries. If you make a mistake, erase the incorrect answer – do not cross it out.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than **one** answer is completed for any question.

All answers must be completed like **this** example: A B C D E

Use pencil only

1	A	B	C	D	E	11	A	B	C	D	E
2	A	B	C	D	E	12	A	B	C	D	E
3	A	B	C	D	E	13	A	B	C	D	E
4	A	B	C	D	E	14	A	B	C	D	E
5	A	B	C	D	E	15	A	B	C	D	E
6	A	B	C	D	E	16	A	B	C	D	E
7	A	B	C	D	E	17	A	B	C	D	E
8	A	B	C	D	E	18	A	B	C	D	E
9	A	B	C	D	E	19	A	B	C	D	E
10	A	B	C	D	E	20	A	B	C	D	E

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Trial Examination 2020

VCE Specialist Mathematics Units 1&2

Written Examination 2

Formula Sheet

Instructions

This formula sheet is provided for your reference. A question and answer booklet is provided with this formula sheet.

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SPECIALIST MATHEMATICS FORMULAS

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	

Vectors in two dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$
$\lvert\underset{\sim}{r}\rvert = \sqrt{x^2 + y^2 + z^2} = r$
$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$

Polar coordinates

$x = r\cos\theta$
$y = r\sin\theta$

END OF FORMULA SHEET

