

 **SPECIALIST MATHS UNITS 3 & 4**

 **TRIAL** **EXAMINATION 2**

 **SOLUTIONS**

 **2021**

## Section A – Multiple-choice answers

**1.** E **6.** D **11.** D **16.** B

**2.** E **7.** E **12.** E **17.** B

**3.** B **8.** C **13.** A **18.** D

**4.** E **9.** A **14.** E **19.** B

**5.** C **10.** D **15.** C **20.** B

## Section A - Multiple-choice solutions

### Question 1



The graph will have three vertical asymptotes  and a non-vertical straight line asymptote of .

Total number of straight line asymptotes is four.

The answer is E.

**Question 2**



So .

Double-check by sketching the function.



The answer is E.

**Question 3**



Sketch the graphs.



The answer is B.

**Question 4**

Let .

 Solve for *x* using CAS.



 

Equating imaginary terms and equating

real terms gives

 

The answer is E.

**Question 5**

Method 1

Let .



Since  must be in the fourth quadrant because the denominator is positive, the real component of the numerator (*y*)is positive and the imaginary component of the numerator (*x*) is negative.

And since  must lie further from the origin than *z*.

So it could only be represented by .

The answer is C.

**Question 5** (cont’d)

Method 2 – geometric approach

Let .

 represents a reflection of *z* in the *x*-axis and lies in the third quadrant.

 represents a rotation of  by anticlockwise and lies in the fourth quadrant.

Since  then dividing  by  will increase the magnitude of . The option in the fourth quadrant corresponding to this is .

The answer is C.

**Question 6**





So we can generate the values in all options except D.

As further confirmation, if 

 

The answer is D.

**Question 7**





The answer is E.

**Question 8**

From the graph, when .

This rules out options A, B and E.

From the graph, when .

This rules out option D.

The answer is C.

**Question 9**



 

The answer is A.

**Question 10**

Note that because we are rotating about the *y*-axis, the terminals of integration must lie on the *y*-axis.

 





The answer is D.

**Question 11**

Method 1

****

****,

 ****



The answer is D.

Method 2

****

****

****



The answer is D.

**Question 12**

Let the angle required be  and using , the unit vector in the positive direction of the

*z-*axis, we have



Closest answer is 56°.

The answer is E.

**Question 13**



The answer is A.

**Question 14**



Draw a diagram.

Method 1



 

The answer is E.

Method 2



The answer is E.

**Question 15**

Method 1 – using a velocity-time graph





 







When 



The answer is C.

Method 2 – using constant acceleration formulas





 

So 



Find where the mass turns by first finding its acceleration.



Find the distance to the stop/turn point from the start point.



 



Solve for *s.*

 

Total distance travelled

  Note that the particle is travelling in a straight line.

The answer is C.

**Question 16**





Around the 7 kg mass

 

Around the *m* kg mass



The answer is B.

**Question 17**

Since the particle is in equilibrium we can use Lami’s theorem.



The closest answer is 4.3 newtons.

The answer is B.

**Question 18**

Use CAS to find the confidence interval (z interval),

.



The answer is D.

**Question 19**



 



The closest answer is 0.23.

The answer is B.

**Question 20**

For Type A fish 

For Type B fish 





 

The closest answer is 0.1241.

The answer is B.

**SECTION B**

# Question 1 (11 marks)

1. 

 

 **(1 mark)**

 **(1 mark)**

1. Method 1

 **(1 mark)**

When .

**(1 mark)**

Method 2

 **(1 mark)**

when .

**(1 mark)**

1. Solve  for *x* (using CAS)

  **** (Note that  is outside the domain of *f* ).



Since  the concavity changes, and so a point of inflection occurs at  where both coordinates are correct to 2 decimal places.

**(1 mark)** – concavity check

**(1 mark)** – correct coordinates



 **(1 mark)** – correct asymptotes

**(1 mark)** – correct shape including

 point of inflection

1. Note that  i.e. *k* is an integer.

Note that the graph drawn in part **d.** shows the graph of *g* when .

The graph of *g* when  is shown below.



The graph of *g* when  is shown below.



The graph of *g* when  is shown below.



The graph of *g* is continuous over its maximal domain for 

**(1 mark)**

1. Using the graphs from part **e**., we see that these asymptotes given by . **(1 mark)**



 **(1 mark)**

**Question 2** (10 marks)

1. Method 1

Use cSolve on CAS to solve.

 **(1 mark)**

 **(1 mark)**

Method 2

The second solution is  **(1 mark)**

 

Since the constant term in  is , then .

So the third solution is . **(1 mark)**

1. 

**(1 mark)**



**(1 mark)**

1. gradient of line *L* joining 

 

**(1 mark)**



 **(1 mark)**

1. *L* is the perpendicular bisector of the line joining .



The line joining  therefore has a gradient of  (ie the negative reciprocal of the gradient of *L).* Its equation is given by





This line and line *L* (given by  from part **d**) intersect at the point  ie they have the same *y*-intercept. **(1 mark)**

So is the midpoint of the line joining .

So , ie .

**(1 mark)**

1.  (using part **b.**)





**(1 mark)**

 **(1 mark)**

**Question 3** (9 marks)

1. 



 **(1 mark)**

1.  (from part **a**.)

 

**(1 mark)**

1. 



 

**(1 mark)**



 

Estimated value of *P* when .

 **(1 mark)**

1. 

 

 **(1 mark)**

1. Method 1

Use deSolve function on CAS, with initial condition , which

. **(1 mark)**

Method 2



So . **(1 mark)**

1. The insecticide level will be ineffective when the concentration drops below

0.05 kg/L.

In the 10 000 L tank, this equates to an amount of insecticide given by

. So when *x* drops below 500, the insecticide will be ineffective.

Solve  for *t*.

 

So after 347 mins (to the nearest minute) the level has dropped below 0.05 kg/L.

To double-check,  ie the level has dropped below 0.05 kg/L

whereas  ie the level is still just above 0.05 kg/L.

**(1 mark)**

1. 



**(1 mark)**

 **(1 mark)**

**Question 4** (11 marks)

1. 



 

**(1 mark)**

So  as required.

**(1 mark)**

1. 

When  is the starting point.

When  is the finishing point.



 **(1 mark)** – correct start and finish points

**(1 mark)** – correct shape and direction indication

1. Method 1 – using parametric equations



**(1 mark)** – correct integral

**(1 mark)** – correct answer

Method 2 – using Cartesian equation For the terminals,

****

**(1 mark)** – correct integral

**(1 mark)** – correct answer

1. 





**(1 mark)**

Solve  for *t*.

Since  **(1 mark)**

A quick sketch of the speed function shows us that the maximum speed occurs when 



 

**(1 mark)**

1. 

 

For a collision, we require

  **(1 mark)**

Solve each equation for *t* where .

 

When  both the camera and the drone are in the same position on top of the prison farm wall (i.e. at ). Therefore they will collide.

**(1 mark)**

**Question 5** (10 marks)



**(1 mark)**

1. 

 

**(1 mark)**



Solve for *t*.

 (correct to 3 decimal places)

 **(1 mark)**

1.  for equilibrium **(1 mark)**



 **(1 mark)**

1. 





**(1 mark)** for 41.3 **(1 mark)**  for 70.1

1. vertical component of initial velocity



height of plane

Take the upward direction as positive.





**(1 mark)**

Solve for *t* where .

 

It takes 1.39 seconds (to 2 decimal places) to hit the ground.

**(1 mark)**

1. horizontal component of initial velocity



**(1 mark)**

**Question 6** (9 marks)

1. 



**(1 mark)**

1.  **(1 mark)**

 



  **(1 mark)**

1. Since , there is good evidence to reject .

**(1 mark)**

1. Find *c* such that





(use inverse norm) **(1 mark)**



So 

Check that the corrected value is greater than 21.8385…

So the smallest value of , correct to 2 decimal places, that can be observed for  **not** to be rejected is 21.84 minutes.

**(1 mark)**

1. For sample 1: 

For sample 2: 

We require 

**(1 mark)**



**(1 mark)**



**(1 mark)**