

 Student Name:………………………………………

### SPECIALIST MATHEMATICS UNITS 3 & 4

### TRIAL EXAMINATION 2

**2021**

#### Reading Time: 15 minutes

Writing time: 2 hours

######  Instructions to students

This exam consists of Section A and Section B.

Section A consists of 20 multiple-choice questions and should be answered on the detachable answer sheet which can be found on page 28 of this exam.

Section B consists of 6 extended-answer questions.

Section A begins on page 2 of this exam and is worth 20 marks.

Section B begins on page 11 of this exam and is worth 60 marks.

There is a total of 80 marks available.

All questions in Section A and B should be answered.

In Section B, where more than one mark is allocated to a question, appropriate working must be shown.

An exact value is required to a question unless otherwise directed.

Unless otherwise stated, diagrams in this exam are not drawn to scale.

The acceleration due to gravity should be taken to have magnitude ** where 

Students may bring one bound reference into the exam.

Students may bring into the exam one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory does not need to be cleared. For approved computer-based CAS, full functionality may be used.

A formula sheet can be found on pages 25-27 of this exam.

##### This paper has been prepared independently of the Victorian Curriculum and Assessment Authority to provide additional exam preparation for students. Although references have been reproduced with permission of the Victorian Curriculum and Assessment Authority, the publication is in no way connected with or endorsed by the Victorian Curriculum and Assessment Authority.

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This Trial Exam is licensed on a non transferable basis to the purchasing school. It may be copied by the school which has purchased it. This license does not permit distribution or copying of this Trial Exam by any other party.**SECTION A – Multiple-choice questions**

**Question 1**

The number of straight line asymptotes of the graph of  is

**A.** 0

**B.** 1

**C.** 2

**D.** 3

**E.** 4

**Question 2**

The maximal domain of the function  is

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 3**

The solutions to  for  are given by

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 4**

If , where , then  is equal to

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 5**

Consider the complex number *z* as shown on the Argand diagram below, where .



The location of  could be represented by

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 6**

For a given complex number *z*, .

Which of the following could  **not** be equal to?

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 7**

With a suitable substitution,  can be written as

1. 
2. 
3. 
4. 
5. 

**Question 8**



The differential equation that could represent the slope field above is

**A. **

**B. **

**C. **

**D. **

**E. **

**Question 9**

A curve has parametric equations  and  for .

An expression for  is

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 10**

The curve given by , is rotated about the *y*-axis to form a solid of revolution.

The volume of the solid can be found by evaluating

1. 
2. 
3. 
4. 
5. 

**Question 11**

The acceleration, in ms-2, of a particle moving in a straight line is given by , where *x* metres is its displacement from a fixed origin *O*.

If the particle has a velocity of  ms-1 when , the speed of the particle when  is

**A.** 0

**B.** 

**C.** 

**D.** 

**E.** 

**Question 12**

The angle that the vector  makes with the positive direction of the *z-*axis is closest to

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 13**

The scalar resolute of  in the direction of  is

**A.** 

**B.**  

**C.** 

**D.** 

**E.** 

**Question 14**

*A*, *B* and *C* are three collinear points with position vectors  and  respectively.

If *B* lies between *A* and *C* and if , then is equal to

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 15**

A 9 kg mass is travelling in a straight line with a momentum of magnitude 20 kg ms-1.

Eight seconds later, the mass is travelling in the opposite direction with a momentum of magnitude 10 kg ms-1.

Assuming that the mass is accelerating at a constant rate, the distance in metres covered by the mass during the eight seconds is

**A. **

**B. **

**C. **

**D. **

**E. **

**Question 16**

Particles of mass 7 kg and *m* kg are attached to the ends of a light inextensible string that passes over a smooth pulley as shown below.



If the acceleration of the 7 kg mass is 2.45 **** downwards, then *m* is equal to

**A.** 2.45 kg

**B.** 4.2 kg

**C.** 7 kg

**D.**  kg

**E.** 51.45 kg

**Question 17**

Forces of magnitude 6, *R* and *W* newtons act on a particle that is in equilibrium, as shown in the diagram below.



The magnitude of *W,* in newtons, is closest to

**A.** 4.2

**B.** 4.3

**C.** 5.0

**D.** 5.5

**E.** 6.0

**Question 18**

For a random sample of 25 football recruits, the mean height is 186 cm.

The standard deviation of the heights of all recruits across this football code is known to be

5 cm.

A 90% confidence interval for the mean of the heights of all recruits across this football code, correct to one decimal place, is closest to

**A.** ****

**B. **

**C. **

**D. **

**E. **

**Question 19**

The heights of giraffes in an area of Africa are known to be normally distributed with a mean of 5.65 metres and a variance of 0.28 metres.

The probability that a sample of seven of these giraffes has an average height of more than 5.8 metres is closest to

**A.** 0.08

**B.** 0.23

**C.** 0.30

**D.** 0.39

**E.** 0.77

**Question 20**

The lengths of a certain type of fish in a lake (Type A) are known to be normally distributed with a mean of 35 cm and a variance of 4. In the same lake, a different type of fish (Type B) has lengths normally distributed with a mean of 31 cm and a variance of 8.

The length of Type A fish caught in the lake is independent of the length of Type B fish caught in the lake.

If one of each type of fish is caught by a fisherman, the probability that the Type B fish is longer than the Type A fish, is closest to

**A.** 0.0028

**B.** 0.1241

**C.** 0.1587

**D.** 0.3694

**E.** 0.8759

**SECTION B**

**Question 1** (11 marks)

Consider the function .

1. Determine the maximal domain *D* and the range of *f*. 2 marks

1. Find the rule forand explain why  for . 2 marks

1. Find the coordinates of any points of inflection on the graph of *f* correct to two decimal places. 2 marks
2. Sketch the graph of  on the set of axes below, labelling any points of inflection with their coordinates and all asymptotes with their equation. 2 marks



Let , where .

1. State the value(s) of *k* such that  is continuous over its maximal domain. 1 mark

The equation of one of the asymptotes of  is given by , where .

1. State the values of *k* for which this asymptote exists and find the rule for the equation of this asymptote in terms of *k*. 2 marks

**Question 2** (10 marks)

Consider the complex polynomial , where .

1. If one of the solutions to  is , find all other solutions. 2 marks
2. Express the solutions to  in polar form. 1 mark
3. Plot your answers from part **b**. on the Argand diagram below. 1 mark



1. State the Cartesian equation of the line *L* which passes through  and the real solution to . 2 marks
2. The line *L* can also be represented by the relation , where .

Find the complex number  in Cartesian form. 2 marks

1. Find the area of the region bounded by the positive direction of the real axis, the relation  and *L*.

Express your answer in the form , where *a*, *b*, and. 2 marks

**Question 3** (9 marks)

The rate at which the population, *P,* of mosquitoes in a jungle is increasing with respect to time *t*, in hours, can be modelled by the differential equation .

**a.** If there are 100 mosquitoes in the jungle when , set up an expression

 containing a definite integral, that can be used to determine the value of *t* when

 the population of mosquitoes reaches 400. 1 mark

**b.** Show that the time taken to reach a population of 400 mosquitoes is 11 hours. 1 mark

**c.** Given that  when , use Euler’s method with a step size of 0.1 to estimate the value of *P* when . Give your answer correct to the nearest

number of mosquitoes. 2 marks

To help combat the rapid increase in mosquitoes, local authorities plan to put an insecticide into the jungle’s lake.

A tank is set up on the bank of the lake. Initially there is 1000 kg of insecticide dissolved in

10 000 L of water. Pure water is pumped into the tank at a rate of 20 L/min. The solution of insecticide and water is kept uniform by stirring and flows out of the tank and into the lake at the rate of 20 L/min.

Let *x* kilograms be the amount of insecticide in the tank after *t* minutes.

**d.** Show that the differential equation representing this situation is given by

 . 1 mark

**e.** Solve the differential equation given in part **d.** to find *x* in terms of *t*. 1 mark

**f.** The insecticide is deemed ineffective if the concentration level in the tank drops

 below 0.05 kg/L.

 How long, correct to the nearest minute, will it take for the insecticide level to be

 ineffective? 1 mark

Fearing the tank on the bank of the lake will not do enough to combat the population of mosquitoes, a second 10 000 litre tank is set up on the other side of the lake.

It initially contains *Q* kg of insecticide. Pure water is pumped into the tank at a rate of

*Y* L/min. The solution is kept uniform by stirring and is pumped out of the tank and into the lake at a rate of *Z* L/min, where .

**g.** If *x* kilograms is the amount of insecticide in the tank after *t* minutes, construct a

 differential equation representing the situation in this second tank, and

 rearrange to express in the form . 2 marks

**Question 4** (11 marks)

A CCTV camera moves along the horizontal top of a perimeter wall of a prison farm.

The position of the camera at time *t,* relative to a control centre at *O*, is given by

 where  is a unit vector in the direction east of the control centre,  is a unit vector in the direction north of the control centre and .

Displacement components are measured in kilometres and time is measured in hours.

**a**. Show thatthe Cartesianequation of the path of the camera is given by . 2 marks

**b.** Sketch the path of the camera on the set of axes below. Label the starting and

 finishing position of the camera with their coordinates and use an arrow to show

 the direction of motion of the camera. 2 marks



**c.** Find the distance that the camera travels during its first hour of motion. Give your

 answer in kilometres, correct to two decimal places. 2 marks

**d.** Find the maximum speed of the camera during its motion. Give your answer in

 kilometres per hour, correct to two decimal places. 3 marks

A drone flies at the same height as the top of the perimeter wall of the prison farm.

 The drone’s path in relation to the control centre is given by

, where  is a unit vector in the direction east of the control centre,  is a unit vector in the direction north of the control centre and  corresponds to the time when the camera started its motion.

Displacement components are measured in kilometres and time is measured in hours.

**e.** Determine whether the drone will collide with the camera. Give reasons for your

 answer. 2 marks

**Question 5** (10 marks)

A 12 kilogram box is placed on a smooth plane inclined at an angle of  to the horizontal ground as shown below.



**a.** Label all the forces acting on the box on the diagram above. 1 mark

**b.** Given the box is initially stationary, find the time taken for it to slide 15 metres down the plane. Give you answer in seconds correct to three decimal places. 2 marks

The box is placed back on the plane and attached to a rope with a tension of 50 newtons, acting as shown below.



**c.** Find the angle , between the rope and the plane, required to keep the box stationary. Give your answer correct to the nearest degree. 2 marks

**d.** If the maximum value that  can take is  and the minimum value that it can take is , find the possible values for *T*, the tension of the rope, in order to keep the box stationary. Give values in newtons, correct to one decimal place. 2 marks

The angle  and the pulling force exerted by the rope are fixed so that the box moves up the plane. The total length of the plane is 25 metres.



When the box reaches the top of the plane its speed is .

At this point, the rope breaks and the box leaves the top of the plane. Assume negligible air resistance on the box, which is subject only to the force of gravity.

**e.** Find the time that it takes the box to hit the ground after it has left the top of the plane. Give your answer in seconds, correct to two decimal places. 2 marks

**f.** How far does the box travel in the horizontal direction between leaving the top of the plane and hitting the ground? Give your answer in metres, correct to two decimal

 places. 1 mark

**Question 6** (9 marks)

The time taken to drive through a large country town in Victoria is known to be normally distributed with a mean time of 26 minutes and a standard deviation of 8 minutes.

Determined to **shorten** the time, the local council puts various measures in place. They then take a sample of 20 cars and find the mean time to be 23 minutes.

1. Write down suitable null and alternative hypotheses, and  respectively, to test

whether the mean time taken to drive through the town has reduced. 1 mark

1. Write down an expression for the *p*-value of the statistical test and evaluate it correct to four decimal places. 2 marks
2. State with a reason whether the null hypothesis should be rejected at the 5% level of significance. 1 mark

The local council wants stronger evidence that the measures put in place have reduced the travel time through their town, so they take a second sample of 20 cars and test at the 1% level of significance.

1. Determine the smallest value of the mean time for this sample that could be observed for **not** to be rejected. Give your answer correct to two decimal places. 2 marks

**e.** The council takes two new independent random samples of 20 cars.

 Find the probability that the means of the two samples taken differ by more than two

 minutes. Give your answer correct to two decimal places. 3 marks

# Specialist Mathematics Formulas

## Mensuration

|  |  |
| --- | --- |
| area of a trapezium |  |
| curved surface area of a cylinder |  |
| volume of a cylinder |  |
| volume of a cone |  |
| volume of a pyramid |  |
| volume of a sphere |  |
| area of a triangle |  |
| sine rule |  |
| cosine rule |  |

## Circular functions

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## Circular functions – continued

|  |  |  |  |
| --- | --- | --- | --- |
| **Function** |  |  |  |
| **Domain** |  |  |  |
| **Range** |  |  |  |

**Algebra (complex numbers)**

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  (de Moivre’s theorem) |  |

**Probability and statistics**

|  |  |
| --- | --- |
| for random variables *X* and *Y* |  |
| for independent random variables *X* and *Y* |  |
| approximate confidence interval for  |  |
| distribution of sample mean  |  |

**Calculus**

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|  |  |
|  |  |
|  |  |
| product rule |  |
| quotient rule |  |
| chain rule |  |
| Euler’s method |  |
| acceleration |  |
| arc length |  |

## Vectors in two and three dimensions Mechanics

|  |  |
| --- | --- |
| momentum |  |
| equation of motion |  |

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