The Mathematical Association of Victoria

Trial Exam 2021

SPECIALIST MATHEMATICS

Written Examination 1

STUDENT NAME _____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of Book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers,
- Students are NOT permitted to bring into the examination room: any technology (calculators or software) notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 15 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the space provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2} where $g = 9.8$.

Question 1 (2 marks)

Consider the curve defined by the relation $axy^3 + bx^3y = 1$ where a and b are real constants.
Find the values of a and b if the gradient of the curve at the point $(1, 1)$ is equal to -1.

TURN OVER

Question 2 (6 marks)

A 20 kg parcel sits on a set of bathroom scales in a stationary lift. The lift begins moving upwards from rest with an acceleration $\frac{1}{3}\sqrt{25-v^2}$ where v ms^{-1} is its speed t seconds after it starts moving.

a. State the maximum speed of the lift. 1 mark

Once the lift reaches its maximum speed the acceleration changes to $-\frac{1}{3}\sqrt{10v-v^2}$ ms^{-2} and the lift slows down.

b. Find, in terms of g and in units of Newton, the magnitude of the force exerted on the scales by the parcel when the lift is slowing down and has a speed of $2\,\text{ms}^{-1}$. 2 marks

c. Find, in terms of g and in units of Newton, the magnitude of the force exerted on the scales by the parcel $\frac{\pi}{2}$ seconds after the lift starts slowing down. 3 marks

Question 3 (3 marks)

Let $z^4 + az^3 + bz^2 - 4z + 6 = 0$, $z \in C$, where a and b are real constants.

Given that $z = i$ is a solution to the equation, find all other solutions.

Question 4 (3 marks)

Bottles of the sports drink *Revitalise* have a volume that varies normally with a mean of 750 ml and a standard deviation of 15 ml. *Revitalise* is sold in packs of four bottles.

Find the approximate probability that the volume of a randomly selected four-bottle pack of *Revitalise* is between 2940 ml and 3000 ml. Give your answer correct to three decimal places.

TURN OVER

Question 5 (5 marks)

Let the function $f(x) = \dfrac{1}{2+e^{-x}}$.

a. Find the equations of all asymptotes of the function. 2 marks

b. Prove that the function has a point of inflection and find the x-coordinate of that point. 3 marks

Question 6 (3 marks)

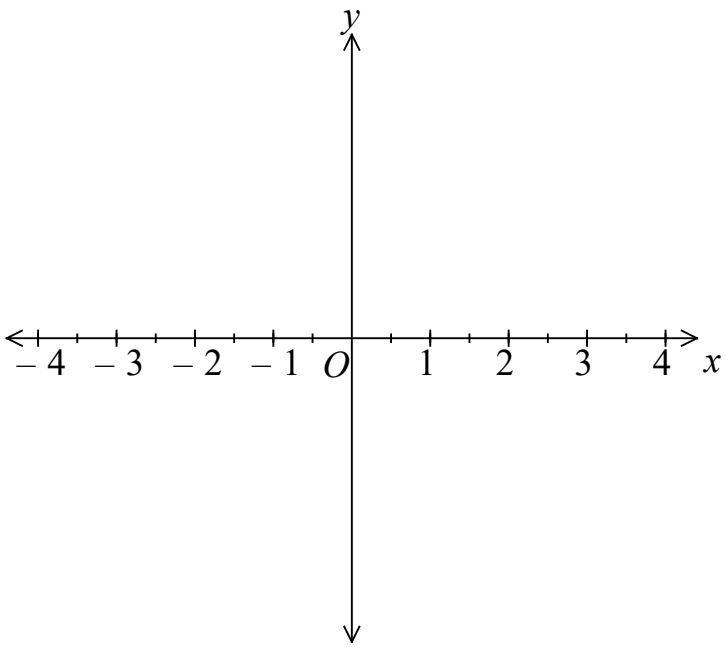
Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan^4(x)\, dx$.

TURN OVER

Question 7 (3 marks)

a. Let the function $f:(-3,\ 3) \to R,\ f(x) = \text{cosec}\left(\frac{\pi x}{3}\right) + 1$.

Sketch the graph of $y = f(x)$ on the axes provided below, labelling any intercepts and endpoints with their coordinates and any asymptotes with their equation. 1 mark



Working space

b. Solve the equation $f(x) = \dfrac{|x+2|}{x+2}$ for x. 2 marks

Question 8 (8 marks)

The position vectors of two particles A and B at time t seconds after they have started moving are given by $\underset{\sim}{r}_A(t) = 3t\,\underset{\sim}{\text{i}} + (t-4)\,\underset{\sim}{\text{j}}$ and $\underset{\sim}{r}_B(t) = \big(6 - 6\cos(\alpha t)\big)\,\underset{\sim}{\text{i}} + 6\sin(\alpha t)\,\underset{\sim}{\text{j}}$ respectively, where α is a positive real constant and $t \geq 0$.

a. Find the times at which it is possible for a collision between the particles to occur. 3 marks

b. Find the values of α for which the particles will collide at the largest time found in **part a.** 2 marks

c. Find the values of $\tan(\alpha)$ if the particles are moving in directions perpendicular to each other at time $t = 2$. 3 marks

TURN OVER

Question 9 (4 marks)

By making the substitution $u = \sqrt{1 + e^{2x}}$ in an appropriate integral, find the arc length of the curve $y = e^x$ from $x = 0$ to $x = \log_e \sqrt{3}$.

Give your answer in the form $\left(a - \sqrt{a}\right) - \frac{1}{a}\log_e\left(b - 6\sqrt{a}\right)$ where *a* and *b* are positive integers.

Question 10 (3 marks)

Let $\sec(\beta) = b$ where $\beta \in \left(\frac{\pi}{2}, \pi\right]$.

Solve $\operatorname{cosec}(2x) = -b$ in terms of β for $x \in [-\pi, \pi]$.

END OF QUESTION AND ANSWER BOOK