

Trial Examination 2021

VCE Specialist Mathematics Units 1&2

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes

Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of booklet

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
7	7	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 8 pages

Formula sheet

Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1 (5 marks)

- a. Solve $2|x - 5| = 10$ for x . 1 mark

- b. Solve $3x - |x| = -4$ for x . 2 marks

- c. Solve $|x^2 - 4x + 2| \geq 2$ for x . 2 marks

Question 2 (8 marks)

a. A sequence is defined by the rule $t_n = t_{n-1} + 5$, $t_2 = 4$.

i. Find the first four terms of the sequence. 1 mark

ii. Find the sum of the first 10 terms of the sequence. 1 mark

b. The third term of an arithmetic sequence is 6 and the sixth term is 15.

Find the 13th term. 2 marks

c. The numbers x , $x - 6$ and $2x - 7$ form the first three terms of a geometric sequence, where $x > 0$.

i. Find the value of x . 2 marks

ii. Find the sum of the infinite geometric sequence. 2 marks

Question 3 (6 marks)

- a. i.** Give the locus of points $P(x, y)$ such that P is a fixed distance of 4 units from the point $(-1, 5)$. 1 mark

- ii.** Express your answer to **part a.i.** using polar coordinates in the form $P(r \cos \theta + h, r \sin \theta + k)$. 1 mark

- b.** Consider the points $A(1, 0)$ and $B(-1, 0)$.

Find the equation of the locus of points $P(x, y)$ satisfying $AP = 4 - BP$. Give your

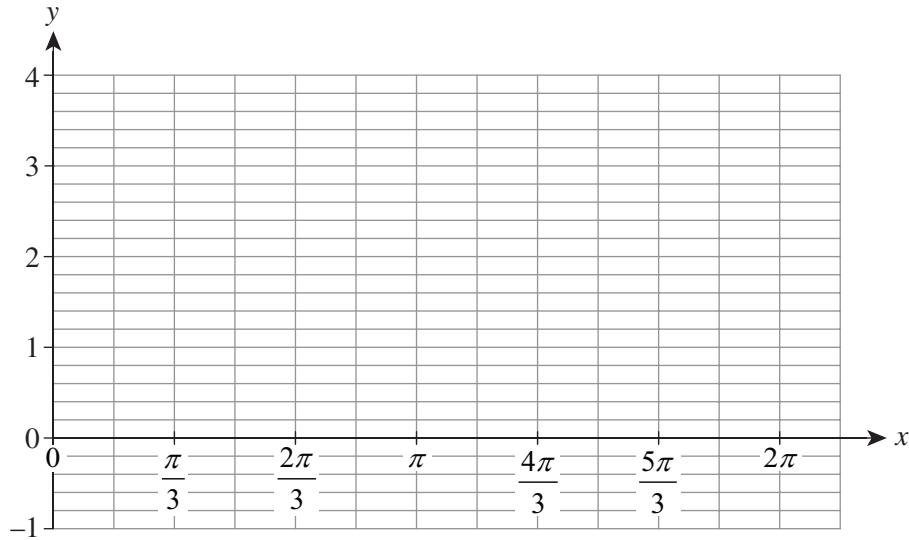
answer in the form $\frac{x^2}{m} + \frac{y^2}{n} = 1$, where $m, n \in Z^+$. 4 marks

Question 4 (4 marks)

a. Let $f : [0, 2\pi] \rightarrow \mathbb{R}$, $f(x) = \frac{3}{2}\cos(x) + 2$.

Sketch the graph of $y = f(x)$ on the axes below. Label the end points and any local minima or maxima with their coordinates.

2 marks



b. If $g(x) = \frac{1}{f(x)}$, sketch the graph of $y = g(x)$ on the axes in **part a**. Label the end points and any local minima or maxima with their coordinates.

2 marks

Question 5 (6 marks)

- a. Given that $2^{x+1} = 20$, prove that x is irrational. 3 marks

- b. Prove by mathematical induction that $11^n - 5^n$ is divisible by 6 for $n \in \mathbb{N}$. 3 marks

Question 6 (6 marks)

The points A and B lie on a cartesian plane such that $\overrightarrow{OA} = \sqrt{3}\underline{i} + 3\underline{j}$ and $\overrightarrow{OB} = m\underline{i} + n\underline{j}$, where $m, n \in R$.

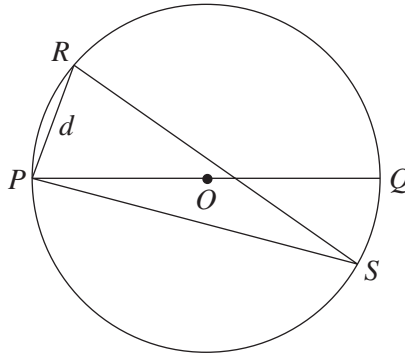
- a.** Find the ratio $\frac{m}{n}$ if $\angle AOB = 90^\circ$. 2 marks

- b. i.** If $m = 0$ and $|\overrightarrow{OA}| = |\overrightarrow{OB}|$, find the possible values for n . 2 marks

- ii.** Hence, find the area of the triangle OAB . 2 marks

Question 7 (5 marks)

The circle centred at O with a radius of r has the diameter PQ . The points R and S lie on the circle as shown in the diagram below. The chord PR is of length d .



- a.** Show that $d = 2r \sin(\angle RSP)$. 3 marks

- b.** Use vectors to show that, if RS passes through O , then the chord PS must be parallel to and equal in length to the chord RQ . 2 marks

END OF QUESTION AND ANSWER BOOKLET

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VCE Specialist Mathematics Units 1&2

Written Examinations 1 & 2

Formula Sheet

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SPECIALIST MATHEMATICS FORMULAS**Mensuration**

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc \sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab \cos(C)$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	

Vectors in two dimensions

$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$
$ \underline{r} = \sqrt{x^2 + y^2 + z^2} = r$
$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

Polar coordinates

$x = r \cos \theta$
$y = r \sin \theta$

END OF FORMULA SHEET