Neap

Trial Examination 2021

VCE Specialist Mathematics Units 1&2

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 19 pages

Formula sheet

Answer sheet for multiple-choice questions

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on the answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1

Given $\underset{\sim}{a} = 2\underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{b} = \underset{\sim}{i} - 3\underset{\sim}{j}$, then $2\underset{\sim}{b} - \underset{\sim}{a}$ is equal to

A. $3\underset{\sim}{i} + \underset{\sim}{j}$
B. $-5\underset{\sim}{j}$
C. $-\underset{\sim}{i} - 2\underset{\sim}{j}$
D. $-2\underset{\sim}{i} - 4\underset{\sim}{j}$
E. $4\underset{\sim}{i} - 5\underset{\sim}{j}$

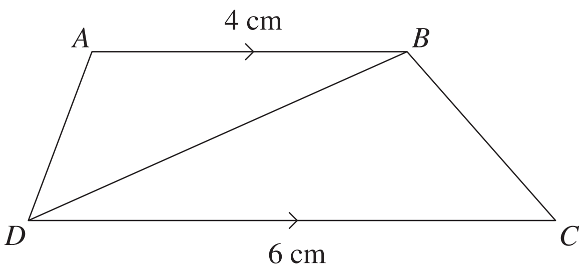
Question 2

If a geometric sequence has $t_2 = 6$ and $t_6 = 24$, then t_4 is equal to

A. 4
B. $6\sqrt{2}$
C. 12
D. 15
E. 144

Question 3

In the trapezium *ABCD*, *AB* is parallel to *DC* and the area of the triangle *ABD* is 24 cm^2, as shown in the diagram below.



The area of the trapezium *ABCD* in cm^2 is

A. 24
B. 36
C. 48
D. 60
E. 72

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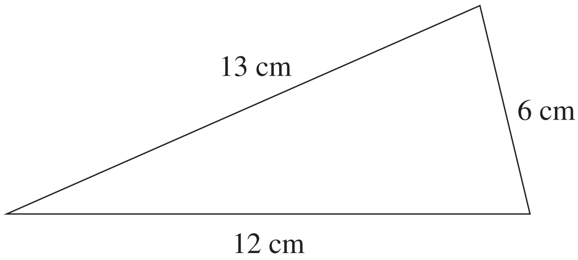
Question 4

Consider the triangle shown below.



The sum of the size of the two smallest angles in the triangle, to the nearest degree, is

A. 67°

B. 86°

C. 90°

D. 94°

E. 113°

Question 5

The graph of $y = \dfrac{x}{x^2 + 3x + 2}$ has a domain of

A. R

B. $R+$

C. $R \setminus \{1, 2\}$

D. $R \setminus \{-2, -1\}$

E. $R \setminus \{-2, -1, 0\}$

Question 6

If $\underset{\sim}{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\underset{\sim}{b} = \begin{bmatrix} 2 \\ m \end{bmatrix}$ and $\underset{\sim}{a} + \underset{\sim}{b}$ is parallel to $6\underset{\sim}{i} - \underset{\sim}{j}$, the value of m is equal to

A. $-\dfrac{5}{2}$

B. 4

C. $\dfrac{3}{2}$

D. –3

E. 6

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Question 7

If $z = 5 + 12i$, then $\text{Re}(z^2)$ is equal to

A. -119

B. 13

C. 25

D. $5 - 12i$

E. 144

Question 8

The vector $2(m\underset{\sim}{i} + m\underset{\sim}{j})$, where $m \in R^+$, is a unit vector if m is equal to

A. 0

B. $\frac{1}{4}$

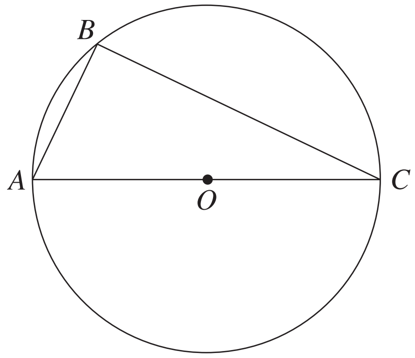
C. $\frac{\sqrt{2}}{4}$

D. $\frac{1}{2}$

E. 1

Question 9

AOC is the diameter of a circle with centre *O*, as shown in the diagram below. *B* is a point on the circumference of the circle.



Let $\overrightarrow{OA} = \underset{\sim}{r}$ and $\overrightarrow{CB} = \underset{\sim}{q}$.

Which one of the following is true?

A. $2\underset{\sim}{r} = -\underset{\sim}{q}$

B. $\underset{\sim}{q} = 2\underset{\sim}{r}$

C. $\underset{\sim}{q}.\underset{\sim}{q} = \underset{\sim}{r}.\underset{\sim}{q}$

D. $\underset{\sim}{q}.\underset{\sim}{q} = 2\underset{\sim}{r}.\underset{\sim}{q}$

E. $\underset{\sim}{q}.\underset{\sim}{q} = -2\underset{\sim}{r}.\underset{\sim}{q}$

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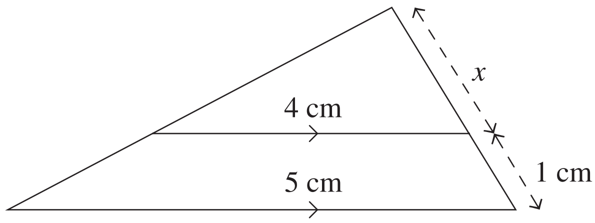
Question 10

Consider the triangle shown below.



The value of x is equal to

A. 0.8 cm

B. 4 cm

C. 4.5 cm

D. 5 cm

E. 10 cm

Question 11

The graph of $y = \frac{1}{2\sin(x)} + k$ will have **no** x-intercepts for

A. $k = R$

B. $k \in [-2, 2]$

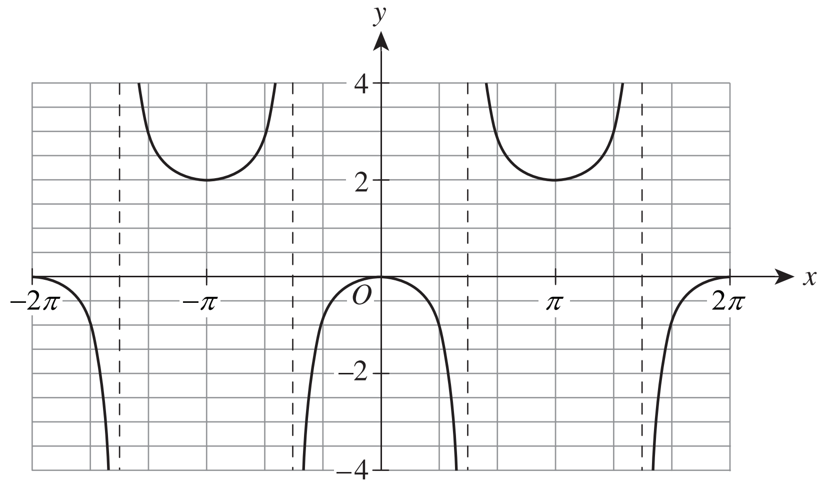
C. $k \in (-2, 2)$

D. $k \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

E. $k \in \left(-\frac{1}{2}, \frac{1}{2}\right)$

Question 12

Consider the graph shown below.



The graph has the equation

A. $y = 1 + \dfrac{1}{\cos(x)}$

B. $y = 1 - \dfrac{1}{\cos(x)}$

C. $y = 1 - \dfrac{2}{\cos(x)}$

D. $y = 1 + \dfrac{1}{\sin(x)}$

E. $y = 1 + \dfrac{2}{\sin(x)}$

Question 13

The sum of the first eight terms of the series $2, 2(1.1), 2(1.1)^2, \ldots$ is equal to

A. $2\left(1.1^8 - 1\right)$

B. $20\left(1.1^8 - 1\right)$

C. $20\left(11^8 - 1\right)$

D. $2\left(1.1^7 - 1\right)$

E. 22

Question 14

If $\bar{z} = 3 - 2i$, then $\text{Im}(z)$ is equal to

A. –3

B. –2

C. 2

D. $-2i$

E. $2i$

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Question 15

The equation $z^2 - 2kiz - 3k = 0$ has exactly one complex solution when k is equal to

A. $3i$

B. 0

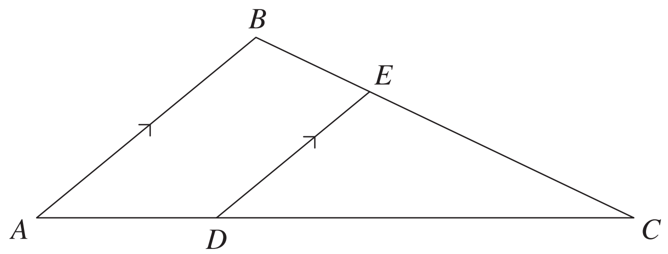
C. 0 or 3

D. 3

E. –3

Question 16

In the diagram below, *AB* and *DE* are parallel lines. The ratio $BE : EC = 2 : 3$.



The ratio of the area of ΔABC to the area of ΔDEC is

A. $\frac{3}{2}$

B. $\frac{9}{4}$

C. $\frac{5}{2}$

D. $\frac{25}{9}$

E. $\frac{25}{4}$

Question 17

If the vectors defined by $\underset{\sim}{a} = \begin{bmatrix} m \\ 1-n \end{bmatrix}$ and $\underset{\sim}{b} = \begin{bmatrix} m \\ n \end{bmatrix}$ are perpendicular, then which one of the following is **not** true?

A. $m = 0$

B. $n = 0$

C. $0 < n < 1$

D. $0 < m < 1$

E. $m \in R$

Question 18

The locus of points (x, y) that are equidistant from the points with coordinates of (1, 2) and (0, –3) is

A. $y = 5x - 3$

B. $y = -x$

C. $y = -\frac{1}{2}x - \frac{1}{2}$

D. $y = -\frac{1}{5}x - \frac{2}{5}$

E. $y = 2x - 3$

Question 19

The ellipse given by $2x^2 + 4ax + y^2 - 2by + c = 0$ has a centre at (2, 1).

The value of c is

A. –1

B. 1

C. 5

D. 8

E. 12

Question 20

The angle between $\underset{\sim}{a} = \sqrt{3}\underset{\sim}{i} + \sqrt{2}\underset{\sim}{j}$ and $\underset{\sim}{b} = \sqrt{2}\underset{\sim}{i} + \sqrt{3}\underset{\sim}{j}$ is θ.

The value of $\tan(\theta)$ is

A. $\frac{1}{24}$

B. $\frac{1}{5}$

C. $\frac{\sqrt{6}}{12}$

D. $\frac{24}{25}$

E. $\frac{2\sqrt{6}}{5}$

END OF SECTION A

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SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

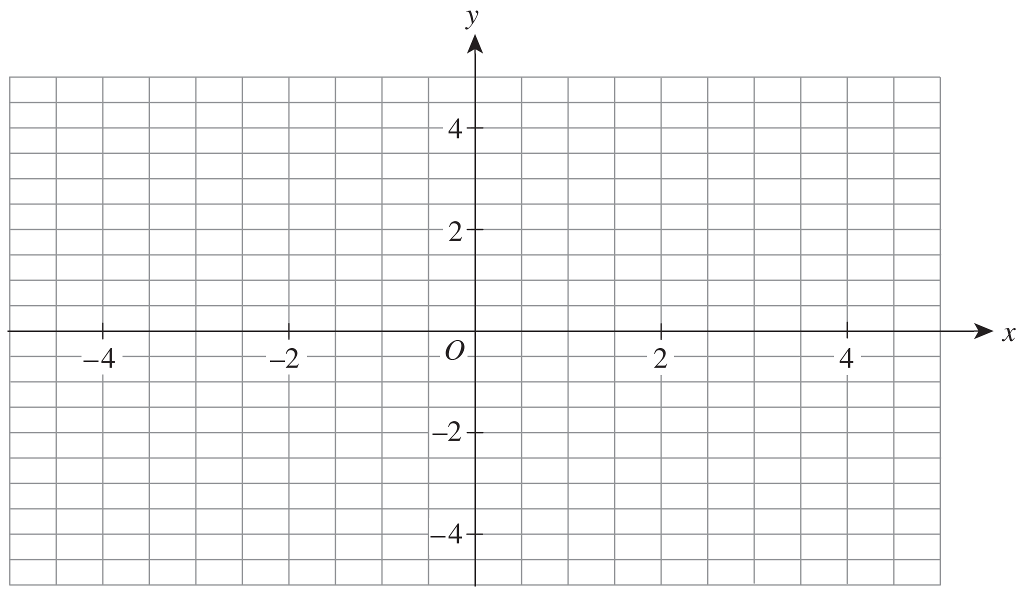
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 (5 marks)

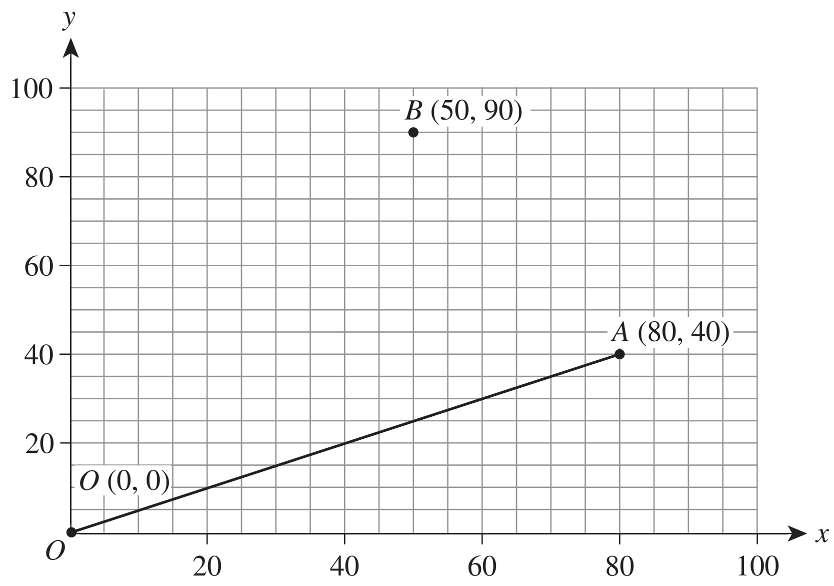
a. Find the cartesian equation for the curve defined by the parametric equations $x = -2\cos(t)$ and $y = 3\sin(t)$, where $t \in \left[0, \frac{3\pi}{2}\right]$. 2 marks

b. Sketch the graph of the relation found in **part a.** on the axes provided below.
Label any axes intercepts and endpoints with their coordinates. 3 marks



Question 2 (13 marks)

The diagram below shows the distance between the main office of a school located at point O and the displacement, measured in metres, to two school buildings at points A and B. A straight line footpath connects the main office to building A and can be expressed as the vector $\overrightarrow{OA} = 80\underset{\sim}{i} + 40\underset{\sim}{j}$.



a. **i.** Find the vector $\overrightarrow{AB}$ in terms of $\underset{\sim}{i}$ and $\underset{\sim}{j}$. 1 mark

ii. Find the distance between A and B. 1 mark

b. Find $\overrightarrow{AB}.\overrightarrow{OA}$ and, hence, find the angle $\angle OAB$. Give your answer correct to the nearest degree. 3 marks

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c. A student is standing at the point $C(60, 30)$.

Find the vectors $\overrightarrow{OC}$ and $\overrightarrow{CA}$ and, hence, state whether the student is standing on the footpath. 2 marks

The school wishes to build a footpath that connects the existing footpath on $\overrightarrow{OA}$ to building B. The new footpath will connect to the existing footpath at point $P(2p, p)$.

d. Find the vector $\overrightarrow{BP}$ in terms of p. 1 mark

e. **i.** Find the coordinates of point P so that the new footpath will have the minimum possible distance. 3 marks

ii. Find the distance a student must walk from the main office to building B via the new footpath. 2 marks

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Question 3 (9 marks)

An animal wildlife sanctuary has land reserved for a kangaroo habitat for an endangered species of kangaroo. Let L_n be the area of the kangaroo habitat, in square kilometres, at the end of year n.

As the population of kangaroos changes, the area of land required also changes. This can be modelled by the following difference equation.

$$L_{n+1} = 0.85L_n + 400 \quad L_{2021} = 1200$$

a. Find the required area of the kangaroo habitat at the end of the year 2022. 1 mark

Let K_n be the number of kangaroos at the end of year n. The kangaroo habitat contains a population of kangaroos, where

$$K_{n+1} = 1.5K_n \quad K_{2021} = 4000.$$

The kangaroo habitat will be overpopulated if there are more than four kangaroos per square kilometre.

b. Show that the kangaroo habitat is expected to be overpopulated in 2022. 2 marks

c. Find the maximum kangaroo population the kangaroo habitat could sustain to avoid overpopulation. 2 marks

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To prevent the kangaroo habitat from becoming overpopulated, a number of kangaroos are moved to other wildlife sanctuaries at the end of each year.

Let P_n be the number of kangaroos in the kangaroo habitat at the end of year n.

Let k be the number of kangaroos that are moved to other wildlife sanctuaries at the end of each year.

The change in the number of kangaroos, from year to year, can be modelled by the following difference equation.

$$P_{n+1} = 1.5P_n - k \quad \mathrm{P}_{2021} = 4000$$

d. How many kangaroos must be moved to other wildlife sanctuaries at the end of each year if the number of kangaroos is to remain constant? 1 mark

e. What is the minimum number of kangaroos that must be moved to other wildlife sanctuaries at the end of each year to ensure that the kangaroo habitat is not overpopulated in 2023? 3 marks

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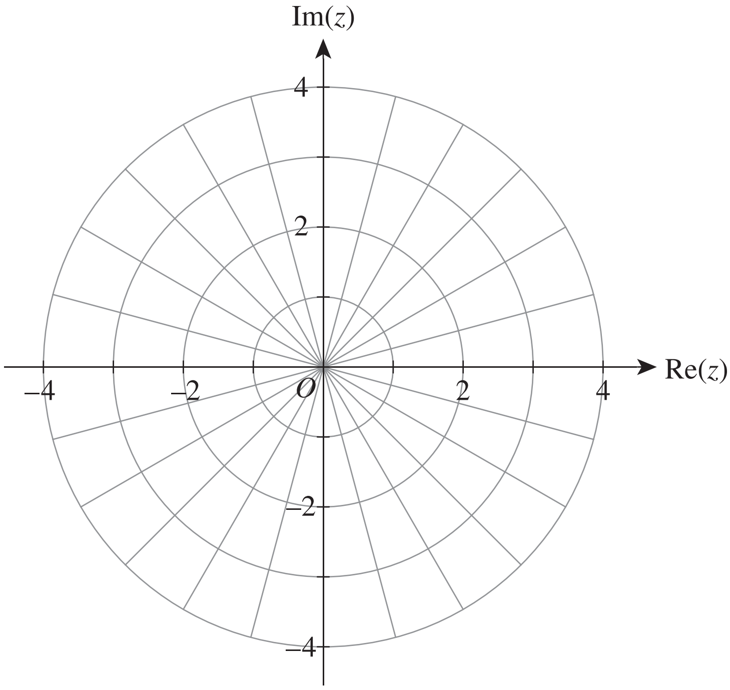
Question 4 (18 marks)

On an Argand diagram, points *O*, *A*, *B* and *C* form a square. The points *A*, *B* and *C* are represented by the complex numbers *u*, *v* and *w* respectively.

a. Given that $u = 2 + 2i$, plot the point *u* on the Argand diagram below. 1 mark



b. **i.** If *u* and *v* are solutions to a quadratic equation of the form $z^2 + bz + c = 0$, where $b, c \in R$, find *v* in cartesian form. 1 mark

ii. Plot *v* on the Argand diagram in **part a.** 1 mark

iii. Find the values of *b* and *c*. 2 marks

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c. **i.** Show that $u = iv$. 1 mark

ii. Consider $u = i^n v$.
State the values of n. 1 mark

d. Find u and v in polar form. 2 marks

e. **i.** Find $|u - v|$. 1 mark

ii. With respect to the square *OACB*, what does $|u - v|$ represent? 1 mark

iii. Hence, find w. Give your answer in cartesian and polar forms. 2 marks

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f. The points O, A, B and C represent the solutions to the quartic equation

$$z^4 + qz^3 + rz^2 + sz = 0.$$

Find the values of q, r, and s. 2 marks

g. Another quartic equation has two real solutions of $z = 0$ and $z = k$, as well as two other complex solutions of the form $z = \alpha \pm \beta i$, where $\alpha, \beta \neq 0$. The four solutions again form the points of a square.

Show that the quartic equation is of the form $z^4 - 2kz^3 + \frac{3k^2}{2}z^2 - \frac{k^3}{2}z = 0.$ 3 marks

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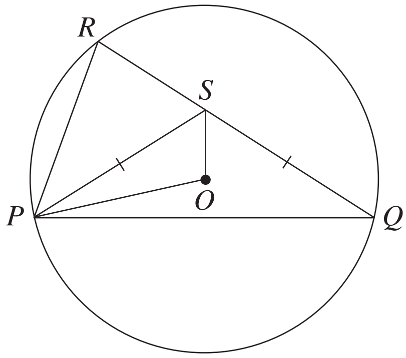
Question 5 (7 marks)

The circle shown below has a centre O. The point S lies on the line RQ. The length of PS is equal to the length of SQ.



a. Prove that $PRSO$ is a cyclic quadrilateral. 4 marks

b. Given $\angle SRP = 60°$ and $OS = \frac{1}{2}OP$, find the ratio of the area of the triangle OSP to the area of the circle. Express your answer in the form $1 : k\pi$, where $k \in Z^+$. 3 marks

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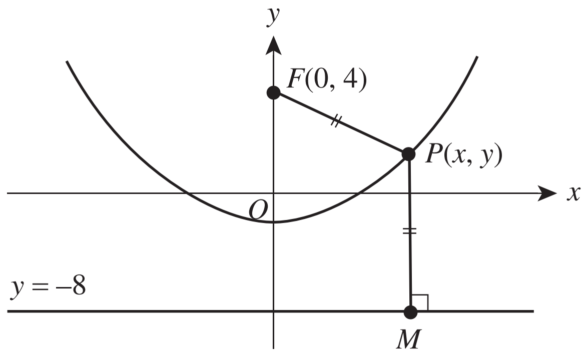
Question 6 (8 marks)

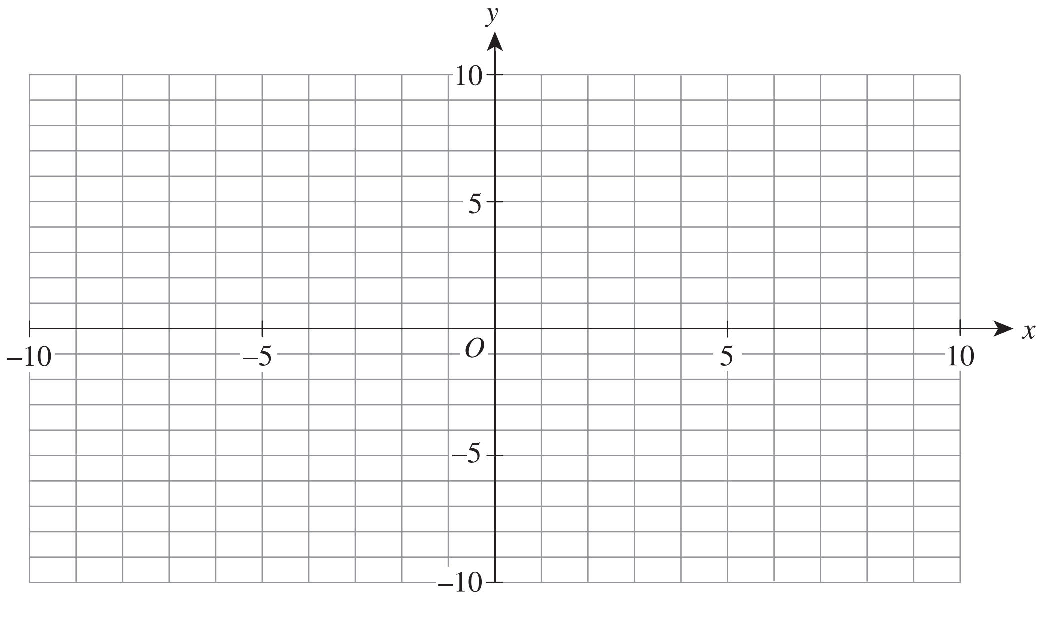
The point M lies on the line $y = -8$. The point $P(x, y)$ is directly above the point M. The point F has coordinates of (0, 4), as shown below.



a. Show that the locus of point $P(x, y)$ when $FP = PM$ is given by the function $f(x) = \frac{x^2}{24} - 2$. 2 marks

b. The function g is defined by the rule $g(x) = \frac{1}{f(x)}$.

Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the axes below. Label any intercepts and intersection points with their coordinates and any asymptotes with their equations. 4 marks



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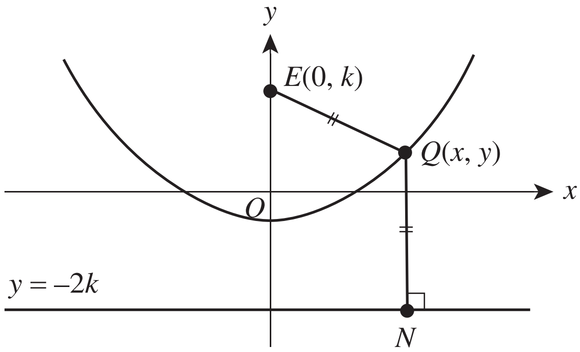
c. The point N lies on the line $y = -2k$. The point $Q(x, y)$ is directly above the point N.
The point E has coordinates of $(0, k)$, as shown below.



The locus of points $Q(x, y)$ when $EQ = QN$ is given by the function $h(x)$.

Find the maximum set of values of k for which the graphs of $y = h(x)$ and $y = \frac{1}{h(x)}$ have exactly two intersection points. 2 marks

END OF QUESTION AND ANSWER BOOKLET

Neap

Trial Examination 2021

VCE Specialist Mathematics Units 1&2

Written Examination 2

Multiple-choice Answer Sheet

Student's Name: ______________________________

Teacher's Name: ______________________________

Instructions

Use a **pencil** for **all** entries. If you make a mistake, **erase** the incorrect answer – **do not** cross it out.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than **one** answer is completed for any question.

All answers must be completed like this example: A B C D E

Use pencil only

1	A	B	C	D	E	11	A	B	C	D	E
2	A	B	C	D	E	12	A	B	C	D	E
3	A	B	C	D	E	13	A	B	C	D	E
4	A	B	C	D	E	14	A	B	C	D	E
5	A	B	C	D	E	15	A	B	C	D	E
6	A	B	C	D	E	16	A	B	C	D	E
7	A	B	C	D	E	17	A	B	C	D	E
8	A	B	C	D	E	18	A	B	C	D	E
9	A	B	C	D	E	19	A	B	C	D	E
10	A	B	C	D	E	20	A	B	C	D	E

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Neap

Trial Examination 2021

VCE Specialist Mathematics Units 1&2

Written Examinations 1 & 2

Formula Sheet

Instructions

This formula sheet is provided for your reference.
A question and answer booklet is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SPECIALIST MATHEMATICS FORMULAS

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule	$c^2=a^2+b^2-2ab\cos(C)$

Circular functions

$\cos^2(x)+\sin^2(x)=1$	
$1+\tan^2(x)=\sec^2(x)$	$\cot^2(x)+1=\operatorname{cosec}^2(x)$
$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$	
$\sin(2x)=2\sin(x)\cos(x)$	

Vectors in two dimensions

$\underset{\sim}{r}=x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k}$
$\lvert\underset{\sim}{r}\rvert=\sqrt{x^2+y^2+z^2}=r$
$\underset{\sim}{r}_1\cdot\underset{\sim}{r}_2=r_1r_2\cos(\theta)=x_1x_2+y_1y_2+z_1z_2$

Polar coordinates

$x=r\cos\theta$
$y=r\sin\theta$

END OF FORMULA SHEET

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