Neap

Trial Examination 2021

VCE Specialist Mathematics Units 3&4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of booklet

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 12 pages

Formula sheet

Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on the answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 (4 marks)

Relative to a fixed origin O, the point A has position vector $\underset{\sim}{a} = \underset{\sim}{i} + \underset{\sim}{j}$ and the point B has position vector $\underset{\sim}{b} = -5\underset{\sim}{k}$. The point C has position vector $\underset{\sim}{c}$ where $\underset{\sim}{c} = \underset{\sim}{a} + \underset{\sim}{b}$.

a. Express $\underset{\sim}{c}$ in terms of unit vectors $\underset{\sim}{i}$, $\underset{\sim}{j}$ and $\underset{\sim}{k}$. 1 mark

b. Prove that *OACB* is a rectangle. 3 marks

Question 2 (3 marks)

Find the set of real values of x for which $|x+2| > 2|x-2|$.

Question 3 (3 marks)

A particle of mass 3 kg moves under a force $\underset{\sim}{F}$ newtons so that its position vector $\underset{\sim}{r}$ at time *t* seconds is given by $\underset{\sim}{r} = \cos(\pi t)\underset{\sim}{i} + \sin(2\pi t)\underset{\sim}{j},\ t \geq 0$.

Find the particle's change in momentum from $t = \frac{1}{2}$ to $t = 1$.

Question 4 (4 marks)

Consider two independent random variables X and Y. The mean of X and the mean of Y are both –2. The variance of X is 1 and the variance of Y is 2.

If the mean and the variance of $aX + bY$ are 2 and 9 respectively, find the value of a and of b, where $a, b \in Z$.

Question 5 (4 marks)

A curve C has the equation $ye^{-x} = x + y^2$.

a. Find $\frac{dy}{dx}$ in terms of x and y. 2 marks

b. Point P lies on curve C and has coordinates (0, 1).

Find the equation of the normal to curve C at point P. 2 marks

Question 6 (4 marks)

a. Show that the equation $\operatorname{cosec}(x)+\cot(x)=2\sin(x)$ can be expressed as $2\cos^2(x)+\cos(x)-1=0$. 2 marks

b. Hence, solve the equation $\operatorname{cosec}(x)+\cot(x)=2\sin(x)$ for $0<x<\pi$. 2 marks

Question 7 (4 marks)

a. Express $\dfrac{3}{(x+1)(x-2)}$ in partial fractions. 2 marks

b. Hence, by solving the differential equation $\dfrac{dx}{dt} = t^2(x+1)(x-2)$, where $x > 2$, show that $\dfrac{x-2}{x+1} = ke^{t^3}$, where k is a constant. 2 marks

Question 8 (3 marks)

A curve C has equation $y = \frac{1}{6}\left(e^{3x} + e^{-3x}\right), 0 \le x \le \log_e(k),$ where $k > 1$.

Show that the length of curve C is $\frac{1}{6}\left(k^3 - \frac{1}{k^3}\right)$.

Question 9 (5 marks)

The functions f and g are defined by

$$f(x) = \arctan\left(\frac{x+1}{1-x}\right)$$

$$g(x) = \arctan(x)$$

where $x < 1$.

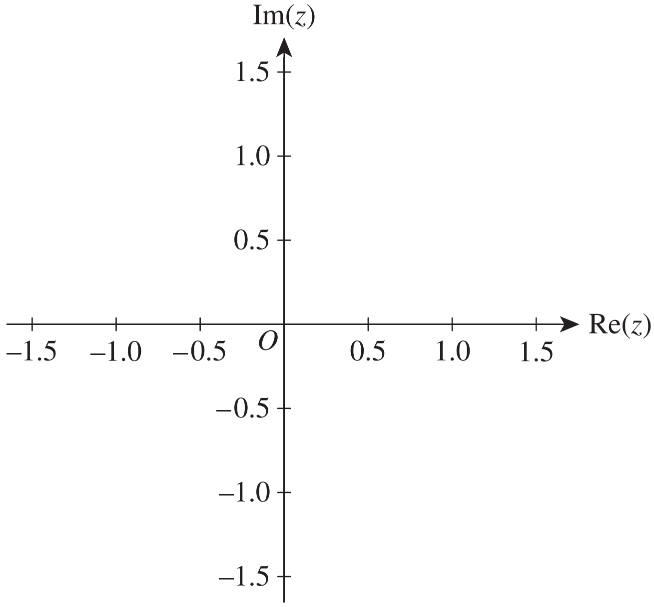
a. Show that $f'(x) = g'(x)$. 3 marks

b. By using an appropriate trigonometric identity, verify the result obtained in **part a.** 2 marks

Question 10 (6 marks)

The roots of the equation $z^3 - 1 = 0$, where $z \in C$, are denoted by 1, w and w^2.

a. Plot the points that represent these roots on the Argand diagram below. 1 mark



b. Show that $1 + w + w^2 = 0$. 2 marks

c. Find a cubic equation with integer coefficients that has roots 1, $1+w$ and $1+w^2$. 3 marks

END OF QUESTION AND ANSWER BOOKLET

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VCE Specialist Mathematics Units 3&4

Written Examinations 1 & 2

Formula Sheet

Instructions

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SPECIALIST MATHEMATICS FORMULAS

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule	$c^2=a^2+b^2-2ab\cos(C)$

Circular functions

$\cos^2(x)+\sin^2(x)=1$	
$1+\tan^2(x)=\sec^2(x)$	$\cot^2(x)+1=\operatorname{cosec}^2(x)$
$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$	$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$
$\cos(x+y)=\cos(x)\sin(y)-\sin(x)\cos(y)$	$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$
$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$	
$\sin(2x)=2\sin(x)\cos(x)$	$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

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Algebra (complex numbers)

$z = x + iy = r\left(\cos(\theta) + i\sin(\theta)\right) = r\text{cis}(\theta)$	
$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \text{Arg}(z) < \pi$
$z_1 z_2 = r_1 r_2 \text{cis}\left(\theta_1 + \theta_2\right)$	$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\text{cis}\left(\theta_1 - \theta_2\right)$
$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)	

Calculus

$\dfrac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \dfrac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\dfrac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \dfrac{1}{a}e^{ax} + c$
$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$	$\int \dfrac{1}{x} dx = \log_e \lvert x\rvert + c$
$\dfrac{d}{dx}(\sin(ax)) = a\cos(ax)$	$\int \sin(ax) dx = -\dfrac{1}{a}\cos(-ax) + c$
$\dfrac{d}{dx}(\cos(ax)) = -a\sin(ax)$	$\int \cos(ax) dx = \dfrac{1}{a}\sin(ax) + c$
$\dfrac{d}{dx}(\tan(ax)) = a\sec^2(ax)$	$\dfrac{d}{dx}(\tan(ax)) = a\sec^2(ax)$
$\dfrac{d}{dx}(\sin^{-1}(x)) = \dfrac{1}{\sqrt{1-x^2}}$	$\int \dfrac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c,\ a > 0$
$\dfrac{d}{dx}(\cos^{-1}(x)) = \dfrac{-1}{\sqrt{1-x^2}}$	$\int \dfrac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\dfrac{x}{a}\right) + c,\ a > 0$
$\dfrac{d}{dx}(\tan^{-1}(x)) = \dfrac{1}{1+x^2}$	$\int \dfrac{a}{a^2 + x^2} dx = \tan^{-1}\left(\dfrac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \dfrac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$
	$\int (ax+b)^{-1} dx = \dfrac{1}{a}\log_e \lvert ax+b\rvert + c$
product rule	$\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
quotient rule	$\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
chain rule	$\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Calculus – continued

Euler's method	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$.
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2} \sqrt{1 + (f'(x))^2}\, dx$ or $\int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$
$\lvert\underset{\sim}{r}\rvert = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$

Mechanics

momentum	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R} = m\underset{\sim}{a}$

END OF FORMULA SHEET