2021 VCE Specialist Mathematics 1 external assessment report

General comments

The 2021 VCE Specialist Mathematics examination 1 comprised nine questions worth a total of 40 marks. Students were not permitted to bring technology or notes into the examination.

Among the highest scoring questions were Questions 1a., 3a., 8a. and 9ai. Questions in which comparatively few students were able to score full marks were Questions 7b. and 8b. and both parts of Question 9c.

Students are reminded to read questions carefully. For example, in Question 4b., many students neglected to give the answer in the required form. In Question 7b., a number of students failed to fully answer the question.

In 'show that' questions, the onus is on the student to demonstrate their full understanding by clearly showing all appropriate steps leading to the given result. This is especially important for questions that are worth one mark only.

Areas of strength included:

- integration (Questions 2, 4 and 7a.)
- implicit differentiation (Question 5)
- understanding the notion of dependence of a set of vectors (Question 6)
- working with parametric equations (Questions 9ai. and 9bi.).

Areas of weakness included:

- poor setting out, often resulting in avoidable errors and incorrect mathematical protocols and statements
- basic arithmetic and algebraic errors
- poor understanding of hypothesis testing and confidence intervals.

Specific information

Note: This report provides sample answers, or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Question 1a.

Mark	0	1	Average
%	20	80	0.8

$$\underset{\sim}{a} = \frac{1}{2}\underset{\sim}{i} + \frac{6}{5}\underset{\sim}{j}$$

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This question was answered well, with students recognising that they needed to use the formula $F = ma$. Some students found the magnitude of acceleration.

Question 1b.

Mark	0	1	2	Average
%	20	14	66	1.5

$$\underset{\sim}{v}(t) = \frac{1}{2}t\underset{\sim}{i} + \left(\frac{6}{5}t - 3\right)\underset{\sim}{j}$$

The majority of students were able to make some progress towards finding the velocity of the particle by either integration or use of a constant acceleration formula.

Students are reminded to be careful with their working. For example, it was common to see the final result written as $\underset{\sim}{v}(t) = \frac{1}{2}t\underset{\sim}{i} - \frac{9}{5}t\underset{\sim}{j}$.

Question 1c.

Mark	0	1	Average
%	42	58	0.6

$$\underset{\sim}{p}(2) = 10\underset{\sim}{i} - 6\underset{\sim}{j}$$

Some arithmetic errors were observed. A number of students gave the magnitude of the momentum.

Question 2

Mark	0	1	2	3	Average
%	22	9	6	63	2.1

$$\log_e(2) + \frac{\pi}{4}$$

Most students understood that they needed to write the integrand as the sum of two rational functions:

$$\int_0^1 \frac{2x}{x^2+1}dx + \int_0^1 \frac{1}{x^2+1}dx = \left[\log_e\left(x^2+1\right)\right]_0^1 + \left[\arctan(x)\right]_0^1$$

Use of a substitution was unnecessary in this situation and in attempting to use a substitution, some students introduced errors into their working.

Question 3a.

Mark	0	1	Average
%	28	72	0.7

$H_0 : \mu = 200$

$H_1 : \mu < 200$

Most students were able to write down the null and alternative hypotheses correctly. The alternative hypothesis was sometimes written with the incorrect inequality ($\mu > 200$ or $\mu \neq 200$) and some idiosyncratic notation was observed.

Question 3bi.

Mark	0	1	2	Average
%	47	13	41	1.0

$$\Pr\left(\bar{X} < 195 | \mu = 200\right) = \Pr\left(Z < \frac{195-200}{\frac{5}{3}}\right)$$
$$= \Pr(Z < -3)$$
$$= 0.001$$

Students were required to use the given information that $\Pr(-3 < Z < 3) = 0.9973$. From this it is found that $\Pr(Z < -3) = \frac{1-0.9973}{2} = 0.00135$.

Question 3bii.

Mark	0	1	Average
%	51	49	0.5

Reject the null hypothesis.

A number of students drew an incorrect conclusion from the p value. This was sometimes due to students confusing the p value (0.001) with the significance level (0.01).

Question 3c.

Mark	0	1	Average
%	44	56	0.6

$$\left(250 - 1.96 \times \frac{10}{5}, 250 + 1.96 \times \frac{10}{5}\right) = (246.08, 253.92)$$

Students frequently used $\mu = 200$ rather than 250. Arithmetic errors were also observed, as was the use of $z = 2$ rather than 1.96 as instructed.

Question 4a.

Mark	0	1	2	3	Average
%	14	22	6	58	2.1

$\frac{\pi^2}{2}$

Most students were able to write down a correct integral for the volume of the solid. Some students were unable to proceed further and incorrect attempts at integration were frequently seen. The most effective method was to use the double angle formula $\sin^2(x)=\frac{1}{2}\left(1-\cos(2x)\right)$.

Question 4b.

Mark	0	1	Average
%	70	30	0.3

$\frac{1}{k}V_S$

Very few students recognised that dilating the graph (and hence the solid) from part a. by a factor $\frac{1}{k}$ yields the graph and solid for part b. Of those who were successful, many did not write their answer in terms of V_S, as instructed.

Question 5

Mark	0	1	2	3	Average
%	14	11	21	53	2.2

$\frac{dy}{dx}=-\frac{1}{10}$

This question was answered well, with the majority of students performing implicit differentiation correctly. Common errors involved not differentiating the constant term to give zero and substituting $x=2$, $y=1$ incorrectly.

Students are reminded that if an expression for $\frac{dy}{dx}$ in terms of x and y is not required, then it may be advantageous to substitute the values for x and y immediately following differentiation.

Question 6

Mark	0	1	2	3	4	Average
%	14	9	19	33	26	2.5

$p \in R \setminus \left\{-\sqrt{5}, \sqrt{5}\right\}$

Most students realised that they first needed to write down and solve a system of linear equations in order to find the values of p for which the set of vectors were linearly dependent. Many students were able to find that $p = \pm\sqrt{5}$ for linear dependence but failed to conclude that $p \in R \setminus \left\{-\sqrt{5}, \sqrt{5}\right\}$ (or equivalent) for independence.

Question 7a.

Mark	0	1	2	3	Average
%	15	6	14	66	2.3

$x(t) = e^{1-\cos(t)}$

The majority of students were able to separate the differential equation correctly and make progress towards the solution. Some sign errors were seen when the trigonometric function was integrated.

Question 7b.

Mark	0	1	2	Average
%	58	34	8	0.5

e^2 when $t = (2k+1)\pi$, $k \in N \cup \{0\}$

The maximum displacement of the particle could be found by inspection. Most students did not attempt or were unable to give the correct times at which the maximum occurred.

Question 8a.

Mark	0	1	Average
%	30	70	0.7

$z = -1 \pm i$

Students could either complete the square or use the quadratic formula to solve the equation. This question was answered well.

Question 8b.

Mark	0	1	2	3	Average
%	50	27	13	10	0.9

$z = 1 \pm \sqrt{5}i$

Students who were successful let $z = x + iy$, leading to $x^2 - y^2 + 2xyi + 2(x - iy) + 2 = 0$. Algebraic errors were often seen in attempts to solve the resulting equations. A number of students assumed that the solutions to part a. were also solutions to part b. and some students confused the complex conjugate with the reciprocal. While it is possible to solve the equation beginning with the polar form $z = r\,\text{cis}(\theta)$, few students took this approach and those who did rarely made any significant progress.

Question 9ai.

Mark	0	1	Average
%	17	83	0.9

$$\left(\frac{x+1}{4}\right)^2 + \left(\frac{\sqrt{3}y}{2}\right)^2 = 1 \Rightarrow \frac{(x+1)^2}{16} + \frac{3y^2}{4} = 1$$

This 'show that' question was answered very well. Students were very comfortable identifying $x = -1 + 4\cos(t)$, $y = \frac{2}{\sqrt{3}}\sin(t)$ and using the trigonometric identity $\sin^2(\theta) + \cos^2(\theta) = 1$ to obtain the required result.

Question 9aii.

Mark	0	1	Average
%	59	41	0.4

$$3y^2 = 4 - \frac{(x+1)^2}{4} \Rightarrow 12y^2 = -x^2 - 2x + 15.$$

In the first quadrant $y \geq 0$ and so the positive square root is selected leading to the required result:

$$y = \frac{\sqrt{3}}{6}\sqrt{-x^2 - 2x + 15}$$

A common error was for students to neglect to justify the choice of sign for the path of the particle in the first quadrant.

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Question 9bi.

Mark	0	1	Average
%	38	62	0.6

The particles collide when $\cos(t)=\frac{\sqrt{3}}{2}$.

This question was answered well, with most students correctly showing that the x components and y components of both particles coincided when $\cos(t)=\frac{\sqrt{3}}{2}$.

Question 9bii.

Mark	0	1	Average
%	48	52	0.5

$$\left(-1+2\sqrt{3},\frac{1}{\sqrt{3}}\right)$$

This question was answered well by students who were successful in answering the previous part. Some students mistakenly gave the point of the collision as $\left(\frac{\pi}{6},\frac{1}{\sqrt{3}}\right)$, confusing the parameter t with the x-value of the point of collision.

Question 9ci.

Mark	0	1	2	Average
%	42	43	15	0.7

$$\frac{d}{dx}\left(8\arcsin\left(\frac{x+1}{4}\right)+\frac{(x+1)\sqrt{-x^2-2x+15}}{2}\right)=\sqrt{-x^2-2x+15}$$

It was necessary to use the product and chain rules as appropriate and then simplify to obtain the required result. For example:

$$\frac{d}{dx}\left(8\arcsin\left(\frac{x+1}{4}\right)+\frac{(x+1)\sqrt{-x^2-2x+15}}{2}\right)$$

$$=\frac{8}{4}\cdot\frac{1}{\sqrt{1-\left(\frac{x+1}{4}\right)^2}}+\frac{1}{2}\sqrt{-x^2-2x+15}+\frac{\frac{1}{2}(x+1)\cdot\frac{1}{2}(-2x-2)}{\sqrt{-x^2-2x+15}}$$

$$=8\cdot\frac{1}{\sqrt{-x^2-2x+15}}+\frac{1}{2}\sqrt{-x^2-2x+15}-\frac{1}{2}\frac{x^2+2x+1}{\sqrt{-x^2-2x+15}}$$

$$=\frac{1}{2}\sqrt{-x^2-2x+15}-\frac{1}{2}\frac{x^2+2x-15}{\sqrt{-x^2-2x+15}}$$

$$=\sqrt{-x^2-2x+15}$$

Students are reminded that in a ‘show that’ question, sufficient evidence must be presented in order for full marks to be awarded. Many students missed steps or made algebraic errors in their working.

Question 9cii.

Mark	0	1	2	Average
%	32	46	22	0.9

$$\frac{2\sqrt{3}\pi}{9}$$

A majority of students realised that the result of Question 9ci. should be used:

$$\frac{\sqrt{3}}{6}\left[8\arcsin\left(\frac{x+1}{4}\right)+\frac{(x+1)\sqrt{-x^2-2x+15}}{2}\right]_1^{2\sqrt{3}-1}$$

$$=\frac{\sqrt{3}}{6}\left(8\arcsin\left(\frac{\sqrt{3}}{2}\right)+\sqrt{3}\sqrt{-\left(12-4\sqrt{3}+1\right)-2\left(2\sqrt{3}-1\right)+15}-8\arcsin\left(\frac{1}{2}\right)-\sqrt{12}\right)$$

$$=\frac{\sqrt{3}}{6}\left(\frac{8\pi}{3}+\sqrt{12}-\frac{8\pi}{6}-\sqrt{12}\right)$$

$$=\frac{\sqrt{3}}{6}\left(\frac{16\pi}{6}-\frac{8\pi}{6}\right)$$

$$=\frac{8\sqrt{3}\pi}{36}=\frac{2\sqrt{3}\pi}{9}$$

Many found the resulting arithmetic to be challenging and were unable to arrive at the correct answer.