Neap

Trial Examination 2022

VCE Specialist Mathematics Units 1&2

Written Examination 1

Suggested Solutions

Question 1 (4 marks)

a. $\hat{p} = \frac{33\,500}{200}$

$= 167.5 \text{ cm}$ A1

b. $200 \times 60\% = 120$ students A1

c. $\Pr(X \geq 1) = 1 - \Pr(X < 1)$ M1

$= 1 - \Pr(X = 0)$

$= 1 - \binom{200}{0}(0.6)^{200}(0.4)^0$

$= 1 - \binom{200}{0}(0.6)^{200}$

$= 1 - (0.6)^{200}$ A1

Question 2 (4 marks)

a. In triangle *ABC*:

$\cos(\alpha) = \frac{8^2 + 7^2 - \left(\sqrt{57}\right)^2}{2 \times 8 \times 7}$

$\cos(\alpha) = \frac{56}{112}$

$\cos(\alpha) = \frac{1}{2}$ M1

$\alpha = \cos^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{3}$ A1

b. In triangle *ACD*:

$DC^2 = 5^2 + 7^2 - 2 \times 5 \times 7 \times \cos(\alpha)$

$DC^2 = 5^2 + 7^2 - 2 \times 5 \times 7 \times \frac{1}{2}$

$DC^2 = 39$ M1

$DC = \sqrt{39}$ A1

*Note: Consequential on working to **Question 2a**.*

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Question 3 (6 marks)

a. $t_3 = r^2 t_1 = 45$

$t_7 = r^6 t_1 = 3645$

$\frac{t_7}{t_3} = \frac{r^6 t_1}{r^2 t_1} = \frac{3645}{45}$

$r^4 = 81$

$r = \pm 3$

Since the sequence is positive:

$\therefore r = 3$ A1

$t_9 = r^2 t_7$

$= 9 \times 3645$

$= 32\,805$ A1

b. **i.** $t_1 = 1$

$t_2 = 2t_1 + 3 = 5$

$t_3 = 2t_2 + 3 = 13$

$t_4 = 2t_3 + 3 = 29$ A1

ii. $S_1 = t_1 = 1$

$S_2 = t_1 + t_2 = 1 + 5 = 6$

$S_3 = t_1 + t_2 + t_3 = 1 + 5 + 13 = 19$

$S_4 = t_1 + t_2 + t_3 + t_4 = 1 + 5 + 13 + 29 = 48$ A1

Note: Consequential on answer to ***Question 3b.i****.*

iii. $S_n = t_1 + t_2 + t_3 + \ldots + t_{n-2} + t_{n-1} + t_2$

$= t_1 + (2t_1 + 3) + (2t_2 + 3) + (2t_3 + 3) + \ldots + (2t_{n-3} + 3) + (2t_{n-2} + 3) + (2t_{n-1} + 3)$ M1

$= t_1 + (2t_1 + 2t_2 + 2t_3 + \ldots + 2t_{n-3} + 2t_{n-2} + 2t_{n-1}) + 3(n-1)$

$= 2S_{n-1} + t_1 + 3n - 3$

$= 2S_{n-1} + 3n - 2$ A1

Question 4 (9 marks)

a. **i.**

$$z + w = 6 + 8i + 5 + 12i$$
$$= 11 + 20i$$

A1

ii.

$$zw = (6 + 8i)(5 + 12i)$$
$$= 30 + 72i + 40i - 96$$
$$= -66 + 112i$$

A1

iii.

$$\frac{\overline{z}}{w} = \frac{6 - 8i}{5 + 12i}$$
$$= \frac{(6 - 8i)(5 - 12i)}{(5 + 12i)(5 - 12i)}$$
$$= \frac{-66 - 112i}{169}$$
$$= -\frac{66}{169} - \frac{112}{169}i$$

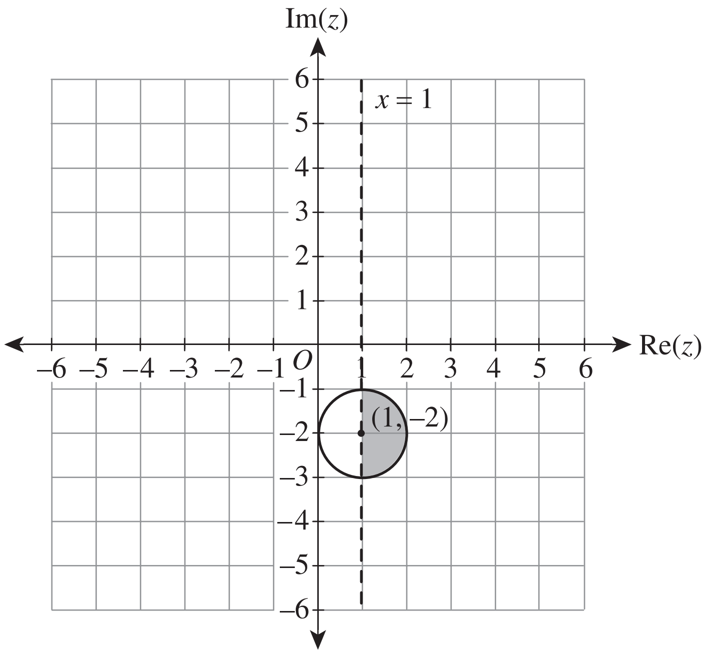
A1

b.

$$|z - 1 + 2i| \le 1$$
$$|x + yi - 1 + 2i| \le 1$$
$$\sqrt{(x - 1)^2 + (y + 2)^2} \le 1$$
$$(x - 1)^2 + (y + 2)^2 \le 1$$

M1

The area described by the inequality is the area in and on the circle centred at (1, –2) with a radius of 1. The combined area is shaded in the following graph.



correct line and circle A1
correct shaded area A1

c. $z = -2 - 2i$

$$r = \sqrt{(-2)^2 + (-2)^2}$$
$$= 2\sqrt{2}$$ M1

$$\tan(\theta) = \frac{-2}{-2}$$
$$= 1$$

Since both real and imaginary components are negative, z is in the third quadrant.

$$\therefore \theta = -\frac{3\pi}{4}$$

Hence, $-2 - 2i = 2\sqrt{2}\text{cis}\left(-\frac{3\pi}{4}\right)$. M1

$$(-2-2i)^{15} = \left(2\sqrt{2}\right)^{15}\text{cis}\left(-\frac{3\pi}{4}\times 15\right)$$
$$= 2^{22}\sqrt{2}\text{cis}\left(-\frac{45\pi}{4}\right)$$
$$= 2^{22}\sqrt{2}\text{cis}\left(\frac{3\pi}{4}\right)$$ A1

Question 5 (4 marks)

a. $\overrightarrow{OA} = 2\underset{\sim}{i} + 13\underset{\sim}{j}$

$$\overrightarrow{OB} = 12\underset{\sim}{i} + 8\underset{\sim}{j}$$
$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$
$$= -\overrightarrow{OA} + \overrightarrow{OB}$$
$$= -2\underset{\sim}{i} - 13\underset{\sim}{j} + 12\underset{\sim}{i} + 8\underset{\sim}{j}$$
$$= 10\underset{\sim}{i} - 5\underset{\sim}{j}$$ A1

b. Let $\overrightarrow{OC} = x\underset{\sim}{i} + y\underset{\sim}{j}$.

Since $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{AB}$:

$$\overrightarrow{OC} \cdot \overrightarrow{AB} = 0$$

$$10x - 5y = 0$$

$$|\overrightarrow{OC}| = \frac{28}{\sqrt{5}}:$$

$$\sqrt{x^2 + y^2} = \frac{28}{\sqrt{5}}$$

$$x^2 + y^2 = \frac{28^2}{5}$$

Combining the two equations about x and y gives:

$$\begin{cases} 10x - 5y = 0 \\ x^2 + y^2 = \dfrac{28^2}{5} \end{cases}$$

Rearranging the first equation gives $2x = y$.

Substituting $2x = y$ into the second equation gives:

$$x^2 + (2x)^2 = \frac{28^2}{5}$$

$$5x^2 = \frac{28^2}{5}$$

$$x = \pm\frac{28}{5}$$

Since C is on AB and both A and B are in the first quadrant:

$$\therefore x = \frac{28}{5}$$ M1

$$y = 2x = \frac{56}{5}$$

The coordinates of C are $\left(\frac{28}{5}, \frac{56}{5}\right)$. A1

*Note: Consequential on answer to **Question 5a**.*

c. $$A = \frac{1}{2}|\overrightarrow{AB}||\overrightarrow{OC}|$$

$$= \frac{1}{2} \times \sqrt{10^2 + (-5)^2} \times \frac{28\sqrt{5}}{5}$$

$$= 70 \text{ units}^2$$ A1

*Note: Consequential on answer to **Question 5a** and working to **Question 5b**.*

Question 6 (9 marks)

a. This is a reciprocal function of $x^2 + 7x + 2k$. If it has two vertical asymptotes, the original function must have two x-intercepts.

$\Delta > 0$ M1

$7^2 - 4 \times 1 \times 2k > 0$

$k < \frac{49}{8}$ A1

b.

$$5x^2 - 20x + 6y^2 + 72y + 206 = 0$$

$$5(x^2 - 4x) + 6(y^2 + 12y) + 206 = 0$$

$$5(x^2 - 4x + 4 - 4) + 6(y^2 + 12y + 36 - 36) + 206 = 0$$ M1

$$5\left[(x-2)^2 - 4\right] + 6\left[(y+6)^2 - 36\right] + 206 = 0$$

$$5(x-2)^2 + 6(y+6)^2 = 30$$

$$\frac{(x-2)^2}{6} + \frac{(y+6)^2}{5} = 1$$ A1

The shape is an ellipse centred at (2, –6). A1

c. $AP = \sqrt{x^2 + (y-5)^2}$

$BP = \sqrt{x^2 + (y+5)^2}$

$|AP - BP| = 2$ implies $AP - BP = 2$ or $BP - AP = 2$.

For $AP - BP = 2$:

$$\sqrt{x^2 + (y-5)^2} - \sqrt{x^2 + (y+5)^2} = 2$$

$$\sqrt{x^2 + (y-5)^2} = 2 + \sqrt{x^2 + (y+5)^2}$$

$$x^2 + (y-5)^2 = 4 + 4\sqrt{x^2 + (y+5)^2} + x^2 + (y+5)^2$$

$$(y-5)^2 - (y+5)^2 = 4 + 4\sqrt{x^2 + (y+5)^2}$$ M1

$$-20y = 4 + 4\sqrt{x^2 + (y+5)^2}$$

$$-5y - 1 = \sqrt{x^2 + (y+5)^2}$$ M1

This only holds true if $-5y - 1 \geq 0$ and $y \leq -\frac{1}{5}$.

$(-5y - 1)^2 = x^2 + (y+5)^2$ M1

$24y^2 - x^2 = 24$

$y^2 - \frac{x^2}{24} = 1$

For $BP - AP = 2$, it can be determined that it is the other branch of the hyperbola with $y \geq \frac{1}{5}$.

Therefore, the shape described by the locus definition is a hyperbola with the equation $y^2 - \frac{x^2}{24} = 1$. A1

Question 7 (4 marks)

a. Expanding the expression by addition formula gives:

$$r\sin(4x+\alpha) = r\sin(4x)\cos(\alpha) + r\cos(4x)\sin(\alpha)$$ M1

$$r\sin(4x)\cos(\alpha) + r\cos(4x)\sin(\alpha) = \frac{3}{2}\sin(4x) + \frac{\sqrt{3}}{2}\cos(4x)$$

$$\begin{cases} r\cos(\alpha) = \dfrac{3}{2} \\ r\sin(\alpha) = \dfrac{\sqrt{3}}{2} \end{cases}$$

Dividing the second equation by the first equation gives:

$$\tan(\alpha) = \frac{\sqrt{3}}{3}$$

Since $r\cos(\alpha) > 0$, $r\sin(\alpha) > 0$, $r > 0$, α is in the first quadrant.

$$\alpha = \frac{\pi}{6}$$ A1

$$r\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$r = \sqrt{3}$$

$$\therefore \frac{3}{2}\sin(4x) + \frac{\sqrt{3}}{2}\cos(4x) = \sqrt{3}\sin\left(4x + \frac{\pi}{6}\right)$$ A1

b.

$$\begin{aligned}\frac{3}{2}\sin(4x) + \frac{\sqrt{3}}{2}\cos(4x) &= \sqrt{3}\sin\left(4x + \frac{\pi}{6}\right) \\ &= \sqrt{3}\sin\left[2\left(2x + \frac{\pi}{12}\right)\right] \\ &= 2\sqrt{3}\sin\left(2x + \frac{\pi}{12}\right)\cos\left(2x + \frac{\pi}{12}\right)\end{aligned}$$ A1

*Note: Consequential on answer to **Question 7a**.*