Neap

Trial Examination 2022

VCE Specialist Mathematics Units 1&2

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 21 pages

Formula sheet

Answer sheet for multiple-choice questions

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on the answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1

Given that $\overrightarrow{OA} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\overrightarrow{OB} = -\underset{\sim}{i} + 5\underset{\sim}{j}$, $2\overrightarrow{BA}$ is equal to

A. $-3\underset{\sim}{i} - 2\underset{\sim}{j}$

B. $-6\underset{\sim}{i} + 4\underset{\sim}{j}$

C. $3\underset{\sim}{i} - 2\underset{\sim}{j}$

D. $6\underset{\sim}{i} - 4\underset{\sim}{j}$

E. $6\underset{\sim}{i} + 4\underset{\sim}{j}$

Question 2

It is known that for a given arithmetic sequence, $t_6 - t_2 = 16$.

For the same sequence, $t_8 - t_5$ is equal to

A. 8

B. 10

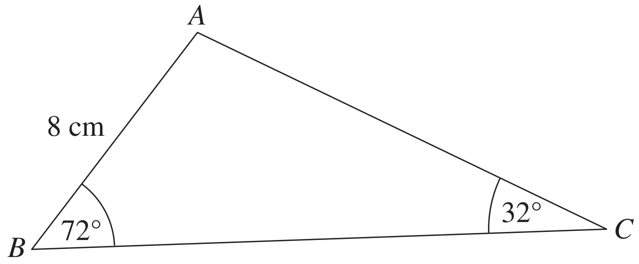
C. 12

D. 14

E. 16

Question 3

Consider the following triangle.



The length of BC, to the nearest centimetre, is

A. 9 cm

B. 10 cm

C. 11 cm

D. 14 cm

E. 15 cm

Question 4

The graph of $y = \dfrac{\sqrt{2x+3}}{x^2 - 9x + 18}$ has a domain of

A. $R \setminus \{3, 6\}$

B. $\left[-\dfrac{3}{2}, \infty\right)$

C. $R \setminus \left\{-\dfrac{3}{2}, 3\right\}$

D. $\left[-\dfrac{3}{2}, 3\right) \cup (3, \infty)$

E. $\left[-\dfrac{3}{2}, 3\right) \cup (3, 6) \cup (6, \infty)$

Question 5

Consider the vectors $\underset{\sim}{a} = -2\underset{\sim}{i} - 3\underset{\sim}{j}$ and $\underset{\sim}{b} = 5\underset{\sim}{i} + k\underset{\sim}{j}$.

What is the value of k if $\underset{\sim}{a} + \underset{\sim}{b}$ is perpendicular to the y-axis?

A. $-\dfrac{10}{3}$

B. 0

C. $\dfrac{3}{2}$

D. 3

E. 5

Question 6

Let $z = 5 + 3i$ be one of the two roots to a quadratic equation.

The product of the two roots to the quadratic equation is

A. 16

B. 34

C. $25 - 9i$

D. $25 + 9i$

E. $34 + 30i$

Question 7

Which one of the following is a unit vector?

A. $\frac{\sqrt{2}}{2}\sin(\theta)\underset{\sim}{i} + \frac{\sqrt{2}}{2}\cos(\theta)\underset{\sim}{j}$

B. $-\underset{\sim}{i} - \underset{\sim}{j}$

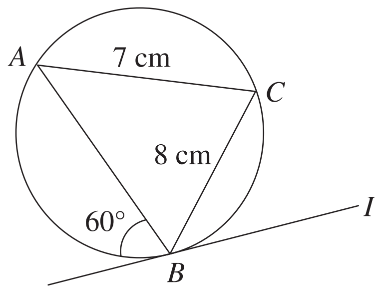
C. $\frac{1}{2}\underset{\sim}{i} + \frac{1}{2}\underset{\sim}{j}$

D. $\frac{1}{5}\left(3\underset{\sim}{i} + 4\underset{\sim}{j}\right)$

E. $\frac{5}{13}\underset{\sim}{i} + \frac{11}{13}\underset{\sim}{j}$

Question 8

A circle contains three chords, *AB*, *AC* and *BC*, as shown in the following diagram. Line *I* is a tangent to the circle at point *B*.



The area of triangle *ABC* is

A. $14\sqrt{3}\ \text{cm}^2$

B. $28\ \text{cm}^2$

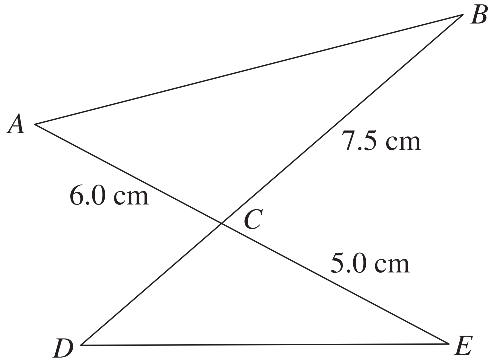
C. $28\sqrt{3}\ \text{cm}^2$

D. $58\ \text{cm}^2$

E. $56\sqrt{3}\ \text{cm}^2$

Question 9

Consider the following shape.



If $\angle B = \angle E$, the length of the line BD is

A. 4.0 cm

B. 8.5 cm

C. 10.0 cm

D. 10.5 cm

E. 11.5 cm

Question 10

Which one of the following identifies the equation(s) of the vertical asymptote(s) of the function $y = \dfrac{1}{x^2 - 2x + 2}$?

A. $x = -2, x = 2$

B. $x = -1, x = 1$

C. $x = 0$

D. $x = 4$

E. no vertical asymptotes

Question 11

If $\tan(\theta) + \dfrac{1}{\tan(\theta)} = 2$, $\sin(\theta) + \cos(\theta)$ is equal to

A. $\pm\sqrt{2}$

B. 1

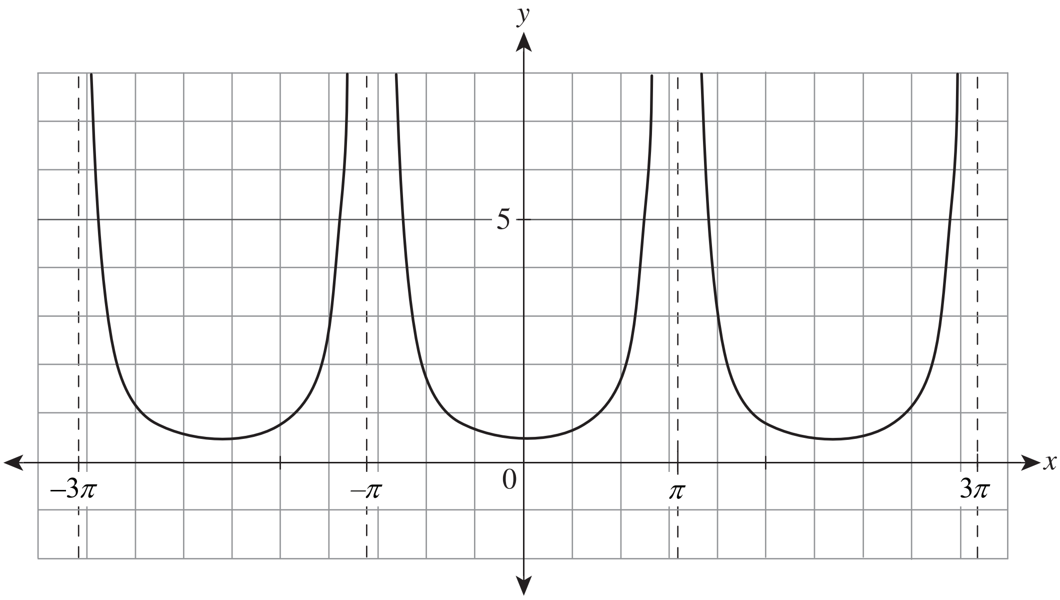
C. $\dfrac{\sqrt{6}}{2}$

D. $\sqrt{2}$

E. 2

Question 12

Consider the following graph.



The equation that best describes this graph is

A. $y = \dfrac{1}{\sin(x) + 1}$

B. $y = \dfrac{1}{\sin(x)} + 1$

C. $y = \dfrac{1}{\cos(x) + 1}$

D. $y = \dfrac{1}{\cos(x)} + 1$

E. $y = \dfrac{1}{\cos^2(x)}$

Question 13

For the sequence $i, i^2, i^3, i^4 \ldots$, where i is an imaginary number, what is the sum of the first 2019 terms?

A. $-i$

B. -1

C. i

D. 1

E. 2

Question 14

The complex number $z = \left(m^2 - m - 2\right) + \left(m^2 - 3m + 2\right)i,$ where $m \in R,$ is on the imaginary axis in an Argand diagram.

The value of m is

A. –2

B. –1

C. 1

D. 2

E. 2 or –1

Question 15

Consider the vectors $\underset{\sim}{a} = (1 - \sin\theta, 1)$ and $\underset{\sim}{b} = \left(\frac{1}{2}, 1 + \sin\theta\right)$. Vector $\underset{\sim}{a}$ is parallel to vector $\underset{\sim}{b}$.

The value of θ is

A. 15°

B. 30°

C. 45°

D. 60°

E. 75°

Question 16

In triangle *ABC*, vertex *A* is at point (–2, 0), vertex *B* is at (2, 0) and vertex *C* is a point that moves so that the lengths of lines *AC*, *AB* and *BC* form an arithmetic sequence.

The shape of the locus of point *C* is a

A. line.

B. parabola.

C. circle.

D. ellipse.

E. hyperbola.

Question 17

If $\sin(\alpha) > \sin(\beta),$ which one of the following statements is true?

A. If α and β are in the first quadrant, $\cos(\alpha) > \cos(\beta)$.

B. If α and β are in the second quadrant, $\tan(\alpha) > \tan(\beta)$.

C. If α and β are in the third quadrant, $\cos(\alpha) > \cos(\beta)$.

D. If α and β are in the fourth quadrant, $\tan(\alpha) > \tan(\beta)$.

E. There is not information to determine the values of cosine and tangent based on the value of sine.

Question 18

If $\underset{\sim}{a} = 3\underset{\sim}{i} + 4\underset{\sim}{j}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} + 2\underset{\sim}{j}$, what is the scalar resolute of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$?

A. $\frac{14}{5}\left(2\underset{\sim}{i} + 2\underset{\sim}{j}\right)$

B. $\frac{14}{5}$

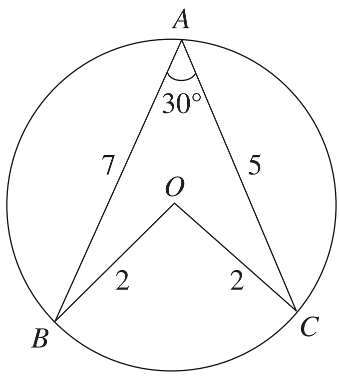
C. $\frac{7}{2}$

D. $\frac{7\sqrt{2}}{2}$

E. $\frac{7}{2}\underset{\sim}{i} + \frac{7}{2}\underset{\sim}{j}$

Question 19

Consider the following diagram.



The area of the shape *ABOC* is

A. $\sqrt{3}$

B. $8.75 - \sqrt{3}$

C. $7\left(\sqrt{6} - \sqrt{2}\right)$

D. 8.75

E. $14\left(\sqrt{6} - \sqrt{2}\right)$

Question 20

The quadratic equation $2x^2 + 3mx + m^2 - m = 0$ has complex roots with a modulus of 1.

The value of m is

A. -7

B. -1

C. $2 - \sqrt{2}$

D. 2

E. $2 + \sqrt{2}$

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 (5 marks)

A company is starting a new project that requires two employees. Anna, Betty and Charles are available; two of them will be assigned to the project. The following information is known.

- If Anna and Betty are assigned to the project, Anna will need to work for 8 days and Betty for 5 days.
- If Anna and Charles are assigned, Anna will need to work for 6 days and Charles for 9 days.
- If Betty and Charles are assigned, Betty will need to work for 10 days and Charles for 6 days.

Assume that they work at a constant rate.

a. **i.** The proportions of the project that can be done individually by Anna, Betty and Charles are x, y and z, respectively.

Write three equations that represent the situation. 2 marks

ii. Write the three equations from **part a.i.** in matrix form. 1 mark

b. The company is considering whether it would be more efficient to have one employee complete the project.

Calculate the number of days it would take Anna, Betty and Charles to each complete the entire project individually. 2 marks

Question 2 (13 marks)

Juanita has \$10 000 that she wants to deposit in a bank account. She is deciding between two bank accounts, *A* and *B*, which have different interest rates.

- Bank account *A* uses simple interest that is defined recursively by the rule $t_n = t_{n-1} + 288$.
- Bank account *B* uses compound interest that is defined recursively by the rule $t_n = 1.021t_{n-1}$.

t_n represents the balance in Juanita's account after n years, and $t_0 = 10\,000$.

a. Determine Juanita's account balance after five years if she chooses bank account *A*. 1 mark

b. Determine Juanita's account balance after five years if she chooses bank account *B*. 1 mark

c. From a long-term perspective, which bank account should Juanita choose to invest her \$10 000? Explain your answer. 2 marks

Juanita will be required to pay tax on any interest earned every year at a rate of 20%, which means she will only get to keep 80% of the interest generated every year in her bank account.

d. **i.** Find Juanita's account balance after tax for the first two years if she chooses bank account *A*. 1 mark

ii. Find Juanita's account balance after tax for the first two years if she chooses bank account *B*. 1 mark

e. Write new recursive rules for bank accounts *A* and *B* that account for tax. 2 marks

After selecting an account in which to deposit her \$10 000, Juanita decides that she also wishes to save progressively. She opens a second bank account and deposits an amount of money, a_1, during the first year. Juanita increases the amount she deposits by d each year so that the amount she deposits each year, a_1, a_2, a_3, …, forms an arithmetic sequence. This new account pays interest at r%. At the end of the nth year, a_1 accumulates to $a_1(1+r)^{n-1}$, a_2 accumulates to $a_2(1+r)^{n-2}$ and so on.

Let Juanita's total saving in this new account after n years be S_n.

f. Express S_n in terms of $S_{(n-1)}$ and a_n. 1 mark

g. Prove that S_n can be written as $S_n = A_n + B_n$, where A_n is a geometric sequence and B_n is an arithmetic sequence. 4 marks

Question 3 (7 marks)

The signal wave travelling between two mobile phone towers can be modelled by the function $A(t) = 2\cos(t) + \sqrt{2}\sin(2t),\ t \geq 0,$ where A is the strength of the wave and t is the length of time that the wave has travelled.

a. Find the maximum and minimum strengths of the wave. 2 marks

b. Prove that $\cos(t) = 0$ and $\sin(t) = -\dfrac{\sqrt{2}}{2}$ when $A(t) = 0$. 2 marks

c. The company that owns the towers finds that the signal is not smooth enough between the two towers because there are too many fluctuations in its strength. A technician fixes the issue and the adjusted signal wave can be modelled by the function $S = 3\cos(2t) + \sqrt{3}\sin(2t)$.

i. Write $S = 3\cos(2t) + \sqrt{3}\sin(2t)$ in the form $S = r\sin(2t - \alpha)$. 2 marks

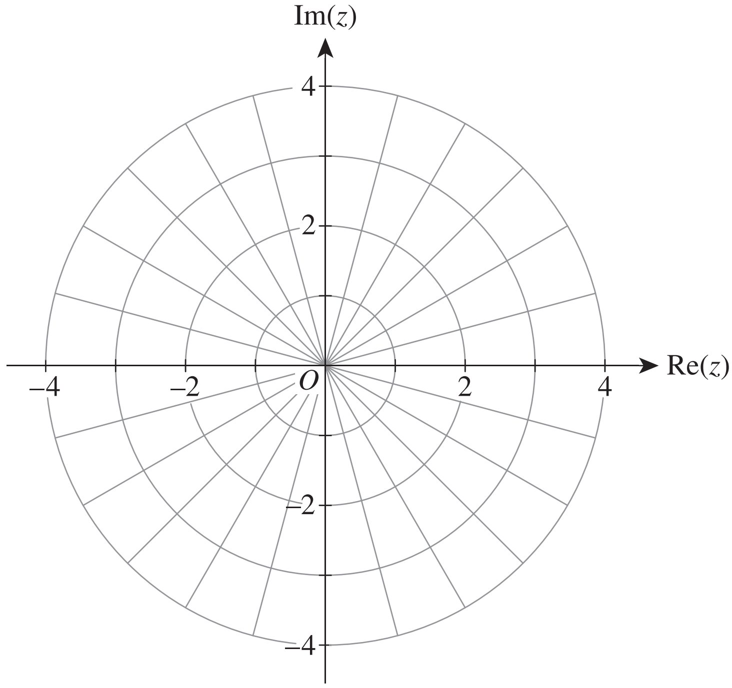
ii. Find the value of t when $S = 0$ for $t \in [0,\ 5]$. 1 mark

Question 4 (15 marks)

Consider the quadratic equation with complex coefficient $z^2 + 2\sqrt{2}iz - 5 = 0$.

a. Using the quadratic formula, find the solutions z_1 and z_2, given that all coefficients in z_1 are negative. 2 marks

b. Plot z_1 and z_2 on the Argand diagram below. 2 marks



c. Given $z_3 = 1 + 2i$ and $z_4 = -2 + i$, plot z_3 and z_4 on the Argand diagram above. 2 marks

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d. It is known that z_3, z_4, z_1 and z_2 correspond to points A, B, C and D, respectively.
Find $\angle ABC + \angle ADC$. 4 marks

e. It is known that $z_5 = (a+i)^2,\ a \in R$.

i. Find the value of a if z_5 is on the imaginary axis. 2 marks

ii. Find the value of a if z_5 is in the fourth quadrant. 3 marks

Question 5 (11 marks)

Points A, B and C have the coordinates (1, 0), (0, 1) and (2, 5), respectively.

a. Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. 2 marks

b. Find $2\overrightarrow{AB}+\overrightarrow{AC}$ and, hence, the modulus of $2\overrightarrow{AB}+\overrightarrow{AC}$. 2 marks

c. Find a unit vector that is perpendicular to $\overrightarrow{BC}$. 2 marks

d. Consider the vectors $\underset{\sim}{a} = \sqrt{3}\underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{b} = \frac{1}{2}\underset{\sim}{i} + \frac{\sqrt{3}}{2}\underset{\sim}{j}$.

It is known that $\underset{\sim}{x} = \underset{\sim}{a} + \left(t^2 - 3\right)\underset{\sim}{b}$ and $\underset{\sim}{y} = -k\underset{\sim}{a} + t\underset{\sim}{b}$, where k and t do not both equal zero and $\underset{\sim}{x}$ is perpendicular to $\underset{\sim}{y}$.

i. Write k as a function of t. 2 marks

ii. Find the minimum value of $\dfrac{k + t^2}{t}$. 3 marks

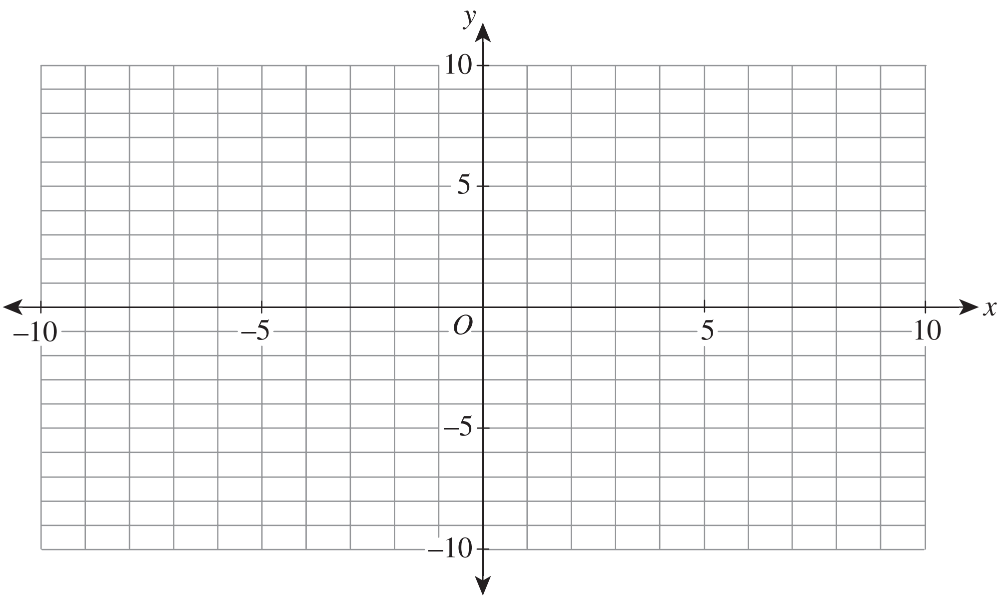
Question 6 (9 marks)

Consider the following parametric equations.

$$\begin{cases} x = 2\cos(\theta) \\ y = \sin(\theta) \end{cases}$$

a. Find the cartesian equation of the shape defined by the parametric equations and describe the shape. 2 marks

b. On the axes below, sketch the graph of the shape found in **part a.** and indicate any axis intercepts. 2 marks



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c. The shape defined by the parametric equations can also be defined by its locus, which has the following characteristics.

- P is a moving point on a circle with the equation $x^2 + y^2 = 4$.
- D is a moving point on the x-axis and PD is always perpendicular to the x-axis.
- M is the midpoint of PD.

Show that the equation of the locus of point M is the same as the parametric equations above. 2 marks

d. Points $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points on the shape found in **part a.** $C(x, y)$ is the midpoint of line AB.

If $y_1 y_2 = -\frac{1}{4} x_1 x_2$, find the equation of the locus of point C. 3 marks

END OF QUESTION AND ANSWER BOOKLET

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Trial Examination 2022

VCE Specialist Mathematics Units 1&2

Written Examination 2

Multiple-choice Answer Sheet

Student's Name: ______________________________

Teacher's Name: ______________________________

Instructions

Use a **pencil** for **all** entries. If you make a mistake, **erase** the incorrect answer – **do not** cross it out.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than **one** answer is completed for any question.

All answers must be completed like this example: A B C D E

Use pencil only

1	A	B	C	D	E	11	A	B	C	D	E
2	A	B	C	D	E	12	A	B	C	D	E
3	A	B	C	D	E	13	A	B	C	D	E
4	A	B	C	D	E	14	A	B	C	D	E
5	A	B	C	D	E	15	A	B	C	D	E
6	A	B	C	D	E	16	A	B	C	D	E
7	A	B	C	D	E	17	A	B	C	D	E
8	A	B	C	D	E	18	A	B	C	D	E
9	A	B	C	D	E	19	A	B	C	D	E
10	A	B	C	D	E	20	A	B	C	D	E

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Trial Examination 2022

VCE Specialist Mathematics Units 1&2

Written Examinations 1 & 2

Formula Sheet

Instructions

This formula sheet is provided for your reference.
A question and answer booklet is provided with this formula sheet.

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SPECIALIST MATHEMATICS FORMULAS

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule	$c^2=a^2+b^2-2ab\cos(C)$

Circular functions

$\cos^2(x)+\sin^2(x)=1$	
$1+\tan^2(x)=\sec^2(x)$	$\cot^2(x)+1=\operatorname{cosec}^2(x)$
$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$	
$\sin(2x)=2\sin(x)\cos(x)$	

Vectors in two dimensions

$\underset{\sim}{r}=x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k}$
$\lvert\underset{\sim}{r}\rvert=\sqrt{x^2+y^2+z^2}=r$
$\underset{\sim}{r}_1\cdot\underset{\sim}{r}_2=r_1r_2\cos(\theta)=x_1x_2+y_1y_2+z_1z_2$

Polar coordinates

$x=r\cos\theta$
$y=r\sin\theta$

END OF FORMULA SHEET

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