Neap

Trial Examination 2023

VCE Specialist Mathematics Units 1&2

Written Examination 2

Suggested Solutions

SECTION A – MULTIPLE-CHOICE QUESTIONS

1	A	B	C	D	**E**	11	A	B	C	**D**	E
2	**A**	B	C	D	E	12	A	B	C	**D**	E
3	A	B	C	**D**	E	13	A	B	C	D	**E**
4	A	B	C	**D**	E	14	**A**	B	C	D	E
5	A	B	**C**	D	E	15	**A**	B	C	D	E
6	A	**B**	C	D	E	16	A	B	C	D	**E**
7	A	**B**	C	D	E	17	A	B	C	**D**	E
8	**A**	B	C	D	E	18	A	B	**C**	D	E
9	A	B	**C**	D	E	19	A	**B**	C	D	E
10	A	**B**	C	D	E	20	A	B	**C**	D	E

Question 1 E

The negation of '$x \neq a$ and $x \neq b$' is '$x = a$ or $x = b$'.

The negation of '$x^2 - (a + b)x + ab \neq 0$' is '$x^2 - (a + b)x + ab = 0$'.

Question 2 A

A Hamiltonian path visits every vertex in a graph exactly once. Only option **A** satisfies this definition.

Question 3 D

The values of k and s are shown in the following table.

k	s
0	0
1	1
2	3
3	7

Therefore, the output value of s is 7.

Question 4 D

The first step of proving a contradiction is assuming that the statement is false, which is equivalent to assuming that the negation of the statement is true. Therefore, the negation of 'at least two solutions' is 'at most one solution'.

Question 5 C

Since $a_1 = 1$ and the common difference is 3:

$$\begin{aligned} a_n &= 1 + 3(n-1) \\ &= 3n - 2 \\ 3n - 2 &= 2005 \\ n &= 669 \end{aligned}$$

Question 6 B

Assuming that the common difference is d:

$$\begin{aligned} b_4 + b_6 &= 2b_1 + 8d \\ -6 &= 2 \times (-11) + 8d \\ d &= 2 \end{aligned}$$

$$\begin{aligned} S_n &= \frac{n}{2}\left(2 \times (-11) + (n-1) \times 2\right) \\ &= n^2 - 12n \\ &= (n-6)^2 - 36 \end{aligned}$$

Hence, the minimum of S_n is -36 and is reached when $n = 6$.

Question 7 B

B is correct. This option is the conditional statement.

A is incorrect. This option is the converse of the statement, which states, 'If I will perform well in the exam, I study eight hours each day'.

C is incorrect. This option is the biconditional of the statement, which states, 'I will perform well in the exam, if and only if I study eight hours each day'.

D is incorrect. This option is the conjunction of the statement, which states, 'I study eight hours each day and I will perform well in the exam'.

E is incorrect. This option is the disjunction of the statement, which states, 'I will perform well in the exam or I study eight hours each day'.

Question 8 A

The number of permutations of the word 'HELLO' is $\frac{5!}{2!} = 60.$ One permutation is correct; therefore, 59 of them are incorrect.

Question 9 C

Since seed 1 must be selected, two of the remaining three seeds need to be selected. Therefore, there are ${}^3C_2 = 3$ different ways.

The three selected seeds are then planted in three garden beds. Therefore, there are $3! = 6$ arrangements.

Hence, there are $3 \times 6 = 18$ different ways in total.

Question 10 B

When 0 is in the unit place, there are $5! = 120$ numbers.

When 1 is in the units place, 2, 3, 4 and 5 can be placed in the tens and 0 cannot be placed in the hundred thousands place. Therefore, there are $4 \times 3 \times 3! = 72$ numbers.

When 2 is in the units place, 3, 4 and 5 can be placed in the tens and 0 cannot be placed in the hundred thousands place. Therefore, there are $3 \times 3 \times 3! = 54$ numbers.

When 3 is in the units place, 4 and 5 can be placed in the tens and 0 cannot be placed in the hundred thousands place. Therefore, there are $2 \times 3 \times 3! = 36$ numbers.

When 4 is in the units place, only 5 can be placed in the tens and 0 cannot be placed in the hundred thousands place. There are $3 \times 3! = 18$ numbers.

The number 5 cannot be in the unit place.

Hence, there are $120 + 72 + 54 + 36 + 18 = 300$ possible numbers.

Question 11 D

Given that $\mathrm{Var}(kx) = k^2\mathrm{Var}(x)$:

$$\begin{aligned}\mathrm{Var}(3x_1, 3x_2, 3x_2) &= 3^2 \times \mathrm{Var}(x_1, x_2, x_3)\\ &= 9 \times 5\\ &= 45\end{aligned}$$

Question 12 D

From the table:

$$\begin{aligned}\mathrm{E}(X) &= -\frac{1}{2}+\frac{1}{6}\\ &= -\frac{1}{3}\end{aligned}$$

$$\begin{aligned}\mathrm{E}(Y) &= \mathrm{E}(2X+3)\\ &= 2\times \mathrm{E}(X)+3\\ &= -\frac{2}{3}+3\\ &= \frac{7}{3}\end{aligned}$$

Question 13 E

By the cosine rule:

$$c^2 = a^2+b^2-2ab\cos(C)$$

$$c^2 = 16+36-2\times 4\times 6\times \cos(120°)$$

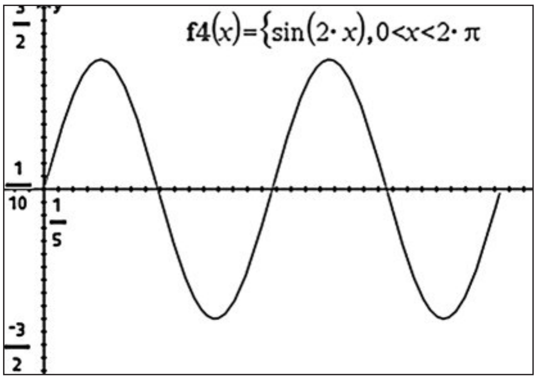
$$c^2 = 76$$

$$c = 2\sqrt{19}$$

Question 14 A

$y=\sin\left(2x+\frac{\pi}{5}\right)=\sin\left(2\left(x+\frac{\pi}{10}\right)\right)$ is translated $\frac{\pi}{10}$ units to the right; therefore, the function becomes $y=\sin(2x)$.

Using a CAS calculator to sketch the graph gives:



Reading from the graph, the function is strictly increasing over the interval $\left(\frac{3\pi}{2}, 2\pi\right)$.

Question 15 A

$$\sin(\alpha)+\cos(\alpha)=-\frac{1}{3}$$

$$(\sin(\alpha)+\cos(\alpha))^2=\frac{1}{9}$$

$$\sin^2(\alpha)+\cos^2(\alpha)+2\sin(\alpha)\cos(\alpha)=\frac{1}{9}$$

$$\begin{aligned}\sin(2\alpha)&=2\sin(\alpha)\cos(\alpha)\\&=\frac{1}{9}-1\\&=-\frac{8}{9}\end{aligned}$$

Question 16 E

E is correct. A unit vector has a length of 1. For (−1, 1), the modulus is $\sqrt{(-1)^2+1^2}=\sqrt{2}$. Hence, this option is not a unit vector.

A is incorrect. For (–1, 0), the modulus is $\sqrt{(-1)^2+0^2}=1$.

B is incorrect. For (cos(x), sin(x)), the modulus is $\sqrt{\left(\cos^2(x)+\sin^2(x)\right)}=1$.

C is incorrect. $\dfrac{\underset{\sim}{a}}{|\underset{\sim}{a}|}$ is always a unit vector; therefore, its modulus is 1.

D is incorrect. For $\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$, the modulus is $\sqrt{\left(-\frac{\sqrt{2}}{2}\right)^2+\left(-\frac{\sqrt{2}}{2}\right)^2}=\sqrt{\frac{1}{2}+\frac{1}{2}}=1$.

Question 17 D

$$\begin{aligned}\overrightarrow{CD}&=\overrightarrow{CA}+\overrightarrow{AD}\\&=\overrightarrow{CA}+\frac{2}{3}\overrightarrow{AB}\\&=\overrightarrow{CA}+\frac{2}{3}(\overrightarrow{CB}-\overrightarrow{CA})\\&=\frac{1}{3}\overrightarrow{CA}+\frac{2}{3}\overrightarrow{CB}\end{aligned}$$

Hence, $\lambda=\frac{2}{3}$.

Question 18 C

From the two asymptotes of the hyperbola ($2x + y = 0$ and $2x - y = 0$), $y = \pm 2x$.

As the formula for asymptotes of a hyperbola is $y - k = \pm\frac{b}{a}(x - h)$, then $a = 1$ and $b = 2$.

It can be determined that the hyperbola has the expression $x^2 - \frac{y^2}{4} = a$.

Since the hyperbola passes through the point (1, 1), substituting (1, 1) into the expression gives:

$$1^2 - \frac{1^2}{4} = a$$

$$a = \frac{3}{4}$$

$$x^2 - \frac{y^2}{4} = \frac{3}{4}$$

$$\frac{4x^2}{3} - \frac{y^2}{3} = 1$$

Question 19 B

The complex number $(m^2 + i)(1 + mi)$ is the product of two individual complex numbers. Since it lies on the real axis, the product of these two individual complex numbers must be a real number. The only possible way for the product of two complex numbers to be real is for the two numbers to be conjugate to each other. Therefore, $m = -1$.

Alternatively, the expression expands to $(m^2 + i)(1 + mi) = m^2 - m + (m^3 + 1)i$.

Since the product is real, the imaginary part is 0. Therefore:

$$m^3 + 1 = 0$$

$$m = -1$$

solve$(m^3+1=0, m)$	$m=-1$

Question 20 C

Deriving the geometric series formula gives:

$$S = i + 2i^2 + 3i^3 + \ldots + 2018i^{2018}$$

$$iS = i^2 + 2i^3 + 3i^4 + \ldots + 2018i^{2019}$$

$$\begin{aligned} S - iS &= (1+i)S \\ &= i + i^2 + i^3 + \ldots + i^{2018} - 2018i^{2019} \end{aligned}$$

Applying the geometric series formula gives:

$$(1-i)S = \frac{i\left(1-i^{2018}\right)}{1-i} + 2018i$$

$$(1-i)S = \frac{2i}{1-i} + 2018i$$

$$(1-i)S = -1 + 2019i$$

$$\begin{aligned} S &= \frac{-1+2019i}{1-i} \\ &= -1010 + 1009i \end{aligned}$$

$i \cdot \left(1-i^{2018}\right)$	$2 \cdot i$
$\frac{2 \cdot i}{1-i}$	$-1+i$
$\frac{-1+2019 \cdot i}{1-i}$	$-1010+1009 \cdot i$

SECTION B

Question 1 (8 marks)

a. The contrapositive of the statement is ‘if at least one of x and y does not equal 0, then $x^2 + y^2 \neq 0$’. M1

Letting $x \neq 0$ and $y = 0$ gives $x^2 + y^2 > 0$.

Therefore, $x^2 + y^2 \neq 0$, meaning that the contrapositive is true.

Hence, the original statement is true. A1

b. If the original statement is false, then $a + b \geq 2$. M1

$$a \geq 2 - b$$

$$a^3 \leq (2-b)^3$$

$$a^3 \geq 8 - 12b + 6b^2 - b^3$$

$$a^3 + b^3 \geq 6b^2 - 12b + 8$$

$$a^3 + b^3 \geq 6(b-1)^2 + 2$$ M1

completeSquare$(6\cdot b^2 - 12\cdot b + 8, b)$

$6\cdot(b-1)^2+2$

Since $6(b-1)^2 + 2 \geq 2$, then $a^3 + b^3 \geq 2$, which contradicts with $a^3 + b^3 = 2$.

Therefore, $a + b \geq 2$ is false and $a + b < 2$ is true. A1

c. When $n = 1, \dfrac{1}{3\times 1} = \dfrac{1}{3}$ is true. M1

Assuming that the equation is true when $n = k$ (where $k > 1$) gives:

$$\frac{1}{1\times 3} + \frac{1}{3\times 5} + \ldots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1}$$ M1

When $n = k + 1$:

$$\begin{aligned}
\text{LHS} &= \frac{1}{1\times 3} + \frac{1}{3\times 5} + \cdots + \frac{1}{(2k-1)(2k+1)} + \frac{1}{(2k+1)(2k+3)} \\
&= \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)} \\
&= \frac{k(2k+3)+1}{(2k+1)(2k+3)} \\
&= \frac{2k^2+3k+1}{(2k+1)(2k+3)} \\
&= \frac{k+1}{2k+3} \\
&= \frac{k+1}{2(k+1)+1} \\
&= \text{RHS}
\end{aligned}$$ A1

Therefore, the equation is true when $n = k + 1$ and, hence, it is true by mathematical induction.

Question 2 (10 marks)

a.

A	B	$A \leftrightarrow B$	$A \rightarrow B$	$B \rightarrow A$	$(A \rightarrow B) \wedge (B \rightarrow A)$
T	T	**T**	T	T	**T**
T	F	**F**	F	T	**F**
F	T	**F**	T	F	**F**
F	F	**T**	T	T	**T**

correct $A \leftrightarrow B$ column A1

correct $(A \rightarrow B) \wedge (B \rightarrow A)$ column A1

Since the $A \leftrightarrow B$ column and the $(A \rightarrow B) \wedge (B \rightarrow A)$ column are the same, $A \leftrightarrow B = (A \rightarrow B) \wedge (B \rightarrow A)$ is true. A1

b.

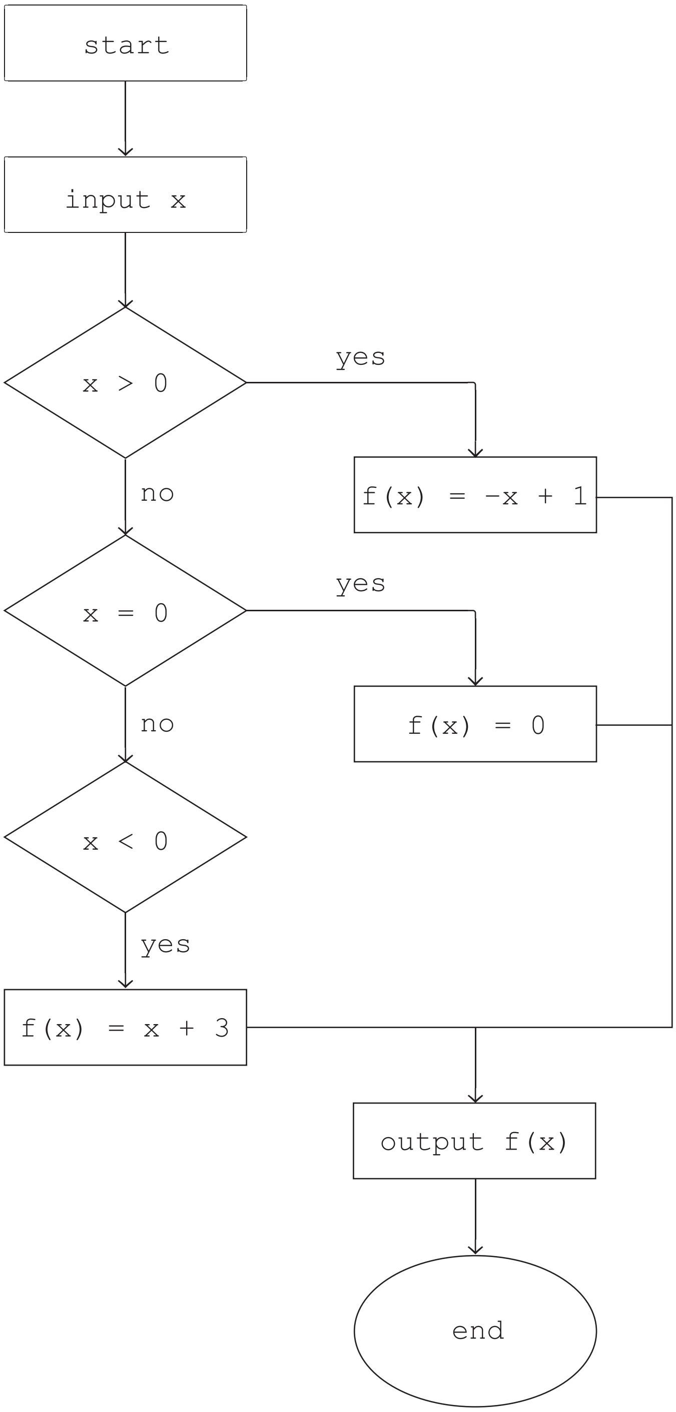
A	B	$\neg A$	$\neg B$	$B \rightarrow A$	$\neg B \rightarrow \neg A$
T	T	F	F	**T**	**T**
T	F	F	T	**T**	**F**
F	T	T	F	**F**	**T**
F	F	T	T	**T**	**T**

correct $B \rightarrow A$ column A1

correct $\neg B \rightarrow \neg A$ column A1

Since the $B \rightarrow A$ column and the $\neg B \rightarrow \neg A$ column are not the same, the converse of $A \rightarrow B$ is not logically equivalent to the contrapositive of $A \rightarrow B$. A1

c.



correct 'yes' and 'no' selection statement A1
correct algorithm flowchart A1

d.

```
input x←1,z←0
    while x≤50
        z←z+2x
        x←x+1
    end while
print z
```

correct while loop A1
correct algorithm A1

Question 3 (11 marks)

a. When $\underset{\sim}{a} \perp \underset{\sim}{b}$, $\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$.

$\sin(\theta) + \cos(\theta) = 0$ M1

Using a CAS calculator to solve for θ gives:

$$\text{solve}(\sin(x)+\cos(x)=0,x)\,|\,\frac{-\pi}{2}<x<\frac{\pi}{2} \qquad x=\frac{-\pi}{4}$$

Hence, $\theta = -\dfrac{\pi}{4}$. A1

b. **i.** $f(x) = \underset{\sim}{c} \cdot (\underset{\sim}{c} + \underset{\sim}{d})$

$= \underset{\sim}{c} \cdot \underset{\sim}{c} + \underset{\sim}{c} \cdot \underset{\sim}{d}$

$= \sin^2(x) + \cos^2(x) + \sin(x)\cos(x) + \cos^2(x)$

Applying trigonometric identity and the double angle formula for sine and cosine gives:

$= 1 + \dfrac{1}{2}\sin(2x) + \dfrac{1}{2}(\cos(2x)+1)$

$= \dfrac{3}{2} + \dfrac{1}{2}(\sin(2x)+\cos(2x))$ M1

Using a CAS calculator gives:

$$\text{tCollect}(\sin(2\cdot x)+\cos(2\cdot x)) \qquad \sqrt{2}\cdot\sin\left(2\cdot x+\frac{\pi}{4}\right)$$

Therefore, $f(x) = \dfrac{3}{2} + \dfrac{\sqrt{2}}{2}\sin\left(2x + \dfrac{\pi}{4}\right)$. A1

ii. $\text{period} = \dfrac{2\pi}{2} = \pi$ A1

Since $-1 \le \sin\left(2x + \dfrac{\pi}{4}\right) \le 1$:

$-\dfrac{\sqrt{2}}{2} \le \dfrac{\sqrt{2}}{2}\sin\left(2x + \dfrac{\pi}{4}\right) \le \dfrac{\sqrt{2}}{2}$

$-\dfrac{\sqrt{2}}{2} + \dfrac{3}{2} \le \dfrac{\sqrt{2}}{2}\sin\left(2x + \dfrac{\pi}{4}\right) \le \dfrac{\sqrt{2}}{2} + \dfrac{3}{2}$

Therefore, the maximum value of $f(x)$ is $\dfrac{3}{2} + \dfrac{\sqrt{2}}{2}$. A1

Note: Consequential on answer to ***Question 3b.i.***

c. **i.** $|\underset{\sim}{m} - \underset{\sim}{n}|^2 = |\underset{\sim}{m}|^2 - 2\underset{\sim}{m} \cdot \underset{\sim}{n} + |\underset{\sim}{n}|^2$

$= |\underset{\sim}{m}|^2 + |\underset{\sim}{n}|^2 - 2(\cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta))$

Since $|\underset{\sim}{m}| = |\underset{\sim}{n}| = 1$:

$|\underset{\sim}{m} - \underset{\sim}{n}|^2 = 2 - 2\cos(\alpha - \beta)$ M1

$$2 - 2\cos(\alpha - \beta) = \left(\frac{2\sqrt{5}}{5}\right)^2$$

$$2 - 2\cos(\alpha - \beta) = \frac{4}{5}$$

$$\cos(\alpha - \beta) = \frac{3}{5}$$ A1

ii. $\sin(\alpha) = \sin\left[(\alpha - \beta) + \beta\right]$ M1

Using the addition formula gives:

$\sin(\alpha) = \sin(\alpha - \beta)\cos(\beta) + \cos(\alpha - \beta)\sin(\beta)$

iii. Since $-\frac{\pi}{2} < \beta < 0$ and $0 < \alpha < \frac{\pi}{2}$, $0 \le \beta \le \frac{\pi}{2}$.

Therefore, $0 < \alpha - \beta < \pi$.

From **part c.i.**, $\cos(\alpha - \beta) = \frac{3}{5}$. Hence:

$$\sin(\alpha - \beta) = \sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$= \frac{4}{5}$$ M1

Since $\sin(\beta) = -\frac{5}{13}$, $\cos(\beta) = \frac{12}{13}$.

$$\sin(\alpha) = \sin(\alpha - \beta)\cos(\beta) + \cos(\alpha - \beta)\sin(\beta)$$

$$= \frac{4}{5} \times \frac{12}{13} + \frac{3}{5} \times \left(-\frac{5}{13}\right)$$

$$= \frac{33}{65}$$ A1

*Note: Consequential on answer to **Question 3c.i.***

Question 4 (11 marks)

a. **i.** $(z_1+z_2)^2 = (2+ai+3-i)^2$

$= \left(5+(a-1)i\right)^2$

$= 25-(a-1)^2+10(a-1)i$ M1

Given that $(z_1+z_2)^2 \in R$:

$a-1=0$

$a=1$ A1

ii. $\dfrac{z_1}{z_2} = \dfrac{2+ai}{3-i}$

$= \dfrac{6-a}{10}+\dfrac{3a+2}{10}i$ M1

$$\begin{cases} \dfrac{6-a}{10}=b \\ \dfrac{3a+2}{10}=\dfrac{1}{4} \end{cases}$$

Using a CAS calculator to solve the equations gives:

$$\text{solve}\left(\begin{cases} \dfrac{6-a}{10}=b \\ \dfrac{3\cdot a+2}{10}=\dfrac{1}{4} \end{cases},\{a,b\}\right) \quad a=\frac{1}{6} \text{ and } b=\frac{7}{12}$$

$a=\dfrac{1}{6}$ and $b=\dfrac{7}{12}$ A1

b. **i.** Let $z = a + bi$.

Since z is in the fourth quadrant, $a > 0$ and $b < 0$.

Since $|z| = \sqrt{5}$, $a^2 + b^2 = 5$.

Given that both a and b are integers:

$\begin{cases} a=1 \\ b=-2 \end{cases}$ or $\begin{cases} a=2 \\ b=-1 \end{cases}$

Therefore, $z = 1 - 2i$ or $z = 2 - i$. A2

ii. When $z = 1 - 2i$:

$z^2 = -3 - 4i$

$$\begin{cases} m=-3 \\ m^2-n=4 \end{cases}$$

Therefore, $m = -3$ and $n = 5$. A1

When $z = 2 - i$:

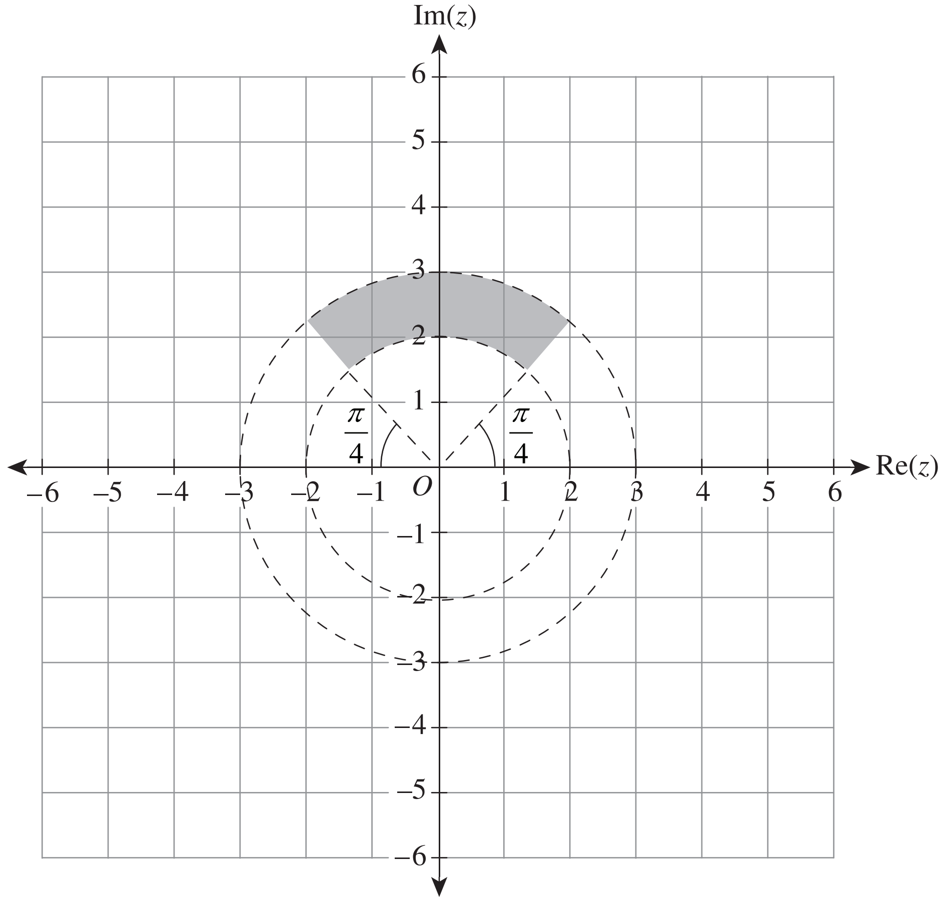
$z^2 = 3 - 4i$

$$\begin{cases} m=3 \\ m^2-n=4 \end{cases}$$

Therefore, $m = 3$ and $n = 5$. A1

Note: Consequential on answer to ***Question 4b.i.***

c.



correct shape A1
correct boundaries A1
correct reflection A1
Note: The information given in the question is about z, but the region required is $\overline{z}$.
Therefore, the shaded region needs to be reflected in the x-axis as shown above.

Question 5 (11 marks)

a. **i.** $A_n = 1500 + 230(n-1)$
$= 1270 + 230n$ A1

ii. $B_n = 2000 \times (1 + 5\%)^{n-1}$
$= 2000 \times 1.05^{n-1}$ A1

b. Letting A_{10} be the sum of John's salary at company A for 10 years gives:

$$A_{10} = \left(1500 \times 10 + \frac{230 \times 9 \times 10}{2}\right) \times 12$$
$$= \$304\,200$$ A1

Letting B_{10} be the sum of John's salary at company B for 10 years gives:

$$B_{10} = \frac{2000 \times \left(1 - 1.05^{10}\right)}{1 - 1.05} \times 12$$
$$= \$301\,869$$ A1

Since $A_{10} > B_{10}$, John should choose company A. A1

Note: Consequential on answers to ***Questions 5a.i.*** *and* ***5a.ii.***

c. Letting C_n be the difference between the monthly salaries from companies A and B gives:

$$C_n = (1270 + 230n) - \left(2000 \times 1.05^{n-1}\right)$$

The maximum difference between the monthly salaries of companies A and B must be found; therefore, C_n must be increasing.

Given that $C_n - C_{n-1} > 0$:

$$C_n - C_{n-1} = 230 - 100 \times 1.05^{n-2}$$
$$230 - 100 \times 1.05^{n-2} > 0$$
$$n < 19.1$$ M1

solve$\left(230-100\cdot(1.05)^{n-2}>0,n\right)$

$n<19.0712506735$

Therefore, C_{19} is the month in which the maximum difference between the monthly salaries of companies A and B occurs.

$$C_{19} = (1270 + 230 \times 19) - \left(2000 \times 1.05^{19-1}\right)$$
$$= \$827$$ A1

Note: Consequential on answers to ***Questions 5a.i.*** *and* ***5a.ii.***

d. From the question, it is known that when $n = 1$, $s_1 = a$. A1

When $n \geq 2$, $s_n = a \times \left(\frac{2}{3}\right)^{n-1} + b \times \left(\frac{3}{2}\right)^{n-2}$.

Hence, $s_n = \begin{cases} a, \ n = 1 \\ a \times \left(\frac{2}{3}\right)^{n-1} + b \times \left(\frac{3}{2}\right)^{n-2}, \ n \geq 2. \end{cases}$ A1

Note: Consequential on answers to ***Questions 5a.i.*** *and* ***5a.ii.***

e. When $b = \dfrac{8a}{27}$, $s_n = a \times \left(\dfrac{2}{3}\right)^{n-1} + \dfrac{8a}{27} \times \left(\dfrac{3}{2}\right)^{n-2}$.

Since a and b are both constant, a can be isolated and s_n can be expressed as a function of n.

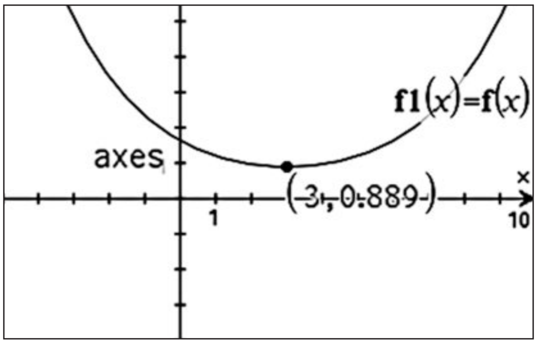
Using a CAS calculator gives:

$$\text{factor}\left(a \cdot \left(\frac{2}{3}\right)^{n-1} + \frac{8 \cdot a}{27} \cdot (1.5)^{n-2}\right) \qquad \frac{a \cdot \left(64 \cdot 9^n + 729 \cdot 4^n\right) \cdot 6^{-n}}{486}$$

$$\text{Define } f(x) = \frac{\left(64 \cdot 9^x + 729 \cdot 4^x\right) \cdot 6^{-x}}{486} \qquad \text{Done}$$

$s_n = \dfrac{a(64 \times 9^n + 729 \times 4^n) \times 6^{-n}}{486}$ M1

Using a CAS calculator to find the minimum of $s_n = \dfrac{a(64 \times 9^n + 729 \times 4^n) \times 6^{-n}}{486}$ gives:



Reading from the graph, there is a minimum at $n = 3$. Therefore, John will earn the least amount of money in the third year of the new scheme. A1

Note: Consequential on answers to ***Question 5d.***

Question 6 (9 marks)

a. **Method 1:**

$$\Pr = {}^5C_2 \times \left(\frac{2}{3}\right)^2 \times \left(1 - \frac{2}{3}\right)^3$$
$$= \frac{40}{243}$$
$$= 0.165$$ A1

Method 2:

Using a CAS calculator gives:

$$\text{binomPdf}\left(5, \frac{2}{3}, 2\right) \qquad 0.164609053498$$

$\Pr = 0.165$ A1

b. Three successful serves can be grouped together. Let S represent the event 'three successful serves in a row'. Let the two unsuccessful serves be S_1 and S_2. Hence, there are three arrangements: S, S_1, S_2; S_1, S, S_2; and S_1, S_2, S. M1

(Note: Since the successful serves are identical to each other and the unsuccessful serves are also identical to each other, arrangement S, S_1, S_2 is the same as S, S_2, S_1; S_1, S, S_2 is the same as S_2, S, S_1; and S_1, S_2, S is the same as S_2, S_1, S.)

Therefore:

$$\Pr = 3\times\left(\frac{2}{3}\right)^3\times\left(\frac{1}{3}\right)^2$$

$$= \frac{8}{81}$$ A1

c. $x = 3$ is equivalent to Vena making two successful serves in a row and one unsuccessful serve.

There are two arrangements. Therefore:

$$\Pr(x = 3) = 2\times\left(\frac{2}{3}\right)^2\times\left(\frac{1}{3}\right)$$ M1

$$= \frac{8}{27}$$ A1

d.

X	0	1	2	3	6
$\Pr(X = x)$	$\frac{1}{27}$	$\frac{2}{9}$	$\frac{4}{27}$	$\frac{8}{27}$	$\frac{8}{27}$

correct X values A1
correct probabilities A1

e. $$\mathrm{E}(X) = 0\times\frac{1}{27}+1\times\frac{2}{9}+2\times\frac{4}{27}+3\times\frac{8}{27}+6\times\frac{8}{27}$$

$$= \frac{86}{27}$$ A1

Note: Consequential on answer to ***Question 6d.***

f. For a sample size of 100:

$$\mathrm{E}(\bar{X}) = \mu$$

$$= \frac{86}{27}$$ A1

Note: Consequential on answer to ***Question 6e.***