Neap

Trial Examination 2023

VCE Specialist Mathematics Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 21 pages

Formula sheet

Answer sheet for multiple-choice questions

Instructions

Write your name and your teacher's name in the space provided above on this page, and on the answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2023 VCE Specialist Mathematics Units 3&4 Written Examination 2.

SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1

The graph of $f(x) = \dfrac{\sqrt{x}}{x^2 - 1}$ has

A. no horizontal asymptote.
B. one inflection point.
C. one stationary point.
D. two vertical asymptotes.
E. two non-vertical asymptotes.

Question 2

Consider the following statement, where n is a natural number.

If n is divisible by 10, then it is divisible by 5.

What is the converse of this statement?

A. If n is not divisible by 10, then it is divisible by 5.
B. If n is not divisible by 10, then it is not divisible by 5.
C. If n is divisible by 10, then it is not divisible by 5.
D. If n is divisible by 5, then it is divisible by 10.
E. If n is not divisible by 5, then it is not divisible by 10.

Question 3

The distance from the origin to the plane $x - 2y + 3z = 5$ is

A. $\frac{5}{14}$

B. $\frac{5}{6}$

C. $\frac{5}{\sqrt{6}}$

D. $\frac{5}{\sqrt{10}}$

E. $\frac{5}{\sqrt{14}}$

Question 4

If $\text{Arg}(5 + ai) = -\frac{3\pi}{4}$, the real number a is

A. $-5\sqrt{2}$

B. -5

C. $-\frac{5}{\sqrt{2}}$

D. 5

E. $5\sqrt{2}$

Question 5

Let $P(z)$ be a polynomial of degree 5 with real coefficients.

If $3 - i$ is a root of $P(z) = 0$, which one of the following **cannot** be correct?

A. $P(3 - i) = 0$

B. $P(3 + i) = 0$

C. $P(z) = 0$ has no real roots.

D. $P(z) = 0$ has one real root only.

E. $P(z) = 0$ has two non-real roots.

Question 6

If $z = \sin(\theta) - i\cos(\theta)$, then z^6 is

A. $-\cos(6\theta) - i\sin(6\theta)$

B. $-\sin(6\theta) + i\sin(6\theta)$

C. $\cos(6\theta) - i\sin(6\theta)$

D. $\cos(6\theta) + i\sin(6\theta)$

E. $\sin(6\theta) - i\cos(6\theta)$

Question 7

The acute angle between the two planes $x - 3y + 2z = 1$ and $6x - 7y + 2z = 5$ is closest to

A. 18.2°

B. 28.6°

C. 34.5°

D. 61.4°

E. 71.8°

Question 8

Using a suitable substitution, $\int_0^1 (x+1)\sqrt{1-x}\,dx$ can be expressed as

A. $-\int_0^1 (u^{\frac{3}{2}} + 2u^{\frac{1}{2}})du$

B. $-\int_0^1 (2u^{\frac{3}{2}} + u^{\frac{1}{2}})du$

C. $\int_0^1 (2u^{\frac{1}{2}} - u^{\frac{3}{2}})du$

D. $\int_0^1 (u^{\frac{3}{2}} + 2u^{\frac{1}{2}})du$

E. $\int_0^1 (2u^{\frac{3}{2}} + u^{\frac{1}{2}})du$

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Question 9

If the function $f(x) = ax^4 - 3x^3 - 5x^2$ has two inflection points, then

A. $a < -\frac{27}{40}$

B. $a > -\frac{27}{40}$

C. $a \le -\frac{27}{40}$

D. $a \ge -\frac{27}{40}$

E. $a \in R \setminus \left\{-\frac{27}{40}\right\}$

Question 10

Which one of the following vector equations does **not** describe a line that passes through the points (1, –2, 2) and (2, 1, –3)?

A. $\underset{\sim}{r} = (1+t)\underset{\sim}{i} + (-2+3t)\underset{\sim}{j} + (2-5t)\underset{\sim}{k}$

B. $\underset{\sim}{r} = (2-t)\underset{\sim}{i} + (1-3t)\underset{\sim}{j} + (-3+5t)\underset{\sim}{k}$

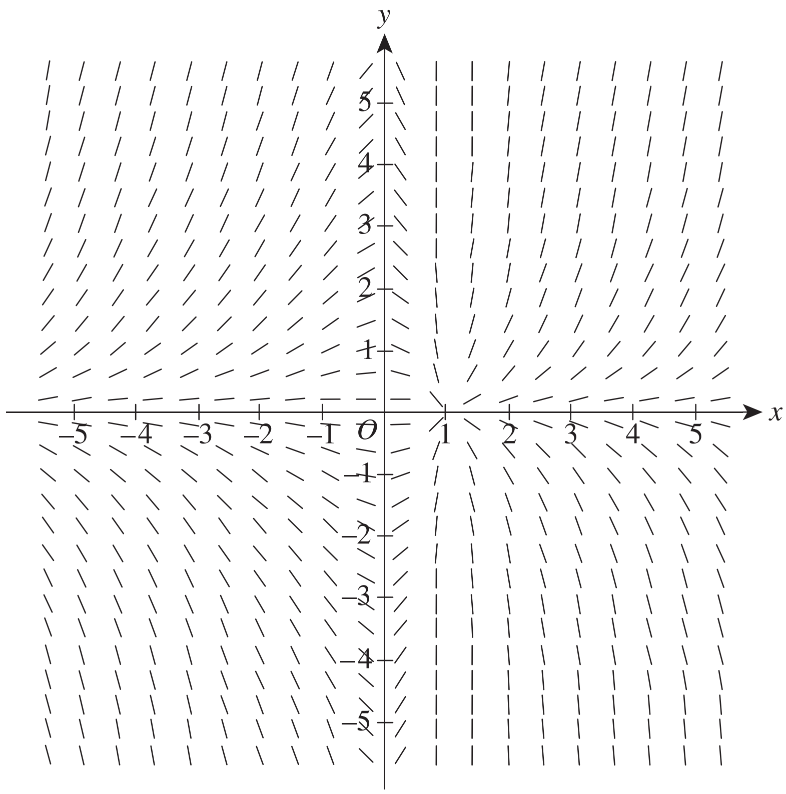
C. $\underset{\sim}{r} = (1+2t)\underset{\sim}{i} + (-2+6t)\underset{\sim}{j} + (2-10t)\underset{\sim}{k}$

D. $\underset{\sim}{r} = (2-t)\underset{\sim}{i} + (1-3t)\underset{\sim}{j} + (-3-5t)\underset{\sim}{k}$

E. $\underset{\sim}{r} = (1-t)\underset{\sim}{i} + (-2-3t)\underset{\sim}{j} + (2+5t)\underset{\sim}{k}$

Question 11

Consider the following direction field.



The direction field represents the solutions of the differential equation

A. $\dfrac{dy}{dx} = \dfrac{xy}{x-1}$

B. $\dfrac{dy}{dx} = \dfrac{xy}{1-x}$

C. $\dfrac{dy}{dx} = \dfrac{x}{y-1}$

D. $\dfrac{dy}{dx} = \dfrac{y}{x-1}$

E. $\dfrac{dy}{dx} = \dfrac{xy}{y-1}$

Question 12

A large container initially contains 20 grams of sugar dissolved in 3 L of water. A solution containing 5 grams of sugar per litre of water is poured into the container at a rate of 2 L per minute, and the mixture in the container is well stirred throughout the pouring. At the same time, 1 L of the mixture flows out of the container per minute.

A differential equation representing the mass, *m* grams, of sugar in the tank at time, *t* minutes, for a non-zero volume of the mixture is

A. $\frac{dm}{dt} = 20 - \frac{m}{3+t}$

B. $\frac{dm}{dt} = 10 - \frac{m}{3-t}$

C. $\frac{dm}{dt} = 10 - \frac{m}{3+t}$

D. $\frac{dm}{dt} = 20 - \frac{2m}{3+t}$

E. $\frac{dm}{dt} = 10 - \frac{2m}{3+t}$

Question 13

Consider the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + p\underset{\sim}{k}$, $\underset{\sim}{b} = 3\underset{\sim}{i} - 2\underset{\sim}{j} + 4\underset{\sim}{k}$ and $\underset{\sim}{c} = \underset{\sim}{i} - 2\underset{\sim}{j} + q\underset{\sim}{k}$, where *p* and *q* are real numbers.

If these vectors are linearly dependent, then

A. $4p + 13q + 28 = 0$

B. $4p - 13q + 28 = 0$

C. $4p + 13q - 28 = 0$

D. $13p - 4q + 28 = 0$

E. $13p + 4q + 28 = 0$

Question 14

In three dimensions, the equation $y = 2x + 1$ represents a

A. line parallel to the *z*-axis.

B. line parallel to the *xy* plane.

C. plane parallel to the *z*-axis.

D. plane parallel to the *xy* plane.

E. plane intersecting all three axes.

Question 15

The curve $y = \arcsin(2x - 1)$ is rotated about the x-axis.

The surface area of the resulting solid of revolution is closest to

A. 1.7

B. 8.0

C. 8.3

D. 16.1

E. 16.8

Question 16

The velocity, v, of a particle travelling in a straight line at position x and time t is given by $v = 2\sin(x)$.

The acceleration of the particle can be given by

A. $\sin(2x)$

B. $4\sin(x)$

C. $2\sin(x)\cos(x)$

D. $2\sin(2x)$

E. $2\cos(2x)$

Question 17

Consider the following algorithm, which has been written in pseudocode.

```
input f(x,y), x0, y0, h, n
    start loop from i = 1 to n
        y = y0 + h × f(x0, y0)
        x = x0 + h
        x0 = x
        y0 = y
    end loop
    print x0, y0
```

If $f(x, y) = x^2 - 3y$, $x_0 = 3$, $y_0 = 2$, $h = 0.2$ and $n = 6$, the algorithm will print

A. $x_0 = 3$, $y_0 = 2$

B. $x_0 = 4$, $y_0 = 4.49235$

C. $x_0 = 4$, $y_0 = 4.99694$

D. $x_0 = 4.2$, $y_0 = 4.49235$

E. $x_0 = 4.2$, $y_0 = 4.99694$

Question 18

A farmer investigates the distribution of the masses of apples in a particular harvest. Using a random sample of 80 apples, the 95% confidence interval for the mean mass of an apple, in grams, is (95.3, 110.6).

The population standard deviation is closest to

A. 3.5

B. 3.9

C. 12.5

D. 14.1

E. 34.9

Question 19

The lifespan of a butterfly species is normally distributed with a mean of 32 days and a variance of 16 days.

The probability that a random sample of six butterflies has an average lifespan of less than 34 days is closest to

A. 0.5204

B. 0.5809

C. 0.6203

D. 0.7668

E. 0.8897

Question 20

If a one-tailed test is performed using a 5% level of significance, then

A. H_0 should not be rejected if $p = 0.005$.

B. H_0 should be rejected if $p = 0.07$.

C. H_0 should be rejected if $p > 0.06$.

D. H_0 should be rejected if $p = 0.04$.

E. H_0 should not be rejected if $p < 0.04$.

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 (7 marks)

Consider the points $A(2, -1, 6)$, $B(4, 8, -1)$, $C(-6, 3, 2)$ and $D(-4, 0, 5)$.

a. **i.** Find a possible vector equation for the line passing through points A and D. 1 mark

ii. Point $E(-10, m, n)$ lies on the line passing through points A and D.
Find m and n. 1 mark

b. Find the equation of the plane passing through points A, B and C. 3 marks

c. Find the angle between the line passing through points A and D and the plane passing through points A, B and C. Give your answer in degrees, correct to one decimal place. 2 marks

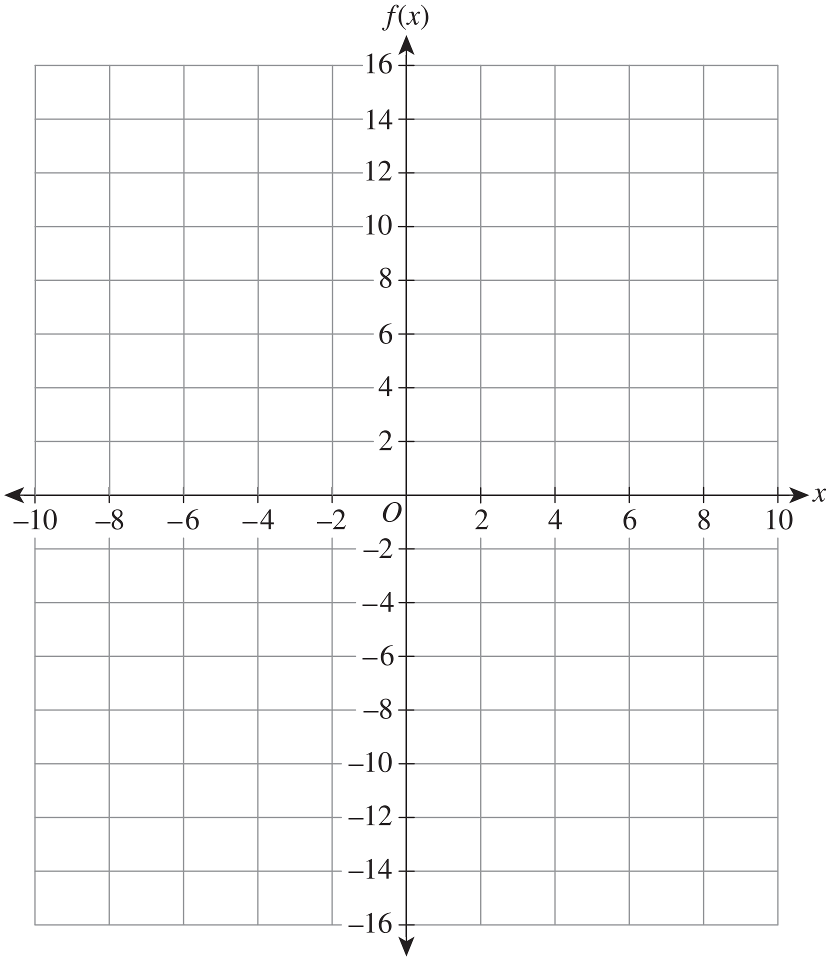
Question 2 (9 marks)

Let $f(x) = \dfrac{x^3 - 2x^2}{x^2 - 5x + 6}$.

a. State the equations of the asymptotes of $f(x)$. 2 marks

b. Show that the graph of $f(x)$ has no points of inflection. 3 marks

c. Sketch the graph of $f(x)$ on the axes below. Label the asymptotes with their equations and the key points with their coordinates. 2 marks

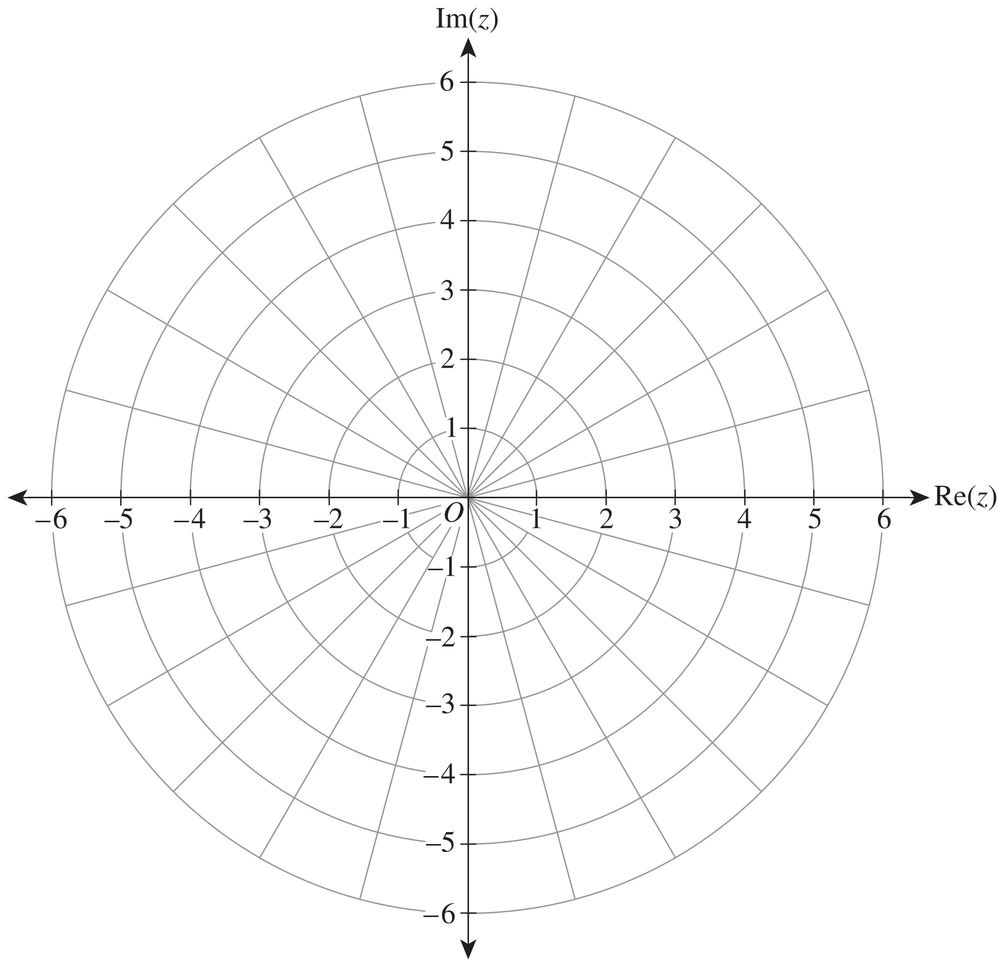


d. Let $g(x)=\dfrac{x^3+px^2}{x^2+qx+r}$, where p, q and r are real constants.

In terms of p, q and r, write down any possible relationship(s) where $g(x)$ has two asymptotes. 2 marks

Question 3 (12 marks)

a. On the Argand diagram below, plot the number $u = 2\sqrt{3} + 2i$. 1 mark



b. Let $u = 2\sqrt{3} + 2i$ be a solution of a quadratic equation with real coefficients.
Find the other solution and hence write down the quadratic equation in expanded form. 2 marks

c. On the Argand diagram above, sketch the circle represented by $|z - u| = 2$. 2 marks

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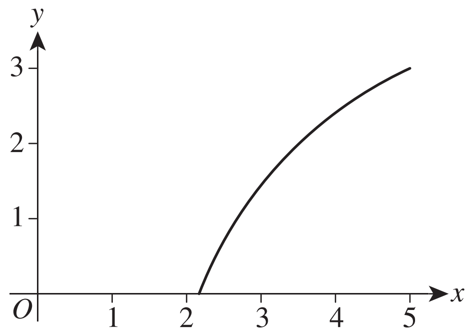
d. Find all the values of k such that the line $y = kx$ intersects the circle $|z - u| = 2$ at two points. 3 marks

e. Find the area of the region bounded by the circle $|z - u| = 2$, the ray $\text{Arg}(z) = \frac{\pi}{3}$ and the real number axis. 4 marks

Question 4 (12 marks)

Part of the graph of $y = \log_e(x^2 - 4)$ for $0 \le y \le 3$ is shown below.



a. Find the arc length of the curve shown. Give your answer correct to four decimal places. 2 marks

The curve shown is rotated about the y-axis to form a solid of revolution that models a container, which has lengths in metres. The container is filled with water to a depth of h metres.

b. Write down a definite integral in terms of y and h for the volume of water in the container in cubic metres. Hence, find an expression for the volume of water in terms of h. 2 marks

c. Find the depth of water in the container when the container is filled to half its volume. Give your answer in metres, correct to two decimal places. 2 marks

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Water leaks out of the container at a rate of $2\sqrt{h}$ cubic metres per minute.

d. How long will it take for the container to be empty if it is initially full? Give your answer in minutes, correct to one decimal place. 4 marks

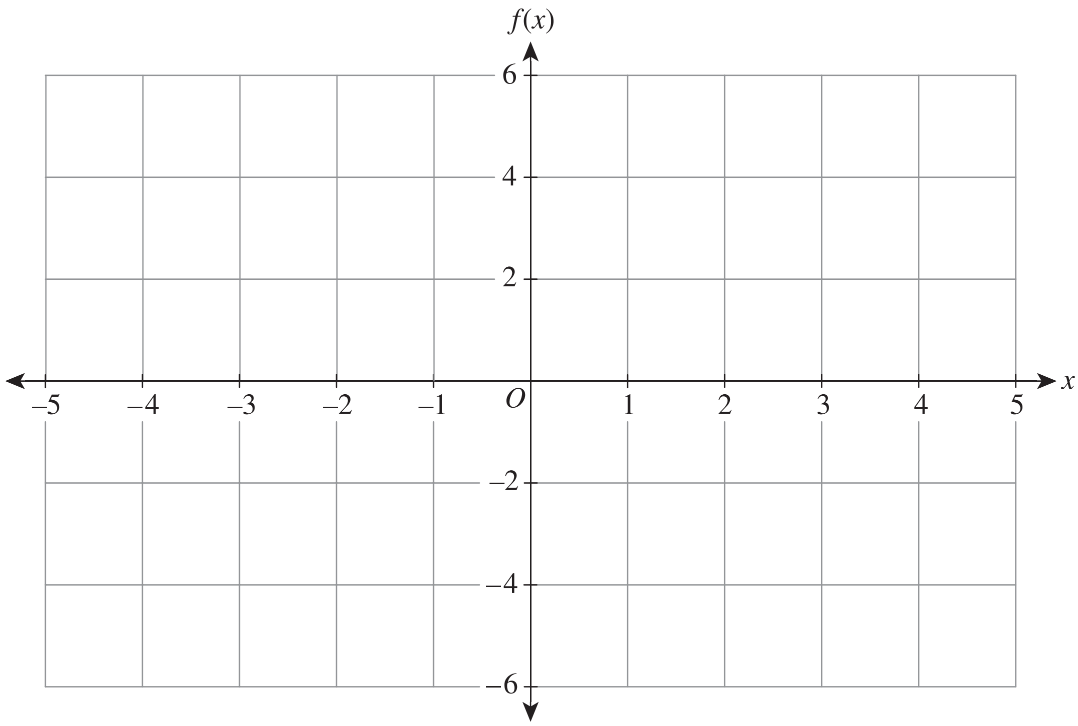
e. At what depth is the water level decreasing at the maximum rate? State the maximum rate. Give your answers correct to two decimal places. 2 marks

Question 5 (11 marks)

Let $f(x) = \dfrac{x}{2\sin(x)+1}$.

a. Write down the equations of all the asymptotes of $f(x)$. 1 mark

b. On the axes below, sketch the graph of $y = f(x)$ for $-5 \le x \le 5$. Label the stationary point(s) and point(s) of inflection with their coordinates, correct to one decimal place, and label the asymptote(s) with their equation(s). 4 marks

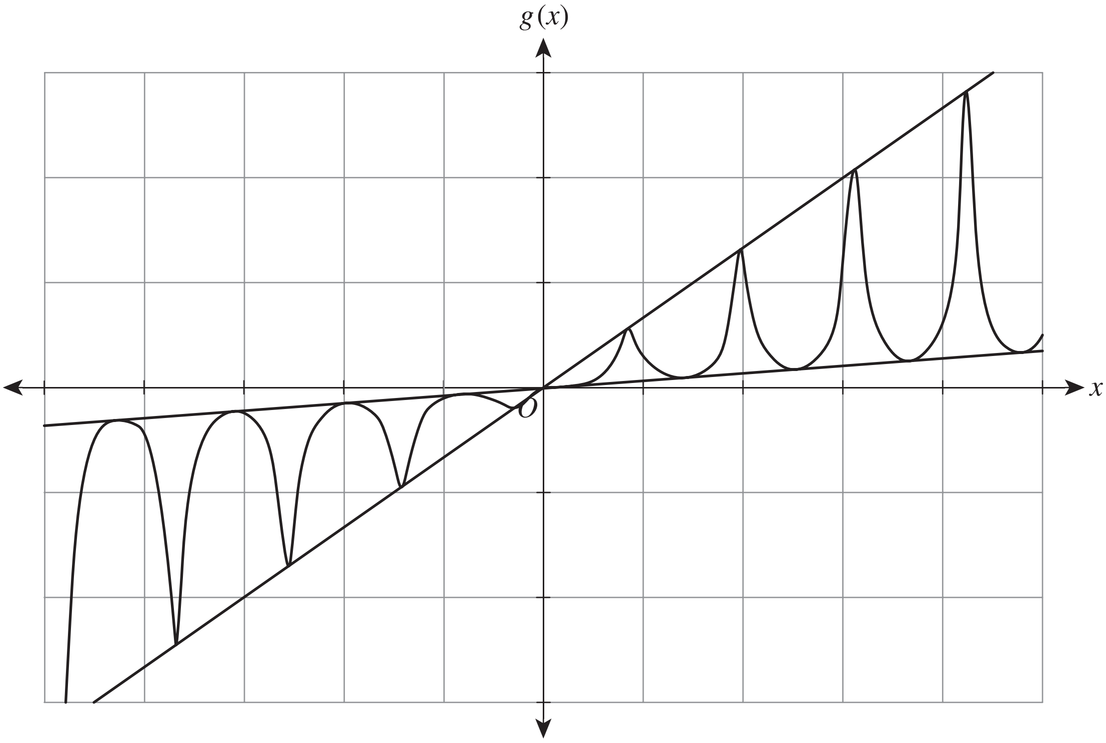


c. Find the values of p such that the only solution of $f(x) = p(x)$ is $x = 0$. 2 marks

d. Consider $g(x) = \dfrac{x}{k\sin(x)+1}$, where $k > 0$.

i. Find the values of k such that $g(x)$ has no asymptotes. 1 mark

ii. The graph of $g(x)$ shown below is between two tangent lines.



If the angle between the tangent lines is 30°, find the value of k. 3 marks

Question 6 (9 marks)

The daily income of a bakery is normally distributed with a mean of \$3200 and a standard deviation of \$430.

a. Find the probability that the bakery's average daily income during a particular week is greater than \$3500. Give your answer correct to three decimal places. 2 marks

b. Find the probability that the bakery's total income in a particular week is greater than \$22 500. Give your answer correct to three decimal places. 2 marks

The daily income of the grocery store next to the bakery is normally distributed with a mean of \$3600 and a standard deviation of \$840.

c. Find the probability that on a particular day the grocery store's daily income is greater than the bakery's daily income. Give your answer correct to three decimal places. 2 marks

d. After a change in management, the grocery store's average daily income is \$3440 for the next 20 days. A one-tailed statistical test using a 5% level of significance is to be carried out to determine the impact of the change in management on the business.

i. Write down the null and alternative hypotheses for this test. 1 mark

ii. Should the null hypothesis be accepted? Give a reason for your answer. 2 marks

END OF QUESTION AND ANSWER BOOKLET

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Trial Examination 2023

VCE Specialist Mathematics Units 3&4

Written Examination 2

Multiple-choice Answer Sheet

Student's Name: ____________________________

Teacher's Name: ____________________________

Instructions

Use a **pencil** for **all** entries. If you make a mistake, **erase** the incorrect answer – **do not** cross it out.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than **one** answer is completed for any question.

All answers must be completed like this example: A B C D E

Use pencil only

1	A	B	C	D	E	11	A	B	C	D	E
2	A	B	C	D	E	12	A	B	C	D	E
3	A	B	C	D	E	13	A	B	C	D	E
4	A	B	C	D	E	14	A	B	C	D	E
5	A	B	C	D	E	15	A	B	C	D	E
6	A	B	C	D	E	16	A	B	C	D	E
7	A	B	C	D	E	17	A	B	C	D	E
8	A	B	C	D	E	18	A	B	C	D	E
9	A	B	C	D	E	19	A	B	C	D	E
10	A	B	C	D	E	20	A	B	C	D	E

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Trial Examination 2023

VCE Specialist Mathematics Units 3&4

Written Examinations 1&2

Formula Sheet

Instructions

This formula sheet is provided for your reference.
A question and answer booklet is provided with this formula sheet.

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Mensuration

area of a circle segment	$\frac{r^2}{2}(\theta - \sin(\theta))$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$	sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
volume of a pyramid	$\frac{1}{3}Ah$	cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Algebra, number and structure (complex numbers)

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\text{cis}(\theta)$	$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	
$-\pi < \text{Arg}(z) \le \pi$	$z_1 z_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$	
$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$	de Moivre's theorem	$z^n = r^n \text{cis}(n\theta)$

Data analysis, probability and statistics

for independent random variables $X_1, X_2 \ldots X_n$	$\text{E}(aX_1 + b) = a\,\text{E}(X_1) + b$ $\text{E}(a_1X_1 + a_2X_2 + \ldots + a_nX_n)$ $= a_1\text{E}(X_1) + a_2\,\text{E}(X_2) + \ldots + a_n\text{E}(X_n)$	
	$\text{Var}(aX_1 + b) = a^2\text{Var}(X_1)$ $\text{Var}(a_1X_1 + a_2X_2 + \ldots + a_nX_n)$ $= a_1^2\text{Var}(X_1) + a_2^2\text{Var}(X_2) + \ldots + a_n^2\text{Var}(X_n)$	
for independent identically distributed variables $X_1, X_2 \ldots X_n$	$\text{E}(X_1 + X_2 + \ldots + X_n) = n\mu$	
	$\text{Var}(X_1 + X_2 + \ldots + X_n) = n\sigma^2$	
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}}, \bar{x} + z\frac{s}{\sqrt{n}}\right)$	
distribution of sample mean $\bar{X}$	mean	$\text{E}(\bar{X}) = \mu$
	variance	$\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$

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Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e \lvert x \rvert + c$
$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$	$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$	$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = a\sec^2(ax)$	$\int \sec^2(ax)dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}(\cot(ax)) = -a\,\text{cosec}^2(ax)$	$\int \text{cosec}^2(ax)dx = -\frac{1}{a}\cot(ax) + c$
$\frac{d}{dx}(\sec(ax)) = -a\sec(ax)\tan(ax)$	$\int \sec(ax)\tan(ax)dx = \frac{1}{a}\sec(ax) + c$
$\frac{d}{dx}(\text{cosec}(ax)) = -a\,\text{cosec}(ax)\cot(ax)$	$\int \text{cosec}(ax)\cot(ax)dx = -\frac{1}{a}\text{cosec}(ax) + c$
$\frac{d}{dx}\left(\sin^{-1}(ax)\right) = \frac{a}{\sqrt{1-(ax)^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\frac{d}{dx}\left(\cos^{-1}(ax)\right) = \frac{-a}{\sqrt{1-(ax)^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}}dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\frac{d}{dx}\left(\tan^{-1}(ax)\right) = \frac{a}{1+(ax)^2}$	$\int \frac{a}{a^2 + x^2}dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$
	$\int \frac{1}{ax+b}dx = \frac{1}{a}\log_e \lvert ax+b \rvert + c$

Calculus – continued

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
integration by parts	$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$
Euler's method	If $\frac{dy}{dx} = f(x, y)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + h \times f(x_n, y_n)$.
arc length parametric	$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$
surface area Cartesian about *x*-axis	$\int_{x_1}^{x_2} 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$
surface area Cartesian about *y*-axis	$\int_{y_1}^{y_2} 2\pi x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$
surface area parametric about *x*-axis	$\int_{t_1}^{t_2} 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$
surface area parametric about *y*-axis	$\int_{t_1}^{t_2} 2\pi x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Kinematics

acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	
constant acceleration formulas	$v = u + at$	$s = ut + \frac{1}{2}at^2$
	$v^2 = u^2 + 2as$	$s = \frac{1}{2}(u+v)t$

VCE_SM34_FS_2023

Vectors in two or three dimensions

$\underset{\sim}{r}(t) = x(i)\underset{\sim}{i} + y(t)\underset{\sim}{j} + z(t)\underset{\sim}{k}$	$\lvert\underset{\sim}{r}(t)\rvert = \sqrt{x(t)^2 + y(t)^2 + z(t)^2}$
	$\dot{\underset{\sim}{r}}(t) = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
for $\underset{\sim}{r}_1 = x_1\underset{\sim}{i} + y_1\underset{\sim}{j} + z_1\underset{\sim}{k}$ and $\underset{\sim}{r}_2 = x_2\underset{\sim}{i} + y_2\underset{\sim}{j} + z_2\underset{\sim}{k}$	vector scalar product $\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = \lvert\underset{\sim}{r}_1\rvert\lvert\underset{\sim}{r}_2\rvert\cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$
	vector cross product $\underset{\sim}{r}_1 \times \underset{\sim}{r}_2 = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (y_1z_2 - y_2z_1)\underset{\sim}{i} + (x_2z_1 - x_1z_2)\underset{\sim}{j} + (x_1y_2 - x_2y_1)\underset{\sim}{k}$
vector equation of a line	$\underset{\sim}{r}(t) = \underset{\sim}{r}_1 + t\underset{\sim}{r}_2 = (x_1 + x_2t)\underset{\sim}{i} + (y_1 + y_2t)\underset{\sim}{j} + (z_1 + z_2t)\underset{\sim}{k}$
parametric equation of a line	$x(t) = x_1 + x_2t \qquad y(t) = y_1 + y_2t \qquad z(t) = z_1 + z_2t$
vector equation of a plane	$\underset{\sim}{r}(s, t) = \text{r}_0 + s\underset{\sim}{r}_1 + t\underset{\sim}{r}_2$ $= (x_0 + x_1s + x_2t)\underset{\sim}{i} + (y_0 + y_1s + y_2t)\underset{\sim}{j} + (z_0 + z_1s + z_2t)\underset{\sim}{k}$
parametric equation of a plane	$x(s, t) = x_0 + x_1s + x_2t, \quad y(s, t) = y_0 + y_1s + y_2t, \quad z(s, t) = z_0 + z_1s + z_2t$
Cartesian equation of a plane	$ax + by + cz = d$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x + y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x - y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\sin(2x) = 2\sin(x)\cos(x)$	
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$
$\sin^2(ax) = \frac{1}{2}(1 - \cos(2ax))$	$\cos^2(ax) = \frac{1}{2}(1 + \cos(2ax))$

END OF FORMULA SHEET