Student Name: ____

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Creating VCE Success

SPECIALIST MATHEMATICS 2023

Unit 4
Key Topic Test 2 – Antidifferentiation applications
Technology Active

Recommended writing time*: 45 minutes
Total number of marks available: 30 marks

QUESTION BOOK

* The recommended writing time is a guide to the time students should take to complete this test. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions

- Students are permitted to bring into the room for this test: pens, pencils, highlighters, erasers, sharpeners and rulers, one CAS and bound reference book
- Students are NOT permitted to bring into the room for this test: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 9 pages.

Instructions

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this test.

Instructions for Section A

- All questions are worth one mark.
- Answer all questions by circling the correct response.
- Marks are not deducted for incorrect answers.
- No marks will be awarded if more than one answer is completed for any question.

Question 1

The approximate area under the curve $y = \sqrt{1-2x}$ between $x = -1$ and $x = \frac{1}{2}$ using the trapezoidal rule is

A. $\sqrt{3}$

B. $4\left(\sqrt{3} + 2\sqrt{2} + 2\right)$

C. $\sqrt{3} + 2\sqrt{2} + 2$

D. $\frac{1}{4}\left(\sqrt{3} + 2\sqrt{2} + 2\right)$

E. $\frac{1}{2}\left(\sqrt{3} + 2\sqrt{2} + 2\right)$

Question 2

For a given function $f(x)$, $f''(x) = \frac{x}{1+x^2}$.

The interval on which the graph of $f(x)$ is concave up is

A. $(0, \infty)$

B. $(-\infty, 0)$

C. $[0, \infty)$

D. $(-\infty, 0]$

E. R

Question 3

Using the trapezoidal rule with an interval size of 1, the approximate value of the integral is $\int_0^3 \left(\frac{1}{3}\right)^x dx$ is

A. 0.877

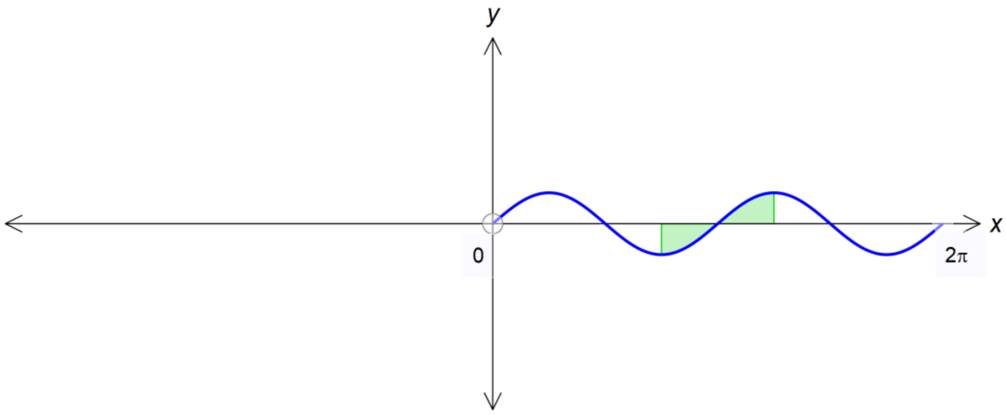
B. 0.963

C. 1.753

D. 1.926

E. 1.976

Question 4
The graph of $y = \sin(2x)$ is given below.



The area of the shaded region is given by

A. $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sin(2x)\ dx$

B. $2\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin(2x)\ dx$

C. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin(2x)\ dx$

D. $\int_{\frac{3\pi}{4}}^{\frac{5\pi}{4}} \sin(2x)\ dx$

E. $2\int_{\pi}^{\frac{3\pi}{4}} \sin(2x)\ dx$

Question 5
The region bounded by the coordinate axes and the graph of $y = \cos^3(x),\ 0 \le x \le \frac{\pi}{2}$ is rotated about the y-axis to form a solid of revolution. It is also rotated about the x-axis to form a solid of revolution.
The different in volume of the solids is given by

A. 0

B. 1

C. 2

D. $\frac{5\pi}{32}$

E. $\frac{5\pi}{16}$

Question 6

The length of the curve defined by parametric equations $x = \sin^2(t)$ and $y = \cos^2(t)$ for $0 \leq t \leq \frac{\pi}{2}$ can be found by calculating

A. $\int_0^{\frac{\pi}{2}} \sin(2t)\, dt$

B. $\int_0^{\frac{\pi}{2}} \sqrt{2\sin^2(2t)}\, dt$

C. $\int_0^{\frac{\pi}{2}} \sqrt{2\sin^2(t)}\, dt$

D. $\int_0^{\frac{\pi}{2}} \sqrt{\sin^4(t) + \cos^4(t)}\, dt$

E. $\int_0^{\frac{\pi}{2}} 1\, dt$

Question 7

The area of the surface generated by revolving the part of the curve $y = 2x^{\frac{1}{3}}$ from (0, 0) to (8, 2) about the x-axis is closest to

A. 25.99

B. 60.86

C. 84.51

D. 163.31

E. 173.04

Question 8

The area of the surface generated by revolving the part of the curve $y = x^3$ from (0, 0) to (3, 27) about the y-axis is closest to

A. 21.97

B. 22.61

C. 173.78

D. 365.23

E. 384.05

SECTION B

Instructions for Section B

- Answer each question in the space provided.
- Please provide appropriate workings and use exact answers unless otherwise specified.

Question 1 (11 marks)

The rate, in liters per hour, at which water fills an empty tank is $\frac{dV}{dt} = 0.048e^{0.4t}$, where *t* is time in hours.

a. Determine the volume, *V*, in the tank after *t* hours.

2 marks

b. Write a rule to find the time taken to fill the tank in terms of the volume, *V* in the tank. Express your answer in the form $\frac{a}{b}\ln\left(\frac{cV+d}{d}\right)$ *where* $a, b, c, d \in Z^+$.

3 marks

The tank is full after $7\ln(6)$.

c. What is the maximum capacity of the tank?
Give your answer correct to one decimal place.

2 marks

The surface area of the tank can be found by rotating the curve $y = \sqrt{x}$ from $x = 1\ to\ x = 2$ about the x-axis.

d. Write down an integral to find the surface area of this tank and hence show that the surface area is $\frac{\pi}{6}\left(27 - 5\sqrt{5}\right)$.

4 marks

Question 2 (11 marks)
A curve is defined by the parametric equations

$x(t) = 3\cos(2t) \ \ and \ \ y(t) = 3\sin(2t), \ 0 \leq t \leq \frac{\pi}{4}$

a. Find $\frac{dy}{dt} \ \ and \ \ \frac{dx}{dt}$ and hence, write down an integral to find the length of the arc of this curve.

3 marks

b. Find the Cartesian equation of the curve.

1 mark

c. Calculate the area, bounded by the curve $x(t) = 3\cos(2t) \ \ and \ \ y(t) = 3\sin(2t), \ 0 \leq t \leq \frac{\pi}{4}$, the line $y = 2\sqrt{2}$ and the y-axis. Answer correct to 2 decimal places.

2 marks

d. The region bounded by the curve $x(t) = 3\cos(2t)$ and $y(t) = 3\sin(2t)$, $0 \leq t \leq \frac{\pi}{4}$, the lines $x = 0$, $y = 0$ and $x = 1$ is rotated about the x-axis.
Calculate the volume of this region.

2 marks

e. The curve $x(t) = 3\cos(2t)$ and $y(t) = 3\sin(2t)$, $0 \leq t \leq \frac{\pi}{4}$, from $x = 0$ to $x = 1$ is rotated about the x-axis.
Find the area of the surface of revolution in exact form.

3 marks

END OF KEY TOPIC TEST

