



SPECIALIST MATHEMATICS 2023

Unit 4

Key Topic Test 4 – Differential equations Technology Active

Recommended writing time*: 45 minutes
Total number of marks available: 30 marks

SOLUTIONS

SECTION A: Multiple-choice questions (1 mark each)

Question 1

Answer: C

Explanation:

$$g(x) = 3x^2$$

$$y_1 = 1 + 3h$$

$$y_2 = 1 + 3h + h \times 3(1 + h)^2$$

$$3.421 = 1 + 3h + h \times 3(1 + h)^2 \rightarrow h = 0.3$$

Question 2

Answer: D

Explanation:

$$\frac{dy}{dx} = y^2$$

$$\int y^{-2} dy = \int dx$$

$$-\frac{1}{y} = x + c$$

$$-\frac{1}{2} = c$$

$$-\frac{1}{y} = x - \frac{1}{2} \rightarrow \frac{1}{y} = \frac{-2x+1}{2} \rightarrow y = \frac{2}{1-2x}$$

Question 3

Answer: E

Explanation:

$$\frac{d\theta}{dt} = k(120 - \theta) \rightarrow \int \frac{d\theta}{120 - \theta} = \int k dt$$

$$-\ln|120 - \theta| = kt + c$$

$$\ln|120 - \theta| = -kt - c$$

$$120 - \theta = e^{-kt-c} \rightarrow \theta = 120 - ae^{-kt}, a \in \mathbb{R}$$

Question 4

Answer: A

Explanation:

$$\int y dy = \int \operatorname{cosec}^2(x) dx \text{ (separation of variables)}$$

Question 5

Answer: B

Explanation:

$$x_0 = \frac{\pi}{3}, y_0 = 2$$

$$x_1 = \frac{\pi}{3} + 0.1, y_1 = 2 + 0.1 \times \cos\left(\frac{\pi}{3}\right) = 2 + 0.1 \times \frac{1}{2} = 2.05$$

Question 6

Answer: D

Explanation:

Check a few points for slopes or sketch on CAS

Question 7

Answer: E

Explanation:

$$\frac{dh}{dt} = -\frac{1}{50h} \rightarrow \int 50h \, dh = -\int dt$$

$$25h^2 = -t + c$$

$$h^2 = -\frac{t}{25} + k$$

Question 8

Answer: C

Explanation:

$$\frac{dQ}{dt} = \frac{18-6t-3}{3-t} \rightarrow Q = \int \frac{15-6t}{3-t} dt$$

SECTION B:**Question 1**

a. $\frac{dy}{dx} = \frac{x+1}{2y}$
 $\frac{dy}{dx} = \frac{0.5+1}{-2} = -0.75$

2 marks

b. $\int 2ydy = \int (x+1)dx$
 $y^2 = \frac{x^2}{2} + x + c$
 $x = 0.5, y = -1 \rightarrow 1 = \frac{1}{8} + \frac{1}{2} + c \rightarrow c = \frac{3}{8}$
 $y^2 = \frac{x^2}{2} + x + \frac{3}{8}$

3 marks

c. $y^2 = \frac{1.5^2}{2} + 1.5 + \frac{7}{8}$
 $x = \pm \frac{\sqrt{14}}{2}$

2 marks

Question 2

a. $\frac{dx}{dt} = \frac{e^{-x}}{1+81t^2}$

$$\int e^x dx = \int \frac{dt}{1+(9t)^2}$$

$$e^x = \frac{1}{9} \tan^{-1} \left(\frac{t}{\frac{1}{9}} \right) + c$$

$$e^x = \frac{1}{9} \tan^{-1}(9t) + c$$

$$t = 0, x = 0 \rightarrow c = 1$$

$$e^x = \frac{1}{9} \tan^{-1}(9t) + 1$$

$$x = \ln \left| \frac{1}{9} \tan^{-1}(9t) + 1 \right|$$

4 marks

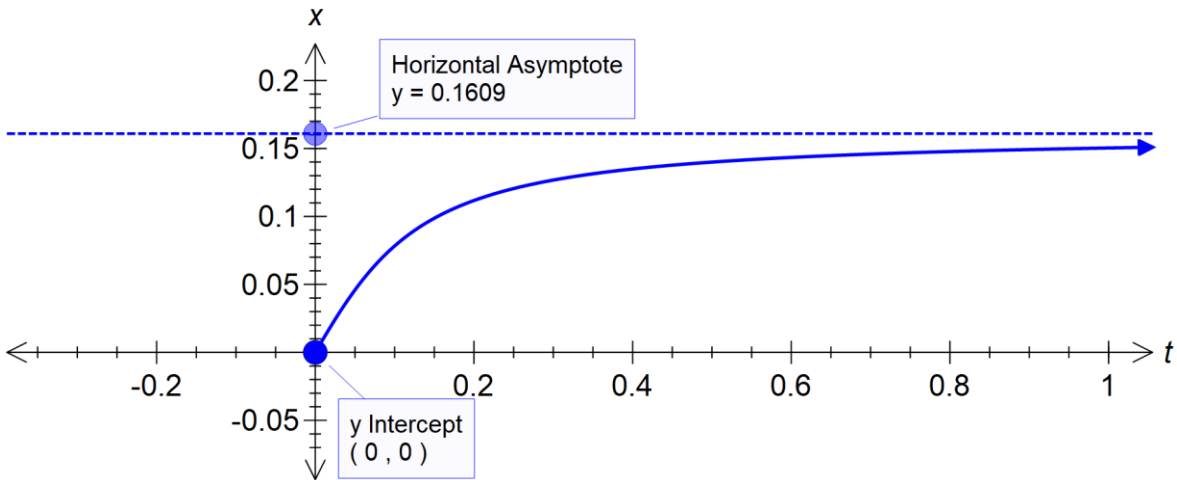
b. $x(10) = \ln \left| \frac{1}{9} \tan^{-1}(90) + 1 \right|$

$$x(10) = 0.1598m$$

$$x(10) = 16 \text{ cm}$$

2 marks

c.



3 marks

Question 3

a. $V = \frac{\pi}{5} \left((h + 6)^{\frac{5}{3}} - 16 \right)$

$$\frac{dV}{dh} = \frac{\pi}{5} \left(\frac{5}{3} (h + 6)^{\frac{2}{3}} \right) = \frac{\pi}{3} (h + 6)^{\frac{2}{3}}$$

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{3}{\pi} (h + 6)^{-\frac{2}{3}} \times -2\sqrt{h}$$

$$\frac{dh}{dt} = -\frac{6}{\pi} \frac{\sqrt{h}}{(h+6)^{\frac{2}{3}}}$$

4 marks

b. $\frac{d^2h}{dt^2} = 0$

$h = 18$

$\text{Max rate of decrease} = -\frac{6}{\pi} \frac{\sqrt{h}}{(h+6)^{\frac{2}{3}}} \Big|_{h=18}$

$\text{Max rate of decrease} = 0.97$

3 marks

END OF KEY TOPIC TEST SOLUTIONS