SPECIALIST MATHEMATICS

Written examination 1

2023 Trial Examination

SOLUTIONS

Question 1 (3 marks)

Consider the following sets of numbers:

$A = \{1, 3, 4, 5, 8\}$, $B = \{2, 3, 6, 7, 8, 9\}$, $C = \{\sqrt{2}, \sqrt{5}\}$, $D = \{\frac{1}{2}, \frac{5}{3}, 2\frac{3}{4}\}$

a. $A \cap B = \{3, 8\}$

1 mark

b. Sets A, B, D

1 mark

c. Sets A, B

1 mark

Question 2 (5 marks)

a. $x^3 + 3x^2 + 2x = x(x + 1)(x + 2)$

2 marks

b. $x^3 + 3x^2 + 2x$ is the product of 3 consecutive natural numbers.
So, either both x and $x + 2$ are multiples of two, or $x + 1$ is a multiple of 2.
Since there are **three** consecutive natural numbers, exactly one of them must be a multiple of 3.
So, $x^3 + 3x^2 + 2x$ must be a multiple of $2 \times 3 = 6$.

3 marks

Question 3 (6 marks)

a.

A	B	C	A'	$A' \wedge B$	$A' \wedge C$	$(A' \wedge B) \vee (A' \wedge C)$	$B \vee C$	$A' \wedge (B \vee C)$
1	1	1	0	0	0	0	1	0
1	1	0	0	0	0	0	1	0
1	0	1	0	0	0	0	1	0
1	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	1
0	1	0	1	1	0	1	1	1
0	0	1	1	0	1	1	1	1
0	0	0	1	0	0	0	0	0

2 marks

b. The truth table in part a. confirms that $(A' \wedge B) \vee (A' \wedge C)$ simplifies to $A' \wedge (B \vee C)$

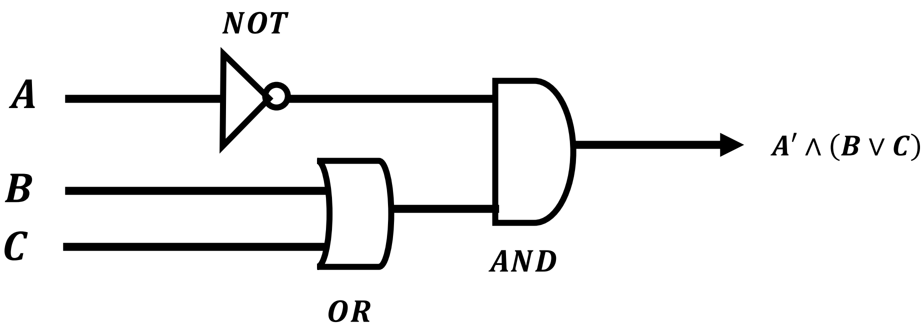
1 mark

c.

. A	B	C	AUTHORITY
1	1	1	**0**
1	1	0	**0**
1	0	1	**0**
1	0	0	**0**
0	1	1	**1**
0	1	0	**1**
0	0	1	**1**
0	0	0	**0**

1 mark

d.



2 marks

Question 4 (5 marks)

a.

$B = \begin{bmatrix} 0 & 2 & -1 \\ \mathbf{3} & 1 & 4 \end{bmatrix}$

1 mark

b.

$$2A^2 = 2 \times \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} = 2 \times \begin{bmatrix} 7 & 21 \\ 14 & 42 \end{bmatrix} = \begin{bmatrix} 14 & 42 \\ 28 & 84 \end{bmatrix}$$

2 marks

c.

$B \times (C + D)$ is a $(2 \times 3) \times (3 \times 1)$ matrix configuration.
Since the columns in matrix B matches the rows in matrix $C + D$, $B \times (C + D)$ exists.
$(C + D) \times B$ is a $(3 \times 1) \times (2 \times 3)$
Since the columns in matrix $C + D$ does not match the rows in matrix B, $(C + D) \times B$ doesn't exist.

$$B \times (C + D) = \begin{bmatrix} 0 & 2 & -1 \\ 3 & 1 & 4 \end{bmatrix} \times \left(\begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix} \right) = \begin{bmatrix} 0 & 2 & -1 \\ 3 & 1 & 4 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \end{bmatrix}$$

2 marks

Question 5 (3 marks)

Down:

$$S_\infty = \frac{2}{1 - 0.8} = 10m$$

1 mark

Up:

$$S_\infty = \frac{1.6}{1 - 0.8} = 8m$$

1 mark

Total:
$10 + 8 = 18m$

1 mark

Question 6 (8 marks)

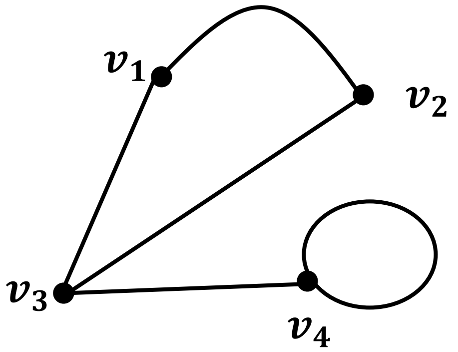
a. $1 + 1 + 0 + 1 = 3$

1 mark

b. $v_4 \text{ to } v_4$ can be done 2 ways, so there must be a loop at v_4

1 mark

c.



2 marks

d. Graph A has an Euler path since the graph has exactly 2 odd vertices. The path must start at an odd vertex and finish at the other odd vertex. So, one of:

$v_3 \text{ to } v_2 \text{ to } v_1 \text{ to } v_3 \text{ to } v_4 \text{ to } v_4$

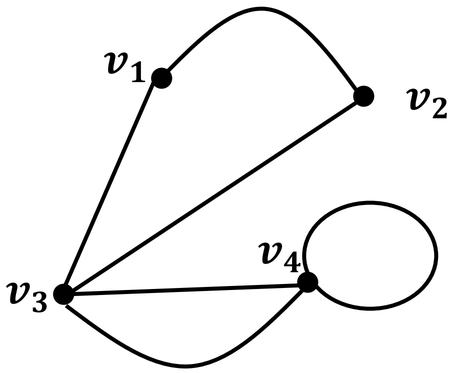
$v_3 \text{ to } v_1 \text{ to } v_2 \text{ to } v_3 \text{ to } v_4 \text{ to } v_4$

$v_4 \text{ to } v_4 \text{ to } v_3 \text{ to } v_1 \text{ to } v_2 \text{ to } v_3$

$v_4 \text{ to } v_4 \text{ to } v_3 \text{ to } v_2 \text{ to } v_1 \text{ to } v_3$

2 mark

e. Graph A does not have an Euler circuit since the graph has some odd vertices (2 in this example). The odd vertices must be joined to create two even vertices. The graph will now have all even vertices, hence an Euler circuit (starting anywhere) will now exist.



One example is:

$v_1 \text{ to } v_3 \text{ to } v_4 \text{ to } v_4 \text{ to } v_3 \text{ to } v_2 \text{ to } v_1$

2 marks

Question 7 (6 marks)

a.

$t_{n+1} = t_n + 6, t_1 = 8, n \geq 1.$

$n = 1, t_2 = 8 + 6 = 14$

$n = 2, t_3 = 14 + 6 = 20$

Arithmetic sequence with $a = 8, d = 6$

$t_n = a + (n-1)d$

$t_{20} = 8 + (20 - 1) \times 6 = 122$

2 marks

b.

$P_{n+1} = 10P_n - 2, P_1 = 12, n \geq 1.$

$n = 1, \quad P_2 = 10P_1 - 2 = 118$

$n = 2, \quad P_3 = 10P_2 - 2 = 1178$

$12, 118, 1178$

1 mark

c.

$U_{n+1} = 5U_n, U_1 = 10, n \geq 1$

$n = 1, U_2 = 5U_1 = 50$

$n = 2, U_3 = 5U_2 = 250$

$n = 3, U_4 = 5U_3 = 1250$

Geometric sequence with $a = 10, r = 5$

$t_n = a \times r^{n-1} = 10 \times 5^{n-1} = 2 \times 5^n$

3 marks

Question 8 (4 marks)

a. $5! = 120$

1 mark

b. $2! \times 4! = 48$

1 mark

c. $Group\ X: 1, Group\ Y: 4\ \binom{5}{1} \times \binom{4}{4} = 5$

$Group\ X: 2, Group\ Y: 3\ \binom{5}{2} \times \binom{3}{3} = 10$

$Group\ X: 3, Group\ Y: 2\ \binom{5}{3} \times \binom{2}{2} = 10$

$Group\ X: 4, Group\ Y: 1\ \binom{5}{4} \times \binom{1}{1} = 5$

Total = 30.

2 marks