

2023 Trial Examination

STUDENT
NUMBER

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Letter

SPECIALIST MATHEMATICS

Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper and/or white out liquid/tape.
- No calculator is permitted in this examination.

Materials supplied

- Question and answer book of 11 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

Instructions

Answer **all** questions in pencil on the answer sheet provided.
 Unless otherwise specified, an **exact** answer is required to a question.
 In questions where more than one mark is available, appropriate working **must** be shown.
 Unless otherwise indicated, the diagrams in this book are not drawn to scale.
 Take the acceleration due to gravity to have magnitude $g \text{ m s}^{-2}$, where $g = 9.8$.

Question 1 (6 marks)

Consider the vectors $\underline{a} = -\underline{i} + c\underline{j} + 3\underline{k}$ and $\underline{b} = 3\underline{i} + 6\underline{j} + d\underline{k}$ where $c, d \in R$

a. Find the value(s) of c given the magnitude of \underline{a} is $\sqrt{14}$.

1 mark

b. If instead, the two vectors are parallel, find the values of c and d .

2 marks

- c. Find the cross product of $\vec{a} = -\vec{i} + c\vec{j} + 3\vec{k}$ and $\vec{b} = 3\vec{i} + 6\vec{j} + d\vec{k}$ hence find the values of c, d and f , (different to either **part a.** or **part b.**), so that \vec{a} and \vec{b} are both perpendicular to $\vec{e} = -2\vec{i} + \vec{j} + f\vec{k}$

3 marks

1 + 2 + 3 = 6 marks

TURN OVER

Question 2 (3 marks)

Given $f(x) = x \tan^{-1}\left(\frac{x}{2}\right)$ find $f'(-2)$, writing your answer in the form $m\pi - n$ where m, n are simplified fractions.

3 marks

Question 3 (3 marks)

Prove, by mathematical induction, that $x^2 + x$ is an even integer for $x \in \mathbb{N}$.

3 marks

Question 4 (6 marks)

Given $\sec^2(2x) = 2$, where x is in the first quadrant, evaluate:

a. x

b. $\operatorname{cosec}(4x)$

c. $\cot(4x)$

2 + 2 + 2 = 6 marks

TURN OVER

Question 5 (6 marks)

Evaluate the following integrals, leaving your answer in exact, simplified form.

a. $\int_0^1 (x \tan^{-1} x) dx$

3 marks

b. $\int_0^{\frac{1}{2}} \left(\frac{2x+1}{\sqrt{1-x^2}}\right) dx$

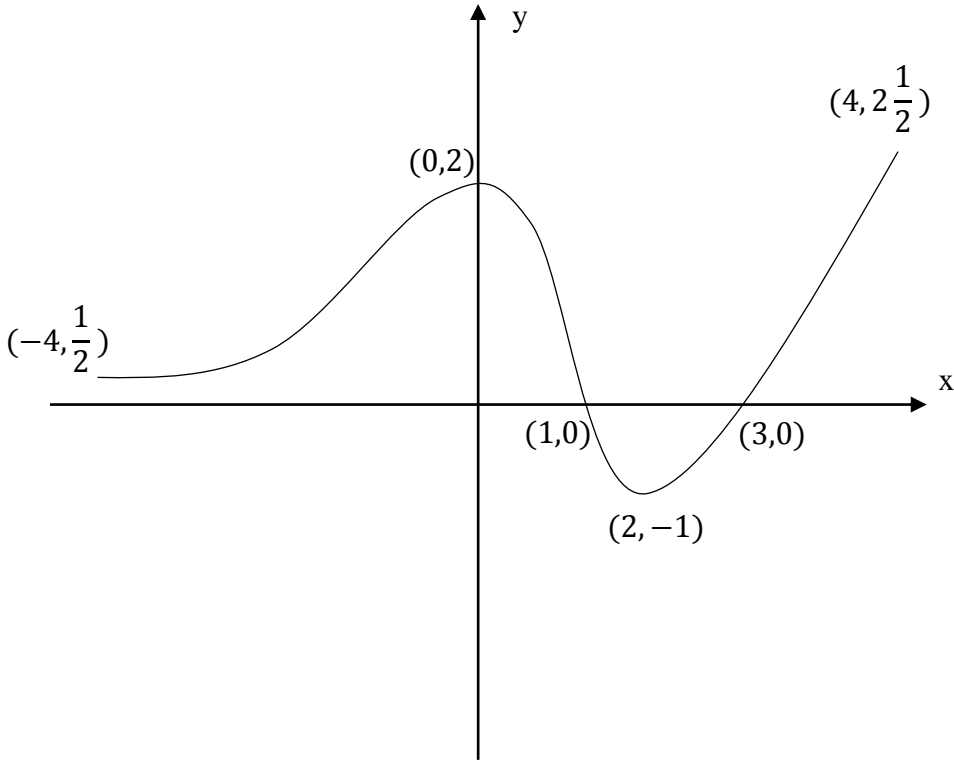
3 marks

3 + 3 = 6 marks

TURN OVER

Question 6 (3 marks)

Sketch on the same axes the graph of $y = \frac{1}{f(x)}$ given the graph of $y = f(x)$ shown below. Label key features of your sketch



3 marks

Question 7 (6 marks)

a. Write $z = -2\sqrt{2} + 2\sqrt{2}i$ in polar form where $z \in \mathbb{C}$.

2 marks

b. Hence find two solutions to $z^2 = -2\sqrt{2} + 2\sqrt{2}i$ in polar form.
(Call these solutions z_1 and z_2).

2 marks

c. Use the identity $\cos 2\theta = 2\cos^2\theta - 1$ to rewrite the solution to z_1 found in **part b.** into the form $z_1 = \sqrt{a + \sqrt{b}} + \sqrt{a - \sqrt{b}}i$, where $a, b, c \in \mathbb{R}$

TURN OVER

2 marks

2 + 2 + 2 = 6 marks

Question 8 (4 marks)

Find the gradients of the tangents to $2x^3 - xy^2 = y$ when $x = -1$.

4 marks

Question 9 (3 marks)

The following may be useful for question 9.

Let Z be an observation from a standard normal distribution.

$\Pr(Z \leq 1.96) \approx 0.975$, $\Pr(Z \leq 2.58) \approx 0.995$, $\Pr(Z \leq 3.00) \approx 0.999$

Suppose that the volume of lemonade delivered to a cup by a vending machine (X) is normally distributed with a mean of 200 mL and a standard deviation of 2.5 mL . Find the percentage chance that a sample of 25 drinks will produce a mean volume of greater than 201.5 mL .

3 marks

END OF QUESTION AND ANSWER BOOK