

Specialist Mathematics

Written Examination 1

2024 Insight Publications Trial Examination

- Reading time: 15 minutes
- Writing time: 1 hour

Approved materials

- Question and Answer book
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and /or correction fluid/tape.

Materials supplied

- Question and Answer book of 15 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All responses must be in English.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

Questions (10 questions, 40 marks) _____ 2–12

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Instructions

- Answer **all** questions in the spaces provided.
- Unless otherwise specified, an **exact** answer is required to a question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where g = 9.8.

Question 1 (3 marks)

Evaluate $\int_{1}^{4} \sqrt{x} \log_{e}(2x) dx$.

Give your answer in the form $a \log_e(2) + b$, where $a, b \in R$.

Question 2 (4 marks)

a. Evaluate $(1+\sqrt{3}i)^5 + (1-\sqrt{3}i)^5$.

2 marks

b. Find the values of k for which $(1+\sqrt{3}i)^k + (1-\sqrt{3}i)^k = 128$, where k is a positive integer.

Question 3 (3 marks)

Consider the curve defined parametrically by

$$x = 2\cos(t) + \cos(2t), y = 2\sin(t) - \sin(2t), \text{ where } 0 \le t \le 2\pi.$$

Find the arc length of the curve from t = 0 to $t = \frac{\pi}{3}$.

Question 4 (3 marks)

Consider the relation $x \cot(y) - 2y^2 = 8$.

Find the gradient of the curve at the point $\left(\frac{\pi^2}{8} + 8, \frac{\pi}{4}\right)$.

Give your answer in the form $\frac{a}{\pi^2 + a\pi + b}$, where $a, b \in R$.

Question 5 (4 marks)

Use mathematical induction to prove that $(\cos(\theta) - \sin(\theta)i)^n = \cos(n\theta) - \sin(n\theta)i$ for all $n \in Z^+$.

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Question 6 (6 marks)

Consider the Cartesian equations of two planes:

 $\Pi_1 : x + 2y - z = 15$ $\Pi_2 : x + y + pz = 5$

a. Given that \prod_{1} and \prod_{2} are perpendicular, show that p = 3.

1 mark

b. Find the Cartesian equation of the plane that is perpendicular to both \prod_1 and \prod_2 , and which also passes through point (1,2,1).

Line *L*, given by the equation $\underline{r}(t) = \underline{i} + \underline{j} + \underline{k} + t(\underline{i} + 3\underline{j} - 6\underline{k}), t \in R$, intersects \prod_{1} at point *A* and \prod_{2} at point *B*.

c. Find the distance *AB*.

1 mark

Question 7 (4 marks)

An ice cube tray with 6 cuboid compartments is filled with water and placed in a freezer to make 6 ice cubes. Each compartment has a 3 cm by 3 cm square base.

The height of each ice cube, *H*, is normally distributed with a mean of 2 cm and a standard deviation of $\frac{1}{6}$ cm. Assume that the height of each ice cube is independent.

a. Find the expected total volume of ice in the ice cube tray in cm³.

Find the standard deviation of the total volume of ice in the ice cube tray in cm Give your answer in the form $\frac{a\sqrt{b}}{c}$ where <i>a</i> , <i>b</i> and <i>c</i> are positive integers.	
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Find the expected surface area of one ice cube in cm^2 .	

Question 8 (4 marks)

The position vector of a particle moving relative to the origin at time *t* seconds is given by $\underline{r}(t) = 2\cos(t)\underline{i} + 4\sin(t)\underline{j} + 2t\underline{k}$ for $t \ge 0$. All distances are measured in metres.

a. Find the maximum and minimum speeds of the particle.

2 marks

b. Find the number of times, between t = 0 and $t = 2\pi$, that the velocity is perpendicular to the acceleration.

Question 9 (5 marks)

A tank initially holds 200 L of water in which 16 kg of salt has been dissolved. Pure water then flows into the tank at a rate of 2 L per minute. The mixture is stirred continuously and flows out of the tank at a rate of 4 L per minute.

a. Show that the differential equation for x, the number of kilograms of salt in the tank after t minutes, is given by

$$\frac{dx}{dt} = -\frac{2x}{100-t}.$$

1 mark

b. Solve the differential equation given in **part a.** to find x as a function of t. Express $(x,y)^2$

your answer in the form $x = \left(\frac{a-t}{b}\right)$), where a and b are positive integers.
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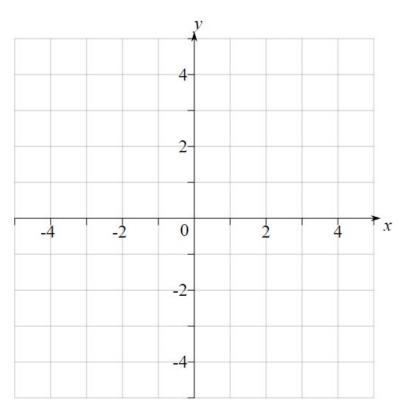
c. Hence, find the amount of salt which has flowed out of the tank over the first 25 minutes and the concentration of salt remaining in the tank after 25 minutes.

2 marks



Question 10 (4 marks)

Sketch the graph of $y = \frac{x^2 - 1}{x^2 - 4}$ on the axes below. Label any axis intercepts and turning points with their coordinates, and any asymptotes with their equation.



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